

24	2	1.41421357
<u>4</u>	1	
96	166	
	<u>96</u>	
	400	
281	281	
	11986	
	11296	
	<u>60968</u>	
	56564	
	<u>111</u>	
	383600	
	282841	

$$\begin{array}{r} 2824 \\ \underline{14} \\ 11296 \end{array}$$

$$\begin{array}{r} 28282 \\ \underline{12} \\ 56564 \end{array}$$

$$\begin{array}{r} 2828423 \\ \underline{3} \end{array}$$

$$\begin{array}{r} 8485269 \end{array}$$

$$\begin{array}{r} 848 \\ 282.84285 \\ \underline{4142135} \\ 141421325 \end{array}$$

$$\begin{array}{r} 10075900 \\ \underline{8985269} \\ 1590731000 \\ \underline{146421325} \\ 17651775 \end{array}$$

$$\frac{1}{1} = 1$$

May 1914

$$+ 1 = 3 \quad \times$$

J. CROUDY

4 KEMPSON
RD. FULHAM
LONDON
S.W. 6.

28 284 280
1 5 2 1 4 8
148989890
228.384270
177274160



Digitized by the Internet Archive
in 2026

BELL'S MATHEMATICAL SERIES FOR SCHOOLS AND COLLEGES

General Editor: WILLIAM P. MILNE, M.A., D.Sc.

$$\begin{array}{r}
 106 \\
 1125 \\
 1130
 \end{array}
 \begin{array}{r}
 3199 \cdot 0336 \\
 25 \\
 \hline
 699 \\
 636 \\
 \hline
 6303 \\
 5325 \\
 \hline
 5625 \\
 11 \\
 \hline
 678
 \end{array}
 \begin{array}{r}
 56 \cdot 175 \\
 1066 \\
 2 \cdot 6 \\
 \hline
 6306 \\
 5325 \\
 \hline
 1125 \\
 15 \\
 \hline
 5625
 \end{array}
 \begin{array}{r}
 106 \\
 20 \\
 \hline
 676 \\
 1065 \\
 1065 \\
 \hline
 5325
 \end{array}$$

ARITHMETIC

Bell's Mathematical Series

FOR SCHOOLS AND COLLEGES.

General Editor : WILLIAM P. MILNE, M.A., D.Sc.

VOLUMES NOW READY.

Crown 8vo, 1s. 6d.

Problem Papers for Preparatory Schools. By T. COOPER SMITH, M.A., late Scholar of University College, Oxford ; Mathematical Master, St. Peter's Court, Broadstairs.

Crown 8vo, 2s. 6d.

Arithmetic. By H. FREEMAN, M.A., sometime Scholar of Christ's College, Cambridge ; Mathematical Master at the Haberdashers' Aske's School, Cricklewood.

Crown 8vo, 2s. 6d.

Statics. Part I. By R. C. FAWDRY, M.A., B.Sc., sometime Scholar of Corpus Christi College, Cambridge ; Head of the Military and Engineering Side, Clifton College.

Crown 8vo, 5s.

The Elements of Non-Euclidean Geometry. By D. M. Y. SOMMERVILLE, M.A., D.Sc., Lecturer in Mathematics, University of St. Andrews.

Others in active preparation.

LONDON : G. BELL AND SONS LTD.

ARITHMETIC

BY

H. FREEMAN, M.A.

SOMETIME SCHOLAR OF CHRIST'S COLLEGE, CAMBRIDGE
MATHEMATICAL MASTER AT THE HABERDASHERS'
ASKE'S SCHOOL, CRICKLEWOOD
AUTHOR OF "EXAMPLES IN PHYSICS"



SPECIMEN.

LONDON
G. BELL AND SONS LTD.

1914

Printed by
MORRISON & GIBB LIMITED
Edinburgh

PREFACE

THE chief object of this book has been to present in a clear and concise form an Arithmetic which, although consisting mainly of examples, should contain just sufficient bookwork to be of use to a pupil when in difficulty. The methods adopted are those generally acknowledged to be the best, and it is believed that they will be helpful to the teacher as well as to the boy. The majority of the examples are original: in order to avoid what might be termed artificial examples, facts from daily life, the newspapers, and many other sources have been adapted to numerical work. Examples have been included in all parts of Arithmetic from the first principles of the subject up to Logarithms and Stocks and Shares. There are, in addition, chapters on Graphs, Use of Formulæ, and Geometrical Applications; these have been included in order to bring out the co-ordination between Algebra, Geometry, and Arithmetic. In the section on Graphs examples on Maxima and Minima have been inserted, and in that on Square Root the use of Pythagoras' Theorem in dealing with numerical examples has been clearly defined. Much attention has been devoted to the explanatory notes on Contracted Methods and Approximations. The use of "Rough Checks" has been insisted upon wherever possible throughout the work.

It has been endeavoured so to arrange the material of the work that the book should be as compact as possible, and it is believed that the size of the book will be a recommendation.

In order that the book may meet the needs of the Scotch Education Department as well as the curricula of the Secondary Schools of England and Wales, the work has been revised in manuscript form by Mr. G. A. Burnett, M.A., B.Sc., Head of

the Mathematical Department, Inverness Royal Academy, to whom I am under deep obligations.

I cannot sufficiently express my gratitude to Dr. W. P. Milne, M.A., for the time and trouble that he has spent in helping me with the book ; his advice on all points has indeed been invaluable. My best thanks are also due to Messrs. H. Lob, B.A., A. Prior, B.A., and W. R. Stokes, B.A., for their criticisms and suggestions while the book was in manuscript, and to Miss I. C. Jacobs for help in that most arduous of tasks—the working out of the examples.

I am indebted to the following authorities for permission to use examples set in their examinations in the Miscellaneous Examples at the end of the book:—The Syndics of the Cambridge University Local Examinations, the Senate of London University, the Education Committee of the London County Council, and the Central Welsh Board.

H. F.

CONTENTS

SECT.	PAGE
1. ADDITION	1
2. SUBTRACTION	4
3. POSITIVE AND NEGATIVE NUMBERS	7
4. MULTIPLICATION	10
5. DIVISION	13
6. PRIME FACTORS	16
7. REDUCTION	17
8. COMPOUND ADDITION AND SUBTRACTION	18
9. COMPOUND MULTIPLICATION AND DIVISION	23
10. DECIMAL FRACTIONS	28
11. METRIC SYSTEM	34
12. AREAS AND VOLUMES.—I	36
13. H.C.F. AND L.C.M.	41
14. VULGAR FRACTIONS	42
15. ADDITION AND SUBTRACTION OF FRACTIONS	45
16. MULTIPLICATION AND DIVISION OF FRACTIONS	50
17. MISCELLANEOUS FRACTIONS	53
18. MISCELLANEOUS DECIMALS	59
19. AVERAGES	63
20. UNITARY METHOD	68
21. AREAS AND VOLUMES.—II	73
22. DENSITY	78
23. GRAPHS	84
24. SQUARE ROOT	96
25. USE OF FORMULÆ	102
26. PROPORTION	106
27. PERCENTAGES	111
28. PROFIT AND LOSS	118

SECT.	PAGE
29. PRACTICE	122
30. APPROXIMATION AND CONTRACTED WORK	127
31. GEOMETRICAL APPLICATIONS	136
32. LOGARITHMS	144
33. SIMPLE INTEREST	152
34. COMPOUND INTEREST	158
35. STOCKS AND SHARES	164
36. MISCELLANEOUS EXAMPLES ON TIME AND WORK	172
TEST PAPERS	178
MISCELLANEOUS EXAMPLES	206
TABLES	224
FOUR-FIGURE LOGARITHMS	228
FOUR-FIGURE ANTILOGARITHMS	230
ANSWERS TO EXAMPLES	i
ANSWERS TO TEST PAPERS	xxiv
ANSWERS TO MISCELLANEOUS EXAMPLES	xxx

ARITHMETIC

§ 1. ADDITION

EXAMPLES. I

Add—

- | | |
|-----------------------------------|---------------------------|
| 1. 35, 83, 79, 5, 12. | 2. 318, 951, 81, 99, 3. |
| 3. 1005, 81, 735, 6. | 4. 10005, 1005, 105, 5. |
| 5. 803, 8003, 80003, 800003. | 6. 197, 3815, 995, 19087. |
| 7. 819, 3919, 6668, 3, 91. | 8. 732, 7302, 73002, 73. |
| 9. 85914, 9523, 951, 10009. | |
| 10. 119532, 235689, 8912, 666777. | |

Find the value of—

11. $83 + 915 + 72 + 9156$.
12. $1009 + 729 + 5861 + 98723$.
13. $562 + 8910 + 30008 + 10906$.
14. $9 + 909 + 90909 + 9090909$.
15. $3572 + 86 + 105 + 100921$.
16. $89132 + 32198 + 7625 + 6 + 3192$.
17. $8562 + 7892 + 40035 + 241827$.
18. $32 + 9 + 803409 + 666 + 900009$.
19. $315 + 79186 + 2009 + 5 + 99999$.
20. $8721 + 56391 + 879197 + 700008 + 95$.
21. A school is divided up into 8 classes. The numbers of boys in each class are 30, 29, 29, 28, 27, 29, 30, 26. How many boys are there in the school?
22. A boy's marks for a week are 5, 8, 7, 9, 15, 19, 0, 8, 11, 17, 15. How many did he obtain altogether?
23. Five towns lie in a straight line. The distance from the first to the second is 17 miles, from the second to the third 18 miles, from the third to the fourth 29 miles, and from the fourth to the fifth 39 miles. What is the distance from the first to the fifth?

24. On six different occasions a boy wins 8, 15, 21, 19, 8, 5 marbles. He had 51 to start with. Find how many he has altogether.

25. Sheets of paper are divided into squares. There are 144 on one sheet, 121 on a second, 100 on a third, 81 on a fourth, and 64 on a fifth. How many squares are there altogether?

26. A greengrocer empties a dozen bags containing apples into a cask. The numbers of apples in the bags are 29, 41, 18, 53, 39, 51, 40, 29, 29, 41, 39, 49. Find how many the cask contains.

27. A train consists of corridor carriages. There are 71, 35, 89, 63, 20, and 109 people in the carriages. How many travelled by the train?

28. A field is divided into sections whose sizes are 561, 89, 72, 199, 95 square links. How many square links are there in the whole field?

29. The runs that a boy makes during a cricket season are 5, 21, 8, 0, 39, 11, 51, 0, 9, 61, 5, 37. Find the number of runs he made altogether.

30. After spending the following amounts at different shops: 9 pence, 7 pence, 3 pence, 8 pence, and one penny—a boy has 11 pence left. What did he start out with?

31. Calculate from the following table the area and population of the world:—

Continent.	Area in thousands of square miles.	Population in millions.
Europe . . .	3750	400
Asia	17000	900
Africa	11500	170
N. America . .	8000	110
S. America . .	6800	35
Oceania	3450	8
Polar Regions .	5000	
Totals . . .	(a)	(b)

32. The amount of naval expenditure of the eight principal Naval Powers of the world is given in the following table. Find the total amounts for 1902 and 1912.

Power.	Naval Expenditure (£).	
	1902.	1912.
United Kingdom	31003977	45075400
United States	16203916	26540019
Japan	3705271	9461817
France	12184683	18090758
Russia	10446392	17681207
Germany	10045000	22609540
Italy	4800000	8566505
Austria	1954617	5841968
Total	(a)	(b)

33. Find the rateable value of land in England and Wales.

Agricultural land	£23732277
Other hereditaments	£191710962
Annual value of Government property	£1736945
Total	<u>£</u>

34. Find the extent in acres, the population, and the number of Irish emigrants from Leinster.

Counties.	Extent in Acres.	Population.	Irish Emigrants.
Carlow	221295	36151	200
Dublin	226821	476909	1039
Kildare	418496	66498	282
Kilkenny	507254	74821	374
King's Co.	493263	56769	307
Longford	257770	43794	442
Louth	201619	63402	416
Meath	578298	64920	293
Queen's Co.	424853	54362	186
Westmeath	434017	59812	209
Wexford	573200	102287	302
Wicklow	499822	60603	208
Leinster	(a)	(b)	(c)

35. Find the total number of students attending evening schools in England and Wales from 1905 to 1911.

Year.	Male Students.	Female Students.	Total Number of Students.
1905	441432	277130	(a)
1906	456049	293442	(b)
1907	442505	294131	(c)
1908	446434	305166	(d)
1909	441913	310443	(e)
1910	453438	314920	(f)
1911	453087	312188	(g)
	(h)	(k)	(l)

36. Fill in columns (a) (b) (c) in the adjoining table.

EMPLOYMENT IN TEXTILE FACTORIES.

Year.	Under 18 years.		Over 18 years.		Total Number of Employees.		
	Males.	Females.	Males.	Females.	Males. (a)	Females. (b)	Total. (c)
1896	82383	153862	306156	482030			
1897	78719	152583	296058	475167			
1898	76335	151604	290797	472587			
1901	71707	148888	290606	481641			
1904	70965	137038	297302	489329			
1907	81270	157502	310983	504821			

§ 2. SUBTRACTION

EXAMPLES. II 2

Subtract the second number from the first :—

- | | | |
|--------------------|----------------|-----------------|
| 1. 89, 35. | 2. 153, 88. | 3. 764, 299. |
| 4. 821, 94. | 5. 983, 394. | 6. 1000, 873. |
| 7. 8931, 3918. | 8. 65321, 752. | 9. 10506, 1591. |
| 10. 101010, 10101. | | |

Find the values of the following :—

11. $832 - 219$. 12. $9151 - 63$. 13. $81725 - 71523$.

14. $11 - 9 + 52$. 15. $83 - 72 + 11$. 16. $753 - 621 + 9321$.

17. $892 + 531 - 291$.

18. Find the excess of 92153 over 76215.

19. By how much is 395192 greater than 71981?

20. Which is greater, $8932 - 5161$ or $10000 - 6173$?

21. What is the remainder when 9532186 is subtracted from ten million?

22. A boy has 39 marbles; he gives 23 away. How many has he left?

23. A man is 60 years old now: how old was he 17 years ago?

24. In the year 1913 a boy was 14 years old. When was he born?

25. A cricketer makes 378 runs in a season; up to the last match he had made 289. How many did he score in the last match?

26. What number must be added to 3719 to make ten thousand?

27. How many times can a man take buckets of water from a tub containing 195 pints if each bucket contains 15 pints?

28. What number must be subtracted from 19325 to leave 9999?

29. Find the value of—

(a) $1715 + 913 - 105$.

(b) The sum of one hundred and ninety, and three hundred and seventy-one, subtracted from a thousand.

(c) $19152 - 951$ added to $1000 - 391$.

(d) The excess of $89 + 52$ over $1095 - 999$.

30. The sum of two numbers is a thousand. One is 399. Find the other.

31. The difference between two numbers is 95 and the larger is 872. What is the smaller?

32. From 493 take 27, from the answer take 28, from the answer take 29. This is 3 sums: do it in 2 sums if you can.

33. The adjoining table gives the number of live stock in the United Kingdom in the years 1910 and 1911. Find the number of sheep.

Live Stock.	1910.	1911.
Horses . . .	2094587	2033216
Cattle . . .	11765433	11866111
Sheep . . .	(a)	(b)
Pigs . . .	3561481	4250012
Totals . . .	48585688	48629146

34. In a certain cricket match the following were the individual scores:—71, 10, 11, 23, 75, 9, 13, 20, 2, 15, 4. The total was 278: find the number of extras.

35. The population of the British Isles in the years 1891, 1901, 1911 is given below. Find the number of females.

Years.	Males.	Females.	Total.
1891	14052901	(a)	29002525
1901	15728613	(b)	32527843
1911	17445608	(c)	36070492

36. The following table gives the number of railway servants in Great Britain and Ireland for the years indicated. Fill in the vacant spaces.

Year.	England and Wales.	Scotland.	Ireland.	Total.
1907 . . .	412804	47040	19470	(a)
1908 . . .	395271	(b)	19040	459753
1909 . . .	394928	45671	(c)	459968
1910 . . .	(d)	46606	19199	463520
1911 . . .	405326	47125	20392	(e)

37. Find the number of kilograms of coffee exported from Ecuador during 1908, and the number of kilograms of rubber during 1910 :—

Year.	Coffee (kg.).	Rubber (kg.).
1906 . . .	2657084	623569
1907 . . .	1143349	468865
1908 . . .	(a)	402375
1909 . . .	3327535	485556
1910 . . .	3938224	(b)
Total . .	14779449	2532961

38. In a ballot of 100,000 members of an Association, 66,299 voted for a motion which was carried by a majority of 58,515. How many voted against the motion, and how many did not vote at all?

§ 3. POSITIVE AND NEGATIVE NUMBERS

(Examples marked with an asterisk may be left till later.)

EXAMPLES. III 3

Find the value of—

1. $-9 + 5$. 2. $-17 + 8$. 3. $-25 + 31$.
4. $+17 - 8$. 5. $+83 - 91$. 6. $-715 + 812$.
7. $-891 + 52 - 31$. 8. $-7214 + 9186 - 5 + 11$.
9. $-83 + 94 - 105$. 10. $-1523 + 91 - 8172$.

*11. How much is (i) a greater than b , (ii), b greater than a , (iii) 10 greater than a , (iv) x greater than 9?

*12. A man has £ x ; he owes £ y . He pays his debt and has £ z given him. How much has he?

13. A boy has 15 marbles. He gives 7 away, receives 11 more, and loses 8. How many has he?

Find the following results (14-16) by drawing diagrams on squared paper :—

14. A ship sails 5 miles north, 8 miles south, 3 miles north, and finally 7 miles south. How far is she from the starting-point (i) north, (ii) south?

15. A boy takes 5 steps forward, 10 backward, 11 forward, and 8 backward. Find his final position (i) measured forward, (ii) measured backward.

16. A man puts £20 in the bank, draws out £15, puts in £9, and then writes out a cheque for £30. If the bank pays out the money for the cheque, how much is the man in debt to the bank?

Show by a diagram that—

$$17. \quad + 8 - 5 + 3 - 11 = -5.$$

$$18. \quad - 8 - 5 - 3 + 11 = -5.$$

$$19. \quad + 8 + 5 + 3 - 11 = +5.$$

$$20. \quad - 8 + 5 - 3 + 11 = +5.$$

$$21. \quad - 8 + 7 - 9 + 10 = 0.$$

$$22. \quad +15 - 13 + 8 - 2 = +8.$$

23. At the end of the first week of term a boy's position in form is 14th. During the next 6 weeks he drops 2 places, rises 4 places, rises 1 place, drops 6 places, rises 3 places, and drops 9 places. What is his position in form at the end of that time? Use squared paper.

24. The rainfall in hundredths of an inch for Margate during January 1907 was 79. It increased by 41 in February, then fell by 54 in March. In April it was 168 more than in the previous month, and it decreased by 24 in May. What was the rainfall for May? Use squared paper.

25. A train goes from A to B along a straight stretch of railway for 60 yards; when it reaches B it reverses its engine and shunts to C, a distance of 11 yards. It then goes forward 159 yards to D, and finally comes back to E, 12 yards from D. How far is E from A?

26. An evening paper published the following figures showing its sale during 6 days in March 1912. The + sign shows an increase over the previous day's sale, and the - sign a decrease:—

March 18.	476522	
„ 19.		+ 2426
„ 20.		+ 21209
„ 21.		+ 13598
„ 22.		- 768
„ 23.		- 90030

What was the sale on March 23?

27. Write down the increase (+) or decrease (−) in the profits of a company during the years 1894–1898. The amounts are reckoned in dollars.

Year.	1893.	1894.	1895.	1896.	1897.	1898.
Profit.	85641	87768	87982	71330	81000	108000

28. Raven Crag is 1000 feet above sea-level; Thirlmere lies 417 feet below Raven Crag. Great How is 509 feet above Thirlmere, and Stybarrow Dodd 1664 feet above Great How. What is the height of Stybarrow Dodd above sea-level?

29. Find the totals for 1911 from the following table showing the imports of Great Britain in corn (cwt.):—

	1910.	Inc. or Dec. in 1911.
Grain from British Empire . .	48116138	+1060300
Grain from foreign countries . .	57106500	− 8215151
Meal and flour from B. E. . .	3204101	+ 557967
Meal and flour from F. C. . .	6756390	− 453326

30. Find the total number of hours of sunshine in Bath during September 1907:—

January	70	June	− 3
February	+ 38	July	+ 57
March	+ 101	August	− 18
April	− 57	September	− 6
May	+ 3		

The + and − signs denote increase or decrease in the number of hours of sunshine over those for the previous month. *E.g.*, April − 57 means that there were 57 hours less sunshine in April than in March.

31. A trader's transactions for the month of January are as follows :—

January 1.	Buys goods of B for	. . .	£265
„ 2.	Sells goods to C for	. . .	125
„ 7.	Pays wages . . .	. . .	8
„ 8.	Pays A . . .	. . .	2
„ 9.	Sells goods to D for	. . .	200
„ 13.	Pays wages . . .	. . .	16
„ „	Pays rent . . .	. . .	30
„ 15.	Pays trade expenses	. . .	40
„ 19.	Buys goods of B for	. . .	120
„ 20.	Pays wages . . .	. . .	16
„ 22.	Sells goods to C for	. . .	150
„ 23.	Pays A . . .	. . .	2
„ 25.	Buys goods of E for	. . .	35
„ 27.	Pays wages . . .	. . .	16
„ 30.	Sells goods to D for	. . .	100.

Find his profit at the end of the month.

§ 4. MULTIPLICATION

When multiplying together two large numbers, the following method is convenient, especially later on when we come to contracted methods.

Example. Find the product of 1752×823 .

[*Rough Check* :— $800 \times 2000 = 1600000$.]

$$\begin{array}{r}
 1752 \\
 823 \\
 \hline
 14016 \\
 3504 \\
 5256 \\
 \hline
 1441896
 \end{array}$$

Place the **first figure** of the multiplier under the **last figure** of the multiplicand : the method is then obvious.

EXAMPLES. IV 4

(*Examples marked with an asterisk may be left till later.*)

Find the value of—

1. 18×5 , 17×3 , 13×9 .

2. 31×8 , 71×4 , 48×5 .

3. 19×9 , 88×5 , 73×7 . 4. 52×8 , 91×9 , 83×5 .
 5. 112×2 , 125×3 , 109×8 .
 6. 18×10 , 19×100 , 15×1000 .
 7. 100×10 , 3000×100 , 10000×1000 .
 8. 53×20 , 81×30 , 910×40 .
 9. 71×50 , 8010×70 , 315×700 .
 10. 53×12 , 810×110 , 5300×1200 .

Find the following products, using factors :—

11. 81×32 . 12. 171×54 . 13. 19×18 .
 14. 359×72 . 15. 166×63 . 16. 86×56 .
 17. 350×350 . 18. 195×121 . 19. 132×132 .

*20. Write down the following in their shortest form, and where possible work out the multiplication :—

- (i) $2 \times 2 \times 2$. (vi) $8 \times 8 \times 8$.
 (ii) $9 \times 9 \times 9 \times 9$. (vii) $y \times y \times y \times y$.
 (iii) $a \times a \times a \times a \times a \times a$. (viii) $x \times x \times x \times x$.
 (iv) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$. (ix) $5 \times 5 \times 5 \times 4 \times 3 \times 3$.
 (v) $7 \times 7 \times 7 \times 7$. (x) $2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 4$.

Find the value of—

21. 891×17 . 22. 351×108 . 23. 783×191 .
 24. 336×715 . 25. 8061×879 . 26. 7123×514 .
 27. 6661×116 . 28. $5321 \times 83 \times 71$. 29. $99 \times 31 \times 41$.
 30. $8721 \times 512 \times 15$. 31. 15^2 ; 18^2 ; 11^3 ; 12^3 ; 2^9 ; 3^7 .
 32. $1^2 + 2^2 + 3^2 + 4^2$. 33. $1^3 + 2^3 + 3^3 + 4^3$.

34. A man earns 5 shillings an hour. How many shillings will he earn in a month, working 8 hours a day except Saturdays and Sundays?

35. There are 17 forms in a school, and in each form there are 29 boys. How many boys are there in the school?

36. There are 1760 yards in a mile. If there are 3 feet in 1 yard, how many feet are there in 15 miles?

37. Find how many pence there are in £175 if there are 12 pence in a shilling and 20 shillings in £1.

38. In a book of 175 pages, each page contains 39 lines: there are 9 words in a line. How many words are there in the book?

39. There are 60 seconds in 1 minute, 60 minutes in 1 hour, and 24 hours in 1 day. How many seconds are there in 1 day?
40. A train travels 39 feet per second: find how many feet it travels in 3 hours.
41. Find the value of 18×51 . Hence write down the value of 180×510 ; 18×5100 .
42. Multiply 7510 by 819, and hence write down the value of 751×80 and 75100×90 by looking at your working.
43. In a certain savings bank 195 depositors put in £2, 395 put in £4, 83 put in £7, and 29 put in £9. How much do they put in together?
44. In the Roman aqueducts 13,504 pipes each brought in daily 2962 cubic feet of water. Find the total amount of water brought in.
45. Reckoning that each person received the same amount of correspondence a year, it is found that in 10 years each inhabitant of the British Isles received 702 letters, 200 post-cards, 235 halfpenny packets, 44 newspapers, and 28 parcels. If the population of the British Isles is 45 millions, how many postal packets did the Post Office deal with in these 10 years?
46. A man puts a penny in the bank. The next day he puts twopence in, the next day fourpence, and so on, doubling the previous day's amount each day. How much will he have in (in pence) at the end of a fortnight?
47. There are 111 Turkish piastres in £1. A man changes £793 into Turkish money. How many piastres should he get?
48. A boy miscopies a question and finds the value of 983×865 instead of 933×865 . By how much is his answer too great?
49. A train takes 9 hours on a journey. For the first two hours its speed is 37 miles per hour, for the next three 39, for the next three 43, and for the last 48. What is the length of the journey?
50. By how much does $5^3 \times 3^2$ exceed $5^2 \times 3^3$? Show that the same result is arrived at by finding the value of $5^2 \times 3^2 \times (5 - 3)$.
- *51. Square 310. Hence square 311, 312, 313, 314, 315. There is a quick way of doing these: find it.
- [Hint :— $(a + b)^2 = a^2 + 2ab + b^2$]

52. Find the following products :—

- (i) 197×25 .
- (ii) 125×83 .
- (iii) 956×99 .
- (iv) 999×999 .

There is a short method for each : obtain the products thus.

§ 5. DIVISION

When large numbers are used as divisors proceed thus :—

Example. Divide 87293 by 152.

$$[\text{Rough Check : } \frac{90000}{150} = 600].$$

$$\begin{array}{r} 574 \\ 152 \overline{) 87293} \\ \underline{1129} \\ 653 \\ \underline{45} \end{array}$$

The subtraction is performed mentally or orally. To obtain the second line in the above working the following process is carried out :—5 times 2 is 10 and 2 to make 12 ; 5 times 5 is 25, add 1, 26, and 1 to make 27 ; 5 times 1 is 5, add 2, which makes 7, and 1 to make 8. This gives us 112 for the second line. Bring down the next figure—9—and proceed as before.

It is useful when there is a remainder, as in this example, to write the result as $574\frac{45}{152}$. The answer is stated thus :—574, and 45 to be divided by 152. $574\frac{45}{152}$ will be better understood when fractions are dealt with.

EXAMPLES. V 5

Divide—

1. 81 by 3, 72 by 8, 144 by 4.
2. 90 by 5, 84 by 6, 84 by 7.
3. 168 by 12, 121 by 11, 720 by 5.
4. 117 by 9, 154 by 7, 690 by 6.
5. 5616 by 9, 82456 by 11, 10236 by 12.

6. 82153 by 8, 71966 by 5, 31209 by 11.
7. 91915 by 9, 31967 by 7, 115015 by 8.
8. 22872 by 11, 812366 by 20, 71532 by 70.
9. 117234 by 500, 81991 by 110, 39152 by 1200.
10. 818979 by 70, 31562 by 120, 124087 by 600.
11. How many times may 9 be subtracted from 81?
12. Find the number of pieces of wood 12 inches long that can be cut from a piece 180 inches long.
13. A cubic foot of iron weighs 450 pounds; how many cubic feet are there in a block of weight 1000 pounds? What weight will be left over?
14. What number must 9 be multiplied by to give 738?
15. There are 16 ounces in 1 pound; how many pounds are there in 2240 ounces?

Divide—

16. 7251 by 36, 821 by 19, 1000 by 17.
17. 32156 by 73, 91919 by 25, 82667 by 91.
18. 10500 by 131, 87625 by 125, 90913 by 721.
19. 32197 by 816, 915328 by 191, 66615 by 811.
20. 1000000 by 625, 333333 by 370, 1958647 by 1131.
21. The product of two numbers is 1221; one is 37. Find the other.
22. What is the smallest number that may be added to 862151 so that it may contain 125 an exact number of times?
23. In a division sum the divisor was 17, the quotient 17, and the remainder 9. Find the dividend.
24. A thousand pence is divided among 25 boys. How much will each receive?
25. A cricketer scores 957 runs in 29 innings. What is his average score in an innings?
26. How many times does a wheel whose circumference is 139 inches turn in rolling 1946 yards, there being 36 inches in 1 yard?
27. A ton of coal costs 19 shillings. How many tons can be bought for 21299 shillings?

28. In a school of 491 boys there are 17 classes. Each class contains the same number of boys with the exception of one which has less boys than any of the others. How many are there in this class?

29. How many times must 53 be added to 151 to give a thousand?

30. In a certain book there are 9 words in a line, 39 lines in a page. There are 42471 words in the book. Find the number of pages in it.

31. Light travels at the rate of 186,000 miles per second. The sun is 93 million miles from the earth. How many seconds does the light from the sun take to reach the earth?

32. There are 1760 yards in 1 mile. Every minute a man walks 88 yards. How long will he take to walk 12 miles?

33. A dollar is worth 52 pence. What is the value in dollars of 16,900 pence?

34. What is the least number that can be subtracted from ten thousand in order that the result may contain 63 an exact number of times?

35. A man can count at the rate of 150 per minute. How long will it take him to count nine hundred thousand?

36. There are 15 telegraph poles in a straight line, the distance between each two being the same. The first pole is 308 yards from the last. Find the distance between any two poles.

37. A train starts from Euston at 8 o'clock a.m. for Cocker-mouth, a distance of 301 miles. If it travels at the rate of 43 miles per hour, at what time will it reach Cocker-mouth?

38. Find the value of $171 \times 1440 \div 15$.

39. Which is the greater, $\frac{10575}{25}$ or $\frac{10946}{26}$?

40. Divide 1540 into two parts, one being ten times as much the other.

41. Find short methods for the following:—

(i) $129975 \div 25$.

(ii) $160000 \div 125$.

(iii) $100000 \div 35$.

§ 6. PRIME FACTORS

If one number divides into another without remainder, the first number is said to be a **factor** of the second.

If a number has no factors except itself and 1 it is said to be a **prime number**. *E.g.* 139, 5, 37 are prime numbers.

48 and 10 are factors of 480. Neither 48 nor 10 is a prime number, for each has factors. When we can find factors of a number such that all these factors are prime numbers we are said to find the **prime factors** of that number.

Example. Resolve 480 into prime factors.

$$\begin{aligned}
 480 &= 2 \times 240. \\
 &= 2 \times 2 \times 120. \\
 &= 2 \times 2 \times 2 \times 60. \\
 &= 2 \times 2 \times 2 \times 2 \times 30. \\
 &= 2 \times 2 \times 2 \times 2 \times 2 \times 15. \\
 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5. \\
 &\text{i.e.}—2^5 \times 3 \times 5.
 \end{aligned}$$

EXAMPLES. VI 6

Find the prime factors of—

1. 360, 990. 2. 86, 390. 3. 462, 1728.
4. 3360, 396. 5. 1690, 8500. 6. 7875, 3575.
7. 4840, 187. 8. 6734, 20919. 9. 19943, 42237.
10. 75582, 67375. 11. 26244, 99225. 12. 161150, 35380.
13. 21322, 382239. 14. 999999, 249860.
- *15. The number $720x4$ is divisible by 8; find the smallest value of x .
16. Which prime factors are common to 169 and 923?
17. How many prime numbers are there less than 50?
18. Find the fifteenth prime number greater than 11.
19. State, without working out, which of the numbers 2, 3, 4, 5 are factors of 144, 7281, 3050, 7274, 81932, 515115.
20. Which of the numbers 2, 3, 4, 5, 6, 8, 9, 11 are factors of 5929, 7326, 3125, 392, 2744, 33957?

§ 7. REDUCTION

EXAMPLES. VII 7

1. How many pence are there in 2s. 4d. ; 3s. 6d. ; 5s. 2d. ; 8s. 4d. ; £1. 12s. 2d. ; £2. 4s. 11d.?

2. How many shillings and pence are there in 41, 175, 29, 38, 62, 95, 117, 384 pence?

3. How many feet and inches are there in 182, 37, 91, 44, 131, 66, 76, 89 inches?

4. How many inches are there in 5 ft. 6 in. ; 2 ft. 3 in. ; 9 ft. 7 in. ; 10 ft. 2 in. ; 1 yd. 1 ft. ; 3 yd. 2 ft. 9 in.?

Reduce—

5. £5. 7s. 6d., £7. 8s. 0d., £10. 4s. 6d., (i) to sixpences, (ii) to pence.

6. 7s. 11d., 8s. 2d., 11s. 4d., 17s., 19s. 11d., (i) to pence, (ii) to farthings.

7. 3 ch., 1 ch. 3 yd., 10 yd. 2 ft., 5 ch. 1 ft., 1 fur., (i) to feet, (ii) to inches.

8. 1 lb., 3 cwt., 1 ton, 3 cwt. 17 lb., 1 ton 7 cwt. 11 oz., to ounces.

9. 7082 farthings to pence ; 15,217 halfpennies to pounds.

10. 173 half-crowns to pounds ; £3. 7s. 4d. to pence.

11. 1712 lb. to cwt. ; 38,215 oz. to quarters.

12. 5 acres to sq. in. ; 11 acres 9 sq. ch. to sq. yards.

13. 17 bushels to pints ; 15,721 pints to bushels.

14. A million cu. in. to cu. yards ; 1 sq. mile to sq. feet.

15. 1 year to seconds ; a million sec. to hours.

16. 26 lbs. (avoir.) to grains ; 8 oz. 16 dwt. to grains.

17. 100 guineas to pence ; a thousand farthings to pounds.

18. £5. 17s. 11½d. to farthings ; 8219 half-crowns to pounds.

19. 42 tons 4 cwt. 1 qr. to pounds ; 1000 oz. to stones.

20. 11 oz. 9 dwt. 21 gr. to grains ; 113 tons 3 cwt. 15 lb. to pounds.

21. A pint of water weighs 20 ounces. Find in pounds the weight of 85 gallons 3 quarts of water.

22. A penny weighs one-third of an ounce. Find the weight of 3 million pennies, and express answer in tons, etc.

23. Find the difference in grains between the weight of a pound of gold and a pound of apples. [*A goldsmith's pound is different from a fruiterer's.*]

24. A cubic foot of water weighs 1000 oz. Find the weight in tons, etc., of 150 cubic yards.

25. Find the number of cubic inches in 3571 cubic yards.

26. 8 farthings weigh 1 ounce. Find the weight of the number of farthings worth £17. 18s. 10d.

27. A halfpenny is an inch across. How many halfpennies, arranged so that each touches the next, would stretch from one corner of a room to another, the distance measured in a straight line being 29 ft. 4 in.? What would be the value of these halfpennies?

28. How many times could a pint can be filled from a bucket of water containing 7 gallons?

29. A man takes 110 steps of 2 ft. 6 in. each minute. How long will he take to walk 10 miles?

30. A train travels at 60 miles an hour. How many feet is this per hour? How many feet per second?

§ 8. COMPOUND ADDITION AND SUBTRACTION

EXAMPLES. VIII 8

Add together—

1. £2. 4s. 5½d., £3. 10s. 3d., £17. 0s. 1½d.

2. £21. 16s. 4½d., 15s. 6¾d., £3. 10s. 2½d.

3. £3. 11s. 9¾d., £7. 8s. 2½d., £19. 4s. 6¾d., £11. 11s. 11¾d.

4. 5 yd. 2 ft. 3 in., 6 yd. 1 ft. 7 in., 1 yd. 0 ft. 3 in.

5. 5 m. 1 fur. 6 ch. 10 yd., 3 fur. 2 ch. 11 yd., 6 m. 7 fur. 1 ch. 1 yd.

6. 3 tons 5 cwt. 2 qr., 6 cwt. 1 qr., 11 tons 17 cwt. 2 qr.

7. 7 bush. 2 pk. 1 gall. 2 qt., 1 pk. 0 gall. 1 qt., 3 pk. 1 gall. 3 qt.

8. 9 a. 6 sq. ch. 300 sq. yd., 10 a. 5 sq. ch. 400 sq. yd.

9. £54. 17s. 9½d., £71. 15s. 9¾d., £259. 18s. 4d.
10. 21 lb. 3 oz. 13 dwt. 19 gr., 25 lb. 7 oz. 10 dwt. 10 gr.
Subtract the second amount from the first:—
11. £17. 5s. 9d., £3. 14s. 9½d.
12. £9. 0s. 1¼d., £3. 17s. 10½d.
13. £253. 11s. 1d., £197. 12s. 2¼d.
14. 5 yd. 0 ft. 3 in., 1 yd. 2 ft. 11 in.
15. 5 tons 10 cwt. 1 qr. 3 lb. 1 oz., 1 ton 0 cwt. 3 qr. 17 lb. 15 oz.
16. 2 fur. 1 ch. 11 yd. 1 ft. 2 in., 9 ch. 20 yd. 2 ft. 9 in.
17. 5 a. 2 r. 980 sq. yd., 4 a. 3 r. 1000 sq. yd.
18. 5 bush. 1 pk. 0 gall. 1 qt., 1 bush. 1 pk. 0 gall. 2 qt.
19. 23 hr. 5 min. 3 sec., 11 hr. 21 min. 19 sec.
20. 25 lb. 7 oz. 10 dwt. 1 gr., 21 lb. 3 oz. 13 dwt. 7 gr.
21. A has £51. 3s. 4½d., B has 2 guineas more than A, and C has half a sovereign less than B. Find how much A, B, and C have altogether.
22. A man has £100. He pays out amounts of £5. 7s., £25. 10s. 6d., £31. 4s. 2½d., and £2. 2s. 0d. How much has he left?
23. A man starts work at 8.45 a.m. and leaves at 6 p.m. If he has an hour and a quarter for dinner and half an hour for tea, how long does he work?
24. To tie up a parcel I use two pieces of string, one 2 ft. 10 in., and the other 5 in. longer than the first. How much string do I use altogether?
25. A lorry loaded with coal weighs 2 tons 5 cwt. The lorry itself weighs 13 cwt. 1 qr. Find the weight of the coal.
26. A park has four sides of lengths 1 mile, 1 mile 3 furlongs 20 yards, 7 furlongs 5 yards, 6 furlongs 1 chain respectively. A man starts to walk right round it: how far will he have to walk?
27. At a concert the following amounts were taken:—210 half-crowns, 250 shillings, and 721 sixpences. What does this amount to altogether?
28. A, B, C, D are four railway stations in order on a certain line. A train takes 1 hr. 13 min. from A to B, 58 min. from B to C, and 1 hr. 5 min. from C to D. It stops 3 min. at B and 5 min. at C. If it start from A at 12 midday, at what time will it reach D?

29. During a certain period a newsagent sold 73 sixpenny magazines, 3901 penny papers, and 6675 halfpenny papers. How much did he take?

30. In a 100 yards race the second man was 6 inches behind the winner and the third man 1 foot 3 inches behind the second. When the winner reached the winning-post, how far had the third man run?

31. A plank is 5 yards long. It is cut into four pieces, the lengths of three of them being 5 ft. 6 in., 2 ft. 9 in., and 5 ft. 7 in. respectively. What is the length of the fourth piece?

32. A plot of land 2 acres in extent is fenced off into 3 sections. The extent of one is 1000 square yards, that of the second is 397 square yards more than the first; how many square yards are there in the third?

33. The first number of a daily newspaper was published on Thursday, January 9, 1913. What will be the number of the copy issued on June 4, 1914?

34. In a certain train the engine weighs 35 tons 2 cwt. 12 lb. The weights of the carriages are 10 tons, 10 tons 3 cwt. 2 qr., 11 tons, 11 tons 14 cwt. 17 lb., 10 tons, 11 tons, and 11 tons 11 cwt. 1 qr. 16 lb. Find the whole weight of the train.

35. A boy miscopied two sums of money, writing 15s. 8d. for £15. 8s., and £12. 11s. for 12s. 11d. Did he make the sum too small or too great on adding up the amounts? How much was he out?

36. From the following table find the weight of each coxswain:—

Oxford.				Cambridge.			
		st.	lb.			st.	lb.
Bow	.	12	2	Bow	.	11	8
2	.	12	7	2	.	12	2
3	.	12	2	3	.	12	0
4	.	12	8	4	.	12	4
5	.	12	13	5	.	13	3
6	.	13	8	6	.	14	6
7	.	13	5	7	.	11	13
Stroke	.	10	13	Stroke	.	10	8
Cox	.			Cox	.		
Total Weight		108	7	Total Weight		107	0

37. Fill in the third column :—

Receipts for 1901-2.			Receipts for 1902-3.			Increase or Decrease.		
£.	s.	d.	£.	s.	d.	£.	s.	d.
31251	16	2	31211	1	5		(a)	
41729	4	8	40007	7	3		(b)	
43823	17	2	45619	11	4		(c)	
40056	19	1	47766	16	2		(d)	
39956	1	9	46682	9	3		(e)	
39998	3	10	41157	8	4		(f)	
40137	8	4	41936	16	8		(g)	

38. Weights of 12 lb. 8 oz. and 9 lb. 10 oz. are put on the two scale-pans of a balance. How much must be taken from the larger weight and added to the smaller so that the pans balance?

39. Fill in the vacant column :—

King's Cross.	Leeds.	Time on Journey.
5.15 a.m.	9.17 a.m.	(a)
7.15 „	11.15 „	(b)
9.45 „	1.20 p.m.	(c)
1.30 p.m.	5.12 „	(d)
1.40 „	6.3 „	(e)
2.20 „	7.11 „	(f)
3.45 „	7.45 „	(g)
6.15 „	10.34 „	(h)
10.45 „	3.33 a.m.	(k)
11.30 „	4.35 „	(l)

40. A surveyor on measuring a road finds the length to be 161 chains 34 links. If the actual length of the road is 2 miles, find his error.

41. Add the following (i) vertically, (ii) horizontally.

£.	s.	d.	£.	s.	d.	£.	s.	d.	
17	3	4	2	7	6	4	11	2	(d)
1912	11	$8\frac{1}{2}$	5	9	9	173	2	$5\frac{1}{2}$	(e)
3	0	$9\frac{1}{2}$	112	19	$2\frac{1}{2}$	89	14	$3\frac{1}{2}$	(f)
15628	19	$2\frac{1}{4}$		7	$8\frac{1}{2}$	1199	19	4	(g)
41	1	11	1235	16	$7\frac{1}{2}$	929	4	$5\frac{1}{4}$	(h)
832	17	$9\frac{1}{2}$	17	17	8	93725	17	$9\frac{1}{2}$	(k)
2	15	$10\frac{1}{4}$	976	6	7	685	5	$11\frac{1}{4}$	(l)
11928	12	$0\frac{1}{2}$	32	9	$6\frac{1}{2}$	1919	1	8	(m)
117	1	5	9	17	11	252	14	4	(n)
1217384	11	$8\frac{1}{2}$	7192	9	4	92	17	$6\frac{1}{2}$	(p)
(a)			(b)			(c)			

42. In the following question (a) and (b) are the sums of the vertical columns, and (c) (d) (e) . . . are found by subtracting the second amount from the first.

£.	s.	d.	£.	s.	d.	
173	19	11	91	8	10	(c)
912836	14	5	2153	15	11	(d)
29	17	6	12	19	9	(e)
119	3	8	108	5	6	(f)
	5	7		3	11	(g)
21729	0	10	1572	19	8	(h)
85	4	8	29	17	11	(k)
7169999	17	2	29599	19	5	(l)
828	8	9	827	16	10	(m)
1576	2	5	32	17	8	(n)
1913	19	3	1911	17	11	(p)
1	17	6		18	7	(q)
912378	7	8	87329	14	9	(r)
7281	16	9	983	2	11	(s)
29	6	11	21	8	8	(t)
2878327	19	5	999915	19	9	(v)
(a)			(b)			

Calculate (a), (b), (c), (d), etc.

§ 9. COMPOUND MULTIPLICATION AND DIVISION

Example I. Find the value of £17. 3s. 5¼d. × 513.

[*Rough Check* :—£17 × 500 = £8500].

£.	s.	d.	farthings.
17	3	5	3
			513
			1539
		384	3 farthings
		2565	
		2949	
	245	9	pence
	1539		
	1784		
89	4		shillings
85			
17			
51			
£8810			

Result :—£8810. 4s. 9¾d.

513 × 3 farthings is 1539 farthings. 1539 farthings = 384 pence and 3 farthings. Find the number of pence in 513 × 5 pence and add in 384. Repeat the process as above.

Example II. Divide £4374. 19s. 7¼d. by 361.

[*Rough Check* :— $\frac{£4000}{400} = £10.$]

£	12	2s.	1¼d.
361	£4374	19s.	7¼d.
	764		
	42	.	.
	20		
Add in 19	859s.		
	37	.	.
	12		
Add in 7	451d.		
	90	.	.
	4		
Add in 1	361	farthings.	

First remainder.

Second remainder.

Third remainder.

Note that we multiply the first remainder—42—by 20 and add in 19; the second remainder—37—by 12 and add in 7; and so on. The division is performed as in § 5.

Example III. How many times is £15. 16s. $8\frac{3}{4}d.$ contained in £649. 5s. $10\frac{3}{4}d.$?

[*Rough Check*:—£600 ÷ £15 = 40.]

£649 5s. $10\frac{3}{4}d.$ = 623323 farthings.

£15 16s. $8\frac{3}{4}d.$ = 15203 farthings.

$$\frac{623323 \text{ farthings}}{15203 \text{ farthings}} = 41 \text{ times.}$$

∴ £15. 16s. $8\frac{3}{4}d.$ is contained in £649. 5s. $10\frac{3}{4}d.$ 41 times.

EXAMPLES. IX 9

Multiply—

1. £3. 4s. 7d. by 18, £7. 5s. 4d. by 11, £9. 10s. $0\frac{1}{2}d.$ by 6.

2. £11. 3s. $4\frac{1}{2}d.$ by 63, £9. 2s. 1d. by 110, £15. 4s. $0\frac{1}{4}d.$ by 24.

3. 3 yd. 1 ft. by 12, 4 yd. 0 ft. 9 in. by 15, 1 yd. 2 ft. 11 in. by 32.

4. 5 cwt. 3 qr. by 15, 3 tons 16 cwt. 1 qr. by 56, 1 ton 0 cwt. 3 qr. by 121.

5. £151. 11s. $9\frac{3}{4}d.$ by 72, £215. 17s. $1\frac{1}{2}d.$ by 45, £1. 19s. $11\frac{1}{2}d.$ by 140.

6. £7. 3s. 1d. by 93, £17. 16s. $2\frac{1}{4}d.$ by 71, £11. 14s. $9\frac{3}{4}d.$ by 113.

7. 7 a. 2 r. 1000 sq. yd. by 29, 5 m. 3 fur. 1 ch. by 173.

8. £45. 17s. $11\frac{1}{2}d.$ by 347, 1 st. 2 lb. 12 oz. by 211.

9. 1 hr. 32 min. 7 sec. by 293, £215. 11s. $2\frac{1}{4}d.$ by 83.

10. 7 bush. 3 pk. 1 gall. by 276, 4 oz. 14 dwt. 7 gr. by 199.

11. £72. 4s. $4\frac{3}{4}d.$ by 412, 1 qr. 17 lb. 15 oz. by 766.

12. 17s. $11\frac{1}{4}d.$ by 1711, 7 yd. 2 ft. 10 in. by 1810, 4 lb. 3 oz. by 3791.

Divide—

13. 14s. 10d. by 8, £1 by 8, £1. 1s. 0d. by 7.

14. £144. 11s. 8d. by 10, £1 by 12, £179. 0s. $9\frac{1}{2}d.$ by 7.

15. 30 yd. 1 ft. 6 in. by 9, 44 yd. 6 ft. 5 in. by 11, 19 yd. 1 ft. 4 in. by 10.

16. £505. 12s. 0*d.* by 24, £22. 3s. 2½*d.* by 21, £2733. 3s. 9*d.* by 30.

17. 2 qr. 7 lb. by 28, 80 tons 2 cwt. by 72, 27 tons 17 cwt. 1 qr. 20 lb. 13 oz. by 25.

18. £60. 10s. 2½*d.* by 17, £378. 5s. 3½*d.* by 31, 12s. 6¾*d.* by 67.

19. 279 yd. 1 ft. 8 in. by 74, 139 tons 15 cwt. 14 lb. by 413.

20. 13 cwt. 1 qr. 12 lb. by 374, 1303 bush. 3 pk. 1 gall. by 171.

21. £2667. 11s. 8*d.* by 269, £2827. 16s. 8*d.* by 893.

22. 862 hr. 22 min. 18 sec. by 218.

23. £12424227. 9s. 2*d.* by 685.

24. 56 tons 6 cwt. 41 lb. by 321.

Find how many times the second quantity is contained in the first :—

25. £97. 10s. 0*d.*, £1. 17s. 6*d.*

26. £35. 3s. 6*d.*, £1. 9s. 3¾*d.*

27. £287. 5s. 4½*d.*, £31. 18s. 4½*d.*

28. £1000. 1s., £2. 19s.

29. £2760. 6s. 3*d.*, £110. 8s. 3*d.*

30. 37 tons 8 cwt., 6 cwt. 2 qr. 20 lb.

31. 2 m. 30 yd., 147 yd. 2 ft. 9 in.

32. 32 lb. 6 oz. 8 dwt., 10 dwt. 16 gr.

33. 114 tons 9 cwt. 1 qr. 20 lb. 14 oz., 3 tons 27 lb. 13 oz.

34. £2065. 15s. 1¼*d.*, £3. 6s. 1¼*d.*

35. Find the cost of 173 books at 1s. 10½*d.* each.

36. A shopkeeper pays 11s. 4½*d.* for a certain article. If he buys 2 gross and sells each for 13s. 6*d.*, what is his total profit?

37. Find the average daily wage (excluding Sundays) of a man whose yearly income is £268. 13s. 2*d.*

38. In a factory there are the same number of men, women, and boys. A man earns 3s. 11*d.*, a woman 3s., and a boy 1s. 5½*d.* per day. The total amount paid out daily in wages is £125. 12s. 6*d.* How many men are there?

39. A circular track is 333 yards round. How many times must a boy run round to complete a mile, and what is the distance from the starting-point to the winning-post?

40. Find the total weight of 371 sacks each containing 3 lb. less than a hundredweight.

41. For letters not exceeding 4 oz. the cost of inland postage is 1*d.*; for every additional 2 oz. $\frac{1}{2}$ *d.* is charged. How much will it cost to send a parcel weighing 1 lb. 10 oz. by letter post?

42. The wheel of an engine revolves 713 times a minute. How long will it take to revolve 5063013 times? How many times does it revolve in $3\frac{1}{2}$ hours?

43. On every £1 over £160 a man pays 9*d.* income-tax. How much does he pay if he earns £275. 10*s.* 0*d.*?

44. A sum of money amounting to £103. 1*s.* 10*d.* was divided among some people. On giving each the same amount it was found that he received £1. 3*s.* 2*d.* How many people were there?

45. What remains after cutting lengths of 4 yd. 2 ft. 3 in. from a roll of cloth 63 yd. long?

46. A boy's hoop is 5 ft. 6 in. round. He rolls it for half a mile. How many times does it turn?

47. Find the total cost of 35 books at 2*s.* 4 $\frac{1}{2}$ *d.* each, 97 at 1*s.* 10 $\frac{1}{2}$ *d.* each, and 351 at 1*s.* 1 $\frac{1}{2}$ *d.* each.

48. A shopkeeper charges 2*s.* 0*d.* per lb. for a certain brand of tea: he makes it up into packages weighing, with the paper, 1 lb. If the paper weighs $\frac{1}{2}$ oz. and is worth nothing to the purchaser, find out of how much a man is defrauded if he buys during a year 45 lb. packages.

49. How often can I take 18*s.* 9*d.* from £14. 1*s.* 3*d.*?

50. A sheet of paper is 2 yd. 1 ft. 3 in. long. Lines of length 7 in. are to be drawn on it at equal distances apart, so that 4 in. is left at each end. If the distance between each pair of lines is 2 in., find how many lines there will be.

51. A dealer buys 17 sheep and 16 cows. He pays £14 12*s.* 6*d.* for each cow. How much does a sheep cost him if he pays out altogether £265. 9*s.*?

52. At a football match it was found that the receipts amounted to £800. 2721 people paid 1*s.*, 1956 people paid 1*s.* 6*d.*, and 871 paid 2*s.* 6*d.* How many paid 6*d.*?

53. A bag contains half-crowns, florins, sixpences, and half-pennies. There is the same number of each coin, and the value of the money in the bag is £23. 3*s.* 10*d.* How many coins are in the bag?

54. Find the weight of 29 wheels each weighing 2 cwt. 3 qr. 1 lb. 8 oz.

55. The total weight of 19 men is 199 st. 12 lb. 17 of the men weigh the same; the other two are respectively 4 lb. heavier and 1 lb. 6 oz. lighter than each of the others. What does the lighter man weigh?

56. Which is cheaper, to buy coal at £1. 3s. 4d. per ton or 1s. 3d. per cwt.? How much will a man save by buying 172 tons at the cheaper rate?

57. 25 francs are worth £1. What is the value in francs of £3038?

58. A picture frame is 1 ft. 3 in. long and 9 in. broad. How many frames can a man make out of a piece of wood 3 yd. 2 ft. 8 in. long? If out of the piece remaining he makes a smaller frame 1 ft. 2 in. long, what will be its breadth?

59. A man sells a certain number of articles for £6. 11s. 3d. If he had sold each for 10d. more he would have received £7. 2s. 11d. How many did he sell?

60. In a certain piece of machinery the pointer moves over a dial 3 times every second. The machine is set working at 11.25 a.m. and is stopped at 5.30 p.m. How many times did the pointer move?

61. The total net tonnages of vessels which arrived and departed from the Port of London during 1911 were 39179153 tons. Find the net tonnage per day. Neglect any remainder there might be, and give your answer as a whole number.

62. One penny per tail was the average price paid by a Rural Council for the destruction of rats, the total sum paid being £70. How many rats were killed?

63. A Gnome engine as used in aeroplanes weighs 2 lb. 12 oz. per horse-power. A man received a crate containing an engine, the weight of the two together being 52 lb. 4 oz. If the crate weighed 11 lb., what was the horse-power of the engine?

64. A rubber company stated in its annual report that the output of rubber was 393511 lb. The amount realised for the sale of this was £81162. Find the price of rubber per lb. to the nearest halfpenny.

65. In 1911 the railway companies of England and Wales employed 405326 men. Their average weekly wage was £1. 7s. 4½d. : what amount was paid out in wages?

66. 1130668 people occupy 576540 acres in Glamorganshire. Find the number of square yards in the county, and then find how many square yards one person occupies. Neglect any remainder.

67. From a certain mine each ton of ore yields on the average 2 oz. 1 dwt. 15 gr. of silver. How much silver can be obtained from 293 tons?

68. A quarter is a heaped measure of 8 bushels. The weight of a bushel of English barley is 60 lb. and that of a bushel of English oats 39 lb. A corn merchant has a stock which includes 3 quarters of oats and 2 quarters of barley. Find the weight in cwt. and lb. of this part of his stock.

69. A degree of longitude is 69 miles 293 yards at a place on the Equator. The longitude of Quito is 80 degrees W. and that of Juba 40 degrees E., both places being on the Equator. Find the distance between them.

70. A sovereign weighs 123 grains. Find the weight in oz., dwt., and gr. of the 30044105 sovereigns coined by the Mint in 1912.

§ 10. DECIMAL FRACTIONS

Since we may write 8 hundreds 3 units 2 tenths 3 hundredths 5 thousandths in the form 803·235, the metric system gives us a convenient method for writing such expressions as 5 Hm. 3 Dm. 2 m. 1 dm. 2 cm. 1 mm. in the form 532·121 m. or 5·32121 Hm.

To **multiply** by ten or powers of ten, move the decimal point the requisite number of places to the **right**—*e.g.*,

$$87\cdot3521 \times 1000 = 87352\cdot1.$$

To **divide** by ten or powers of ten, move the decimal point the requisite number of points to the **left**—*e.g.*,

$$87\cdot3521 \div 1000 = \cdot0873521.$$

When adding or subtracting lengths, etc., in the metric system, or numbers involving decimals, write down the numbers so that the decimal points are in the same vertical line.

Example I. Add 35·05, 410, 8·053, 993·721.

$$\begin{array}{r}
 35\cdot05 \\
 410\cdot \\
 8\cdot053 \\
 993\cdot721 \\
 \hline
 1446\cdot824
 \end{array}$$

Example II. Find the excess of 171·92 yd. over 83·715 yd.

$$\begin{array}{r}
 171\cdot92 \\
 83\cdot715 \\
 \hline
 88\cdot205 \text{ yd.}
 \end{array}$$

Example III. Find the value of $80\cdot315 \times 721\cdot31$.

[*Rough Check*:— $80 \times 700 = 56000$.]

Multiply 80315 by 72131 as if no decimal point existed.

$$\begin{array}{r}
 80315 \\
 72131 \\
 \hline
 562205 \\
 160630 \\
 80315 \\
 240945 \\
 80315 \\
 \hline
 5793201265
 \end{array}$$

Since the rough check tells us that the answer is about 56000, we know that the decimal point must come between the 2 and the 0. The answer is therefore

$$57932\cdot01265.$$

Example IV. Divide 16·60152 by ·5321.

Multiply both dividend and divisor by 10.

[*Rough Check*:— $166 \div 5 = 33\cdot\text{.....}$]

Divide 1660152 by 5321 as if no decimal point existed, and insert the decimal point by means of the rough check as in multiplication:—

$$\begin{array}{r}
 312 \\
 5321 \overline{) 1660152} \\
 \underline{6385} \\
 10642 \\
 \underline{}
 \end{array}$$

The answer is 31·2.

Example V. Divide 9·284 by ·000781.

[*Rough Check*:— $9 \div \frac{8}{10000} = 10000$ roughly.]

$$\begin{array}{r} 120153 \\ 781 \overline{) 9284} \\ \underline{1474} \\ 1200 \\ \underline{4190} \\ 285 \end{array}$$

The answer is **12015·3.....**, the dots denoting that there is a remainder.

Note.— $836255 \div 5 = 1·67251$ **exactly.**
 $= 1·6725$ correct to 4 decimal places.
 $= 1·673$ 3
 $= 1·67$ 2
 $= 1·7$ 1
 $= 2$ **the nearest whole number.**

EXAMPLES. X 10

Write down—

1. $83·5 \times 10$, $91·52 \times 100$, $·318 \times 1000$, $8·2 \times 10000$.
2. $91·5 \div 10$, $8·001 \div 100$, $·05 \div 10$, $·005 \div 1000$.
3. $·69 \times 100$, $·69 \div 100$, $6·9 \div 100$, $·0069 \times 100$.
4. $72·1 \div 100$, $·721 \div 100$, $72·1 \times 1000$, $72·1 \div 1000$.

Add—

5. $83·5$, $72·91$, $·00087$.
6. $·315$, $81·76$, $·005$.
7. $875·329$, $89·329$, $·875$.
8. $120·05$, $·009$, $976·325$, $·876$.

Find the difference between—

9. $89·317$ and $9·428$.
10. $110·631$ and $·00915$.
11. $1000·01$ and $863·954$.
12. $897·21$ and $976·005$.

Find the value of—

13. $35·5 - 8 + 91·5$.
14. $87·25 - 6·3971 + 11·82$.
15. $·005 - ·076 + ·099 - ·0082$.
16. $·42 + ·013 - ·0005 + 1·1 - ·062$.
17. $·8721 + 973·214 - ·87295$.

18. $3000 - 300 + 30 - 3 + \cdot 3 - \cdot 03 + \cdot 003$.

19. $65298\cdot 315 - 7969\cdot 9876 + 315\cdot 8115$.

20. $195976\cdot 7215 - 119732\cdot 6 + 953\cdot 00087$.

21. Write down $\cdot 0008$ Dm. in metres, $\cdot 0008$ m. in Dm.

22. Point out the mistakes (if any) in the positions of the decimal points in the following products:—

(i) $2\cdot 8 \times 7\cdot 3 = 204\cdot 4$. (iv) $81\cdot 3 \times \cdot 009 = 731\cdot 7$.

(ii) $11\cdot 5 \times \cdot 08 = 92$. (v) $25100 \times \cdot 0006 = 1\cdot 506$.

(iii) $\cdot 1 \times \cdot 1 \times \cdot 1 = \cdot 1$. (vi) $(\cdot 09)^4 = \cdot 6561$.

23. Find as a decimal of £1 the difference between £07219 and £18611.

24. AD is a ruler, and B and C are two points on it. If $AD = 1$ foot, $AB = 1\cdot 72$ inches, $CD = 8\cdot 04$ inches, find BC.

25. The contents of two vessels, one containing $793\cdot 5$ cubic centimetres and the other $814\cdot 75$ cubic centimetres, are poured into a vessel containing 120 cubic centimetres. How many litres are there now in the third vessel? 1 litre = 1000 cubic centimetres.

26. The rainfall for a certain week was:—Monday, $\cdot 40$ in.; Tuesday, $\cdot 02$ in.; Wednesday, $\cdot 45$ in.; Thursday, $\cdot 28$ in.; Friday, $\cdot 58$ in.; Saturday, $\cdot 04$ in. Find the total rainfall.

27. Represent on squared paper the following facts:—A is 12·0 feet west of O, B is 13·7 feet east of A, C is 15·8 feet west of B, D is 20·5 feet east of C. Find the distance of (i) A west of D, (ii) O west of C.

28. Express in thousands of pounds the difference between £197521 and £3884652.

29. On the scale-pans of a balance are weights of 11·09 grams and 13·31 grams. What weight must be taken from the latter and put on the former so that the scale-pans balance?

30. The mean temperature for January 1902 in an English town was $37\cdot 8^\circ$ F. It fell $2\cdot 4^\circ$ during February, rose $6\cdot 7^\circ$ during March, rose $2\cdot 4^\circ$ during April, rose $\cdot 9^\circ$ during May. Illustrate this by a diagram, and find the mean temperature for May.

Find the value of—

31. $83\cdot 1 \times 7$, $8\cdot 31 \times \cdot 7$, $\cdot 831 \times \cdot 07$.

32. $915\cdot 2 \times 78$, $9\cdot 152 \times \cdot 78$, $\cdot 09152 \times \cdot 078$.

33. $1.05 \times .09$, 105×900 , $.0105 \times 90$.

34. 873.6×11.5 , 8.736×1.15 , $.008736 \times 115$.

Find the following products, checking the position of the decimal point :—

35. $819.1 \times .09$, 3.5×1.1 , $.097 \times .12$.

36. 5.17×1.3 , 99.86×120 , $.87 \times .8$.

37. $65.2 \times .005$, 81.3×900 , $7.5 \times .0011$.

38. $53 \times .17$, $.0095 \times 21.1$, $20.01 \times .18$.

39. $.0087 \times .315$, $.871 \times 31$, $.0915 \times .086$.

40. 44.2×87.1 , $66.4 \times .41$, $153.4 \times .0091$.

41. $(.2)^2$, $(.03)^2$, $(.001)^2$.

42. $(.01)^3$, $(.002)^3$, $(1.1)^4$.

Find the value of—

43. $814.11 \div 3$, $81.411 \div .03$, $.81411 \div 300$.

44. $.3267 \div 11$, $32.67 \div .0011$, $.03267 \div 1.1$.

45. $.03717 \div .007$, $37.17 \div 700$, $37170 \div .7$.

46. $27.36 \div 11.4$, $.002736 \div .00114$, $273600 \div .114$.

Find the following quotients, checking the position of the decimal point :—

47. $64.89 \div .09$, $856.4 \div 4$, $.3591 \div 70$.

48. $46.5 \div 12$, $.0472 \div .04$, $6.225 \div 1.2$.

49. $.0333 \div .0037$, $.289 \div 17$, $.0441 \div 210$.

50. $.018704 \div 28$, $149.81 \div .71$, $122.5 \div .0035$.

51. $132506.22 \div .00231$, $.3721 \div .61$, $.01936 \div 440$.

52. $102.9 \div .00168$, $1587.6 \div 5.0625$, $1392 \div 11.6$.

53. $.46875 \div .00125$, $37.845 \div 290$, $6389.76 \div 1040$.

54. $281.37084 \div 579$, $29019.69 \div 36.5$, $3364.2 \div .0054$.

Find the value of the following :—

55. $7.315 \div 3$ correct to 2 decimal places.

56. $819.72 \div .51$ 2

57. $10 \div 7$ 3

58. $973\cdot25872 \div 11\cdot6$ 4

59. $374\cdot1172 \div 158$ 3

60. $71\cdot01 \div 0\cdot951$ 5

61. A cubic foot of water weighs $62\cdot5$ lb. Express in lbs. the weight of 719 cubic feet.

62. A metre = $39\cdot37$ inches. Find the number of inches in (i) 1 Km., (ii) 725 Dm.

63. Reduce $15\cdot275$ yards to inches.

64. Express $\pounds 1\cdot7298$ as pence.

65. The average weekly wage of a clerk in a certain office is 21s. $3\cdot12d$. There are 25 such clerks. How much is paid out in wages per week?

66. A kilogram = $2\cdot2$ lb. How many kilograms are there in 1 ton?

67. The amount of available space for seating in a hall is a thousand square feet. Each seat takes up 255 square inches. How many seats will the hall contain?

68. On a certain day the value of $\pounds 1$ in French money was $25\cdot22$ francs. A week later it had risen to $25\cdot27$ francs. How much did a man lose by exchanging $\pounds 1973$ on the former day instead of on the latter?

69. There are $2\cdot54$ cm. in 1 inch. How many cm. are there in 1 mile? How many inches (correct to 2 decimal places) are there in 1 metre?

70. To obtain the circumference of a circle its diameter is multiplied by $3\cdot1416$. What error in length do I make if I obtain the circumference of a circle of 15 cm. diameter by multiplying the diameter by $3\cdot14$?

71. On a certain day 2055000 rupees were transferred from Bombay: the value of a rupee was $16\cdot031d$. Find the value of the amount transferred in $\pounds$. s. d.

72. In a hydro-electric installation there is a yearly output of 51272580 units at a cost of $10\cdot45d$. per unit. To what does this amount? Answer correct to the nearest penny.

§ 11. METRIC SYSTEM

EXAMPLES. XI 11

Express in m., cm., and mm. (where possible)—

1. 83.15 Km. ; 3.275 m. ; 19.3612 Hm.
2. 1729 mm. ; 81.237 Dm. ; 5 Km.

Express in gm.—

3. 8 Kg. ; 7.219 Hg. ; 831.572 Dg.
4. 4 Hg. 2 gm. ; 1 Kg. 2 Dg. 3 dg. 1 mg.

Express in fr. and c.—

5. 817.2 fr. ; 31.295 fr. ; 7.008 fr. ; .07 fr.
6. 8172 c. ; 31295 c. ; 7008 c.

Express in Dl.—

7. 8641 l. ; 30.75 dl. ; 792.056 Hl.
8. Add 12 Km. 3 Hm., 3 Hm. 95 m., 3 Km. 21 m., giving answer in Km.
9. Add 5 Hl. 3 Dl., 9 Dl. 5 l., 752 l., giving answer in Hl. and l.
10. Add 75 fr. 31 c., 897 c., 32.9 fr., giving answer in fr. and c.

11. Subtract—

- | | |
|----------------------------------|-----------------------------|
| (i) 8 Hm. from 3152 m. | (ii) 95 c. from 8 fr. 25 c. |
| (iii) 3.725 gm. from 1 Hg. 2 gm. | (iv) 9.728 cm. from 1 m. |

Find the value of—

12. 5 dollars 20 cents $\times$ 11 ; 3m. 5 cm. $\times$ 90.
13. 8 Hl. 35 l. $\times$ 71 ; 21 Kg. 35 gm. $\times$ 66.
14. 1 fr. 35 c. $\times$ 181 ; 2 dm. 1 mm. $\times$ 74.
15. 3 Km. 250 m. $\times$ 376 ; 8 dollars 5 cents $\times$ 719.
16. 628 fr. 20 c. $\div$ 12 ; 1 Km. 24 m. $\div$ 8.
17. 11 Hl. 5 l. $\div$ 5 ; 62 dollars 50 cents $\div$ 25.
18. 1377 m. 85 cm. $\div$ 17 ; 493 Km. 232 m. $\div$ 29.

Find correct to the nearest centime—

19. 89 fr. 5 c. $\div$ 28 ; 1128 fr. 35 c. $\div$ 97.

Find correct to the nearest centimetre—

20. 8 Km. 9 m. $\div 19$; 182 Km. 35 m. $\div 176$.

Find the cost of—

21. 8 Hl. 2 l. of spirits at 112 fr. per Hl.

22. 7500 gm. of tea at 3 fr. 80 c. per Kg.

23. 7 m. 35 cm. of silk at 6 fr. 5 c. per metre.

24. 9 m. 85 cm. of rubber at 2 fr. 5 c. per metre.

25. 756 Hl. of wine at 55 fr. 50 c. per l.

26. How many mm. are there in the difference between 1 Km. and 1 m.?

27. 1 inch = 2.54 cm. How many mm. are there in 1 yard?

28. There are 453.6 gm. in 1 lb. How many Kg. and gm. are there in 1 cwt.?

29. A sovereign is worth 25 fr. 24 c. How many fr. and c. are there in £750?

30. A litre being the same as a cubic decimetre, how many litres are equivalent to 539 cubic metres?

31. Find to the nearest centime the cost of 185 m. 95 cm. of silk at 6 fr. 5 c. per m.

32. What is the price of ribbon per m. when 303 m. 20 cm. cost 197 fr. 8 c.?

33. Express 2.04 times 1 m. 5.75 cm. as cm.

34. The cost of 11 m. 5 cm. of material is 38 fr. 40 c. Find correct to the nearest 5 cm. the length of material that can be bought for 100 fr.

35. A cu. cm. of brass weighs 8.3 gm. What will be the weight of a brass rod whose volume is 15.5 cu. cm.?

36. The weight of 10 cu. cm. of cork is 2.5 gm. What is the volume of a cork weighing 4 gm.?

37. What length will be left over from a rod 3.72 m. long after cutting 28 equal lengths from it?

38. A book of 534 pages is (without the cover) 2.67 cm. thick. Find the thickness of each leaf.

39. 29 five-centime pieces when placed so that each one touches the one next to it stretch a distance of 72.5 cm. Find the diameter of one of the coins.

40. Two boys measure a rod of length of 3 ft. 4 in. in cm. One takes 1 in. to be equal to 2.54 cm. and the other to be equal to 2.5 cm. Find the difference in their measurements of the rod.

41. A South American State exported in 1903 2642220 Kg. of coffee. If a Kg. = 2.2 lb., how many tons, etc., is this?

42. A motor-bicycle consumes petrol at the rate of a gallon per 21.2 miles. How many gallons were used in a ride of 487 miles?

43. In Ghent a gardener works 15 hours for 2 fr. 50 c. Taking a franc as equal to 10d., what would he be paid in English money (i) for 15 hours, (ii) per hour?

44. It is often said that "a pint of pure water weighs a pound and a quarter." 1 cu. cm. of water weighs a gm. Do these two facts correspond? 453.6 gm. = 1 lb., and 4.536 litres = 1 gallon.

45. .9144 m. = 1 yd. The St. Gothard tunnel is 8 miles 1330 yards long. Express this length in Km.

§ 12. AREAS AND VOLUMES.—I

In this chapter we shall only treat of simple areas and volumes involving measurements in the Metric System. For the subject more generally treated refer to § 20.

Areas—

Area of rectangle	= (length × breadth).
..... parallelogram	= (base × height).
..... triangle	= $\frac{1}{2}$ (base × height).
..... trapezium	= $\frac{1}{2}$ (sum of parallel sides × distance between these sides).
..... circle	= π (radius) ² .
	[Take $\pi = 3.14$.]

Volumes—

Volume of rectangular block	= (length × breadth × height).
..... right prism or cylinder	= (area of base × height).
..... cone	= $\frac{1}{3}$ (area of base × height).

It is very important to notice the fact that lengths are measured in LINEAR UNITS, areas in SQUARE UNITS, and volumes in CUBIC UNITS.

Such a statement as "The area of a circle of radius 3 cm. is $\pi \times 3^2$ " is incorrect: the units employed should always be stated.

Always keep the lengths to be multiplied in the same linear unit, so that the area will be in the corresponding square unit; similarly, work in corresponding units when finding the volume.

When finding the area of a figure of irregular shape, draw the figure on paper ruled preferably in square centimetres; count the number of whole squares, and estimate the portions of squares remaining.

EXAMPLES. XII 12

1. Find the areas (in sq. m.) and perimeters (in cm.) of the following rectangles:—

- | | |
|-----------------------------|-------------------|
| (i) length 3 m., | breadth 25 cm. |
| (ii) 5 m. 3 cm., | 10 dm. |
| (iii) 6 Hm. 2 m., | 8 m. |
| (iv) 8 Km. 1 m., | 35 m. |
| (v) 3 m. 1 cm. 3 mm., | 5 cm. 9 mm. |

2. By drawing the figures and measuring the heights, find the areas of the following triangles:—

- | | |
|--------------------------------------|----------------------|
| (i) sides 5 cm., 6 cm., 7 cm. | (answer in sq. cm.). |
| (ii) 4 cm., 5.2 cm., 3.8 cm. | (..... sq. mm.). |
| (iii) 9.2 cm., 8.3 cm., 10 cm. | (..... sq. m.). |
| (iv)05 m., .08 m., .07 m. | (..... sq. mm.). |
| (v) 51 mm., 62 mm., 73 mm. | (..... sq. cm.). |

3. Draw the parallelograms ABCD and find the area in sq. cm., where—

- | | | |
|------------------------|----------------|-------------------------|
| (i) AB = 8.35 cm., | AD = 9.25 cm., | $\angle A = 30^\circ$. |
| (ii) AB = .1 m., | AD = .06 m., | $\angle A = 45^\circ$. |
| (iii) AB = 78 mm., | AD = .08 m., | $\angle A = 60^\circ$. |
| (iv) AB = 2 cm. 5 mm., | AD = .5 dm., | $\angle A = 30^\circ$. |
| (v) AB = 3.9 cm. | AD = 75 mm., | $\angle A = 45^\circ$. |

4. Find the areas of the circles whose radii are—

- | | |
|---------------|----------------------|
| (i) 3 cm. | (answer in sq. cm.). |
| (ii) 45 mm. | (..... sq. cm.). |
| (iii) 2.7 cm. | (..... sq. m.). |
| (iv) .005 Dm. | (..... sq. m.). |
| (v) 1.073 m. | (..... sq. mm.). |

5. The area of a board is 500 sq. cm. The width is 25 cm. What is the length?

6. Find the width of the cover of a book 18 cm. long if its area is 216 sq. cm.
7. A sheet of paper contains 560 sq. cm. Its length is .28 m. Find its width in m.
8. A field is 278 m. long and 110 m. wide. What is its area in (i) sq. m., (ii) ares, (iii) hectares?
9. Find the width of a field 1.4363 Ha. in extent and .271 Km. long.
10. The area of a postage stamp is 5 sq. cm. How many mm. long is it if its width is 20 mm.?
11. A room is 13 m. long, 10 m. broad, and 5 m. high. How many sq. m. of paper will the walls require?
12. Find the number of sq. m. of paper required to cover the walls of a room 14 m. 8 cm. long, 10 m. broad, and 4 m. 73 cm. high.
13. How many tiles, area 25 sq. cm., will be required to cover the floor of a room 6.8 m. by 6.3 m.?
14. A courtyard 25 m. by 10 m. 80 cm. is paved with tiles. There are 6750 tiles: what is the area of each?
15. From a sheet of paper 30 cm. long and 20 cm. wide sufficient strips are cut to cover the upper surface of a flat rod 2 cm. wide. What is the length of the rod?
16. How much paper will be required to cover a box in the shape of a cube 1 m. side?
17. A sheet of copper is 7.5 m. square. How many 10 cm. strips can be cut from it?
18. There are 217 pages in a book. Each page is 17.8 cm. by 12 cm. How many sq. m. of paper are there in the book?
19. An inch = 2.54 cm. How many sq. cm. are there in a sq. in.?
20. How many sq. in. are there in a sq. m. if 39.37 in. = 1 m.?
21. Find the area of a gravel path 1 m. wide round the outside of a rectangular grass plot 50 m. by 35 m.
22. A garden is in the shape of a square, 95 m. side. There is a grass plot in the centre, round the outside of which is a path 2.5 m. wide. Find the area of the plot.

23. A border 1 m. 50 cm. wide is left round the outside of a rectangular carpet placed on the floor of a room 13 m. 75 cm. by 12 m. 90 cm. Find the area of the carpet.

24. How many ares are there in a circular piece of ground, radius 15 m.?

25. Express the area of a circular disc of zinc of 8·7 cm. radius in sq. mm.

26. From a sheet of copper 8·5 cm. long and 7·2 cm. broad a circular disc of radius 4 cm. is cut. How much metal will be left over?

27. Find the area of a border 1 m. wide round the outside of a circular plot 16 m. in diameter.

28. The glass of a picture frame measures 28 cm. by 35 cm. How much wood 2·5 cm. wide would be required to frame the picture?

29. The diameter of a circular photograph frame is 20 cm. and the diameter of the glass 17 cm. Find the area of the wood of which the frame is composed.

30. Find the cost of painting the walls of a room 12·1 m. long, 11 m. broad, and 5 m. high, at 7d. per sq. m.

31. Find the volumes in cu. cm. of the following rectangular blocks—

- (i) length 5 cm., breadth 3 cm., height 2 cm.
- (ii) length 8 cm. 5 mm., breadth 15 mm., height 15 mm.
- (iii) length 7·25 dm., breadth 3·8 dm., height 5·7 dm.
- (iv) length 3 m., breadth 2 m., height 12 mm.
- (v) length 1 m. 2 dm. 5 cm., breadth 9 dm. 5 cm., height 15 mm.

32. Find the volumes of the circular cylinders—

- (i) radius of base 1 cm., height 5 cm. (answer in cu. cm.).
- (ii) 5 cm., 9 dm. (..... cu. cm.).
- (iii) 9 mm., 15 cm. (..... cu. dm.).
- (iv) 09 m., 15 m. (..... cu. cm.).
- (v) 008 cm., 36 cm. (..... cu. mm.).

33. Find the volumes of cones of the same dimensions as the cylinders in the above question.

34. 1 litre = 1 cu. dm. Find the number of (i) litres, (ii) hectolitres in 1 cu. m.

35. How many cu. cm. are there in 58.25 Hl.?
36. Find the volume of a board 5.3 m. long, 2.75 m. wide, and 35 mm. thick. Give answer in cu. cm.
37. Cubical blocks, edge 9 cm., are piled one on top of another so that the volume of the whole set of blocks is 10935 cu. cm. How many blocks are there?
38. The area of cross-section of a piece of wire is 1 sq. mm. The volume of the wire is .5 cu. cm. How long is the wire?
39. Find the area of cross-section in sq. mm. of a wire 25 cm. long whose volume is .225 cu. cm.
40. A tank is 2.5 m. long and 1 m. broad. 10 Hl. of water are poured in. Find the depth of the water in cm.
41. How many litres will a glass vessel, radius of base 1 dm., height 1 m., contain?
42. From a cube of wood, side 49 cm., as many cubes, side 70 mm., as possible are cut. How many of these cubes will there be?
43. A wineglass is in the shape of an inverted cone, radius of base 3 cm., height 7 cm. How much liquid will it hold? Give answer in litres.
44. 1 cu. cm. of water weighs 1 gm. Find the weight of water in Kg. that a cistern whose dimensions are 3 m. by 1.5 m. by 1.3 m. will hold.
45. Find the cubical contents of a cylindrical vessel, height 65 cm., radius of base 3.2 cm.
46. When filled to the brim a glass cylinder will hold .98125 l. The radius of the base is 2.5 cm. Find the height of the vessel.
47. How much water will be left over after filling a jar 15.3 cm. high on a base 12 cm. square from a vessel containing 3 litres?
48. A leaden cylinder radius of base 4.2 cm. and height 13.75 cm. is melted down. What will be the height of a rectangular block on a base 5.5 cm. by 7 cm. made from the resulting metal?

§ 13. H.C.F. AND L.C.M.

Example. Find the H.C.F. and L.C.M. of 296, 111, 4070.

Find the prime factors of each.

$296 = 2 \times 148.$	$111 = 3 \times 37.$
$= 2 \times 2 \times 74.$	$4070 = 2 \times 2035.$
$= 2 \times 2 \times 2 \times 37.$	$= 2 \times 5 \times 407.$
$= 2^3 \times 37.$	$= 2 \times 5 \times 11 \times 37.$

The highest common factor is 37.

The lowest common multiple is $2^3 \times 3 \times 5 \times 11 \times 37$.

It is inadvisable to multiply the factors out.

EXAMPLES. XIII 13

Find the H.C.F. and L.C.M. of the following by factors :—

- | | | |
|-------------------------|-------------------------|------------------|
| 1. 72, 48. | 2. 55, 75. | 3. 90, 243. |
| 4. 105, 77. | 5. 35, 56, 98. | 6. 144, 72, 120. |
| 7. 15, 42, 75. | 8. 81, 72, 90, 729. | |
| 9. 110, 132, 55, 1210. | 10. 52, 390, 169, 676. | |
| 11. 160, 704, 320, 176. | 12. 187, 289, 850, 374. | |

Find the H.C.F. and L.C.M. of—

- | | | |
|----------------------------|--------------------------|-----------------|
| 13. 276, 529. | 14. 1189, 2929. | 15. 936, 32175. |
| 16. 2006, 12051. | 17. 20525, 35303. | |
| 18. 111407, 366300. | 19. 19404, 32670. | |
| 20. 1529, 130382, 16819. | 21. 42237, 37791, 81510. | |
| 22. 17280, 13121, 19845. | 23. 1404, 3042, 7293. | |
| 24. 98131, 592841, 900210. | | |

25. What is the smallest number divisible by every number from 5 to 17 inclusive?

26. Find the shortest piece of wood that can be cut into equal lengths either 1 ft. 5 in., 1 ft. 6 in., or 2 ft. long.

27. What is the smallest sum of money that could be paid either in half-crowns, florins, or pence?

28. Tea is to be made up into 4 oz. packets, 16 oz. packets, 28 oz. packets, or $4\frac{1}{4}$ lb. packets. Find the smallest quantity of tea that can be used.

29. A rupee = 1s. 4d. and an Indo-Chinese dollar = 1s. 9d. Find the smallest sum of money that can be paid in either an exact number of rupees or an exact number of dollars.

30. A courtyard is to be paved with square tiles of the same size so that only complete tiles are used. The courtyard is 119 ft. 2 in. by 46 ft. 9 in.: find the greatest possible length of side that a tile can have.

31. The top of a table 2 ft. 6 in. by 2 ft. 3 in. is inlaid with squares at least an inch side. In how many ways may this be done?

32. Three bells start tolling together: they toll at intervals of 3, 5, 12 seconds respectively. If they start at 6 a.m., at what time will they be tolling together again?

33. Three boys have pieces of string of lengths 10 cm., 10.5 cm., and 11 cm. Find the smallest possible length they can each measure exactly. Give answer in m.

34. Find the smallest possible number of equal cubes which can be made (without wasting any material) from a rectangular block of wood 8.4 m. by 1.1 m. by .75 m.

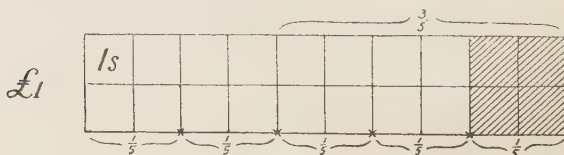
35. Three different kinds of tobacco cost 5d., 5½d., 6d. per oz. What is the least amount of money I can lay out which will buy me an exact number of ounces of any of the kinds of tobacco?

36. A circular track is 440 yards round. Three men start walking together at rates of 4, 5, 6 miles per hour round the track. How long will it be before they are all at the starting-point again?

§ 14. VULGAR FRACTIONS

Squared paper may be used to great advantage in examples such as the following:—

Example I. Find the value of $\frac{3}{5}$ of £1.

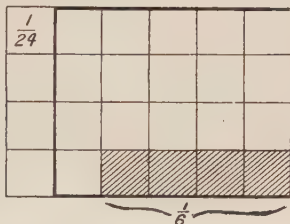
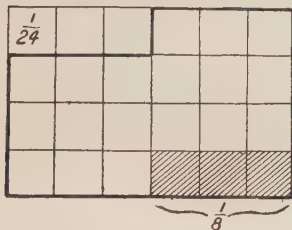


The method is as follows:—Draw a rectangle 10 units by 2 units; divide the rectangle into 20 squares corresponding to the 20 shillings in one pound. If the rectangle represents £1, each square will represent 1 shilling. Then one-fifth of the rectangle, *i.e.* one-fifth of £1, will be the same as 4 small squares, *i.e.* 4 shillings.

$$\text{Then } \frac{1}{5} \text{ of } £1 = 4 \times 1s. = 4s.$$

$$\therefore \frac{3}{5} \text{ of } £1 = 3 \times 4s. = 12s.$$

Example II. Which is the greater, $\frac{7}{8}$ or $\frac{5}{6}$?



Draw two rectangles 6 units by 4 units and divide each into 24 squares corresponding to the L.C.M. of the denominators 8 and 6. Dividing the first rectangle into 8 parts, $\frac{1}{8}$ of the rectangle is represented by 3 squares and $\frac{7}{8}$ by 21 squares. Similarly in the second rectangle $\frac{1}{6}$ of the figure is represented by 4 squares and $\frac{5}{6}$ by 20 squares. Therefore $\frac{7}{8}$ is greater than $\frac{5}{6}$.

EXAMPLES. XIV 14

Find the value of the following:—

1. $\frac{1}{3}$ of 1s., $\frac{1}{5}$ of £1, $\frac{8}{5}$ of £1, $\frac{3}{4}$ of 1 foot, $\frac{1}{10}$ of 1 min.
2. $\frac{7}{10}$ of £1, $\frac{3}{4}$ of 1 min., $\frac{1}{3}$ of 2s. 6d., $\frac{1}{8}$ of £1, $\frac{1}{9}$ of 1 yd.
3. $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{1}{12}$, $8\frac{3}{5}$, $7\frac{1}{10}$, $5\frac{3}{10}$ of £1, in £. s. d.
4. $\frac{1}{3}$, $\frac{5}{9}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{16}$ of 1 hour in min. and sec.
5. $\frac{1}{10}$, $\frac{3}{100}$, $\frac{9}{5}$, $\frac{180}{50}$ of 1 m. in cm.
6. $\frac{5}{7}$ of 4 guineas.
7. $\frac{4}{11}$ of £12. 2s.
8. $\frac{5}{32}$ of 12 lb.
9. $\frac{21}{3}$ of 4 cwt. 12 lb.
10. $\frac{2}{7}$ of £38. 12s. 11d.
11. $\frac{8}{9}$ of £15. 4s. 6d.
12. $\frac{93}{6}$ of £1000.
13. $\frac{8}{11}$ of 3 m. 2 fur. 4 ch.
14. $\frac{7}{15}$ of 3 Kg. 9 Hg. 3 gm.
15. $\frac{13}{17}$ of £9. 8s. 0 $\frac{3}{4}$ d.

16. What fractions of £1 are the following sums :—3d., 2s., 3s. 4d., 2s. 6d., 16s., 16s. 8d.?

17. Express as fractions of 1 cwt. :—8 lb., 14 lb., 27 lb., 56 lb., 3 oz.

18. Express the following as mixed numbers :—

- (i) $\frac{28}{11}$. (ii) $\frac{35}{8}$. (iii) $\frac{99}{37}$. (iv) $\frac{188}{5}$.
 (v) $\frac{891}{72}$. (vi) $\frac{1000}{71}$. (vii) $\frac{8214}{5139}$. (viii) $\frac{7156}{365}$.
 (ix) $\frac{20021}{8191}$. (x) $\frac{100000}{361}$.

19. Express the following as improper fractions :—

- (i) $1\frac{2}{7}$. (ii) $3\frac{9}{19}$. (iii) $14\frac{1}{21}$. (iv) $11\frac{5}{7}$.
 (v) $20\frac{1}{365}$. (vi) $1\frac{260}{1649}$. (vii) $71\frac{5}{93}$.
 (viii) $29\frac{51}{52}$. (ix) $51\frac{49}{711}$. (x) $253\frac{3}{7}$.

The following examples (20–29) should be worked by drawing diagrams on squared paper :—

20. Show that $\frac{1}{2} = \frac{4}{8} = \frac{8}{16}$.

21. Prove that $\frac{4}{5}$ and $\frac{8}{10}$ are the same.

22. By drawing a rectangle consisting of 48 squares, find which other fractions are the same as $\frac{4}{6}$.

23. Show that $\frac{5}{8}$ is greater than $\frac{3}{5}$.

24. Find which of the following are equivalent fractions :—
 $\frac{9}{12}$, $\frac{16}{24}$, $\frac{3}{4}$, $\frac{54}{72}$.

25. Express $\frac{4}{5}$ with denominators 10, 15, 25, 30.

26. Express as twelfths $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.

27. Express $\frac{2}{7}$ with denominators 28, 35, 49.

28. Find which is the greatest and which the least of
 $\frac{3}{4}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{7}{8}$.

29. Show that $\frac{26}{45}$ is less than $\frac{3}{5}$ but greater than $\frac{5}{9}$.

Reduce the following to their lowest terms, expressing where possible improper fractions as mixed numbers :—

30. $\frac{15}{35}$, $\frac{14}{35}$, $\frac{70}{35}$, $\frac{24}{36}$, $\frac{18}{32}$.

31. $\frac{28}{30}$, $\frac{30}{32}$, $\frac{54}{72}$, $\frac{63}{70}$, $\frac{100}{75}$.

32. $\frac{51}{34}$, $\frac{95}{38}$, $\frac{441}{210}$, $\frac{66}{121}$, $\frac{143}{52}$.

*33. $\frac{5 \times 7}{10 \times 14}$, $\frac{3 \times a}{a \times 9}$, $\frac{17 \times x}{x \times 34}$, $\frac{2^2 \times 3}{2 \times 3^2}$, $\frac{2^2 \times a}{2 \times a^2}$.

*34. $\frac{2^3 \times 5 \times 3}{3^2 \times 5 \times 2^3}$, $\frac{8 \times 5}{2^3 \times 10}$, $\frac{a^2 \times b}{b^3}$, $\frac{9 \times a^2 \times 5}{3 \times a \times 5^3}$.

35. $\frac{190}{76}$, $\frac{169}{520}$. 36. $\frac{345}{375}$, $\frac{319}{508}$.

38. $\frac{88725}{98735}$, $\frac{426}{1584}$. 39. $\frac{663}{310284}$.

41. $\frac{7076}{3782}$. 42. $\frac{16819}{10395}$.

37. $\frac{252}{1062}$, $\frac{1484}{742}$.

40. $\frac{9669}{16115}$.

§ 15. ADDITION AND SUBTRACTION OF FRACTIONS

Example I. Find the value of $7\frac{1}{24} + 3\frac{2}{3} + 1\frac{5}{6} + 1\frac{1}{2}$.

$$\begin{aligned} 7\frac{1}{24} + 3\frac{2}{3} + 1\frac{5}{6} + 1\frac{1}{2} &= 11\frac{1+16+20+12}{2 \times 2 \times 2 \times 3} \\ &= 11\frac{49}{2 \times 2 \times 2 \times 3} \\ &= 11\frac{13}{8} \\ &= 11 + 1\frac{5}{8} \\ &= 12\frac{5}{8}. \end{aligned}$$

Add the whole numbers together: express the common denominator in prime factors and perform any cancelling at the step indicated above.

Example II. Subtract $2\frac{5}{9}$ from $3\frac{1}{18}$.

$$\begin{aligned} 3\frac{1}{18} - 2\frac{5}{9} &= 1\frac{1-10}{2 \times 3 \times 3} \\ &= 1\frac{-9}{2 \times 3 \times 3} \\ &= 1 - \frac{1}{2} \\ &= \frac{1}{2}. \end{aligned}$$

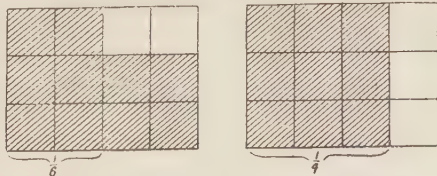
In cases of simple addition and subtraction squared paper can be used to advantage, thus:—

Example III. Find the value of—

(i) $\frac{5}{6} + \frac{3}{4}$.

(ii) $\frac{5}{6} - \frac{3}{4}$.

Draw two rectangles each 4 units by 3 units so as to obtain 12 squares in each rectangle (12 being the l.c.m. of 6 and 4).



Divide each into 12 squares. Then $\frac{1}{6}$ of the first rectangle will be two squares and $\frac{1}{4}$ of the second 3 squares.

$$\begin{aligned} \therefore \frac{5}{6} + \frac{3}{4} &= 10 \text{ squares} + 9 \text{ squares} \\ &= 19 \text{ squares}. \end{aligned}$$

But each square is $\frac{1}{12}$ of the rectangle.

$$\therefore \frac{5}{6} + \frac{3}{4} = \frac{19}{12} \\ = 1\frac{7}{12}.$$

$$\text{Similarly } \frac{5}{6} - \frac{3}{4} = 10 \text{ squares} - 9 \text{ squares} \\ = 1 \text{ square.} \\ = \frac{1}{12}.$$

EXAMPLES. XV 15

The following examples (1-5) should be worked by drawing diagrams on squared paper:—

1. Add $\frac{5}{8}$ to $\frac{1}{4}$.
2. What total length is the result of adding $\frac{1}{2}$ in., $\frac{1}{3}$ in., and $\frac{1}{6}$ in.?
3. Find the sum of $2\frac{1}{5}$ cm., $3\frac{1}{10}$ cm., 1 cm.
4. Show that $\frac{3}{16} + \frac{5}{8} = 1\frac{3}{8}$.
5. Add $\frac{1}{12}$, $\frac{5}{24}$, $\frac{1}{6}$.

Find the value of—

6. $\frac{3}{5} + 1\frac{2}{5} + \frac{9}{10}$. 7. $5 + 3\frac{1}{3} + 4\frac{1}{27}$. 8. $1\frac{3}{8} + 5\frac{4}{5} + \frac{3}{40}$.
9. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$. 10. $17\frac{1}{2} + 12\frac{5}{6} + 9\frac{1}{3}$.
11. $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000}$. 12. $17\frac{3}{17} + 1\frac{1}{34} + 2\frac{1}{51}$.
13. A boy spends $\frac{1}{3}$ of his money at one shop and $\frac{1}{6}$ at another. What fraction did he spend? If he started with 2s., how much money has he left?
14. A man walks $\frac{1}{20}$ of his journey and then rides $\frac{3}{5}$ in a train. His journey is from London to St. Albans, a distance of 20 miles. How much farther has he to go when he gets out of the train?
15. A man starts business with £1000. At the end of the first year he has increased his capital by $\frac{1}{20}$, and at the end of the second year he has increased this amount by $\frac{1}{4}$. How much has he at the end of the second year?
16. On three consecutive days a draper sells $25\frac{1}{2}$ yd., $20\frac{3}{8}$ yd., and $15\frac{1}{4}$ yd. of cloth all cut off the same roll. He finds he has $3\frac{3}{8}$ yd. left. What was the original length of the roll?

17. The prices of shares in three Rubber Companies are £ $3\frac{9}{32}$, £ $6\frac{1}{4}$, £ $2\frac{5}{8}$. How much money (in £. s. d.) will be necessary to buy a share in all three Companies?

18. The expenses in working a certain gold-mine show an increase of $1s. 5\frac{31}{100}d.$ per ton for 1912 over 1911. In 1913 there is a further increase of $\frac{19}{20}d.$ per ton over the expenses for 1912. Find the total increase per ton.

19. The fees in a certain school are £7. 10s. per term for each boy ; $\frac{2}{3}$ of the full fee for each brother. A man sends his 4 sons to the school. How much does he pay if two of the boys are at the school for 5 years and two for 6 years 1 term and they all start at the same time?

20. Prove that $\frac{1}{7} + \frac{1}{9} = \frac{16}{63}$. Use this fact to find the value of $5\frac{16}{63}$ guineas.

21. The legal weight of a penny is $\frac{1}{3}$, of a halfpenny $\frac{1}{5}$, and a farthing $\frac{1}{10}$ of an ounce avoirdupois. A man has an equal number of pence, halfpence, and farthings ; the total value is $8\frac{1}{4}d.$ What should they weigh altogether?

22. A gallon of spirit which is $\frac{3}{5}$ water is added to 3 gallons containing $\frac{3}{50}$ water in each gallon. $\frac{1}{10}$ gallon of water is added to this mixture. How much water is there in the mixture now?

23. Standard silver consists of $\frac{37}{40}$ fine silver ; Britannia silver consists of $\frac{23}{24}$ fine silver. Which is the more valuable?

24. I pay away $\frac{1}{10}$ of my money to A and $\frac{3}{16}$ to B. What fraction of my money have I paid away? What fraction have I left?

25. A man read $\frac{3}{8}$ of a book on one day, and $\frac{4}{5}$ of the remainder on another day. How much of the book did he read altogether?

The following examples (26–28) should be worked by drawing diagrams on squared paper :—

26. Find how much (i) $\frac{11}{16}$ is greater than $\frac{3}{8}$, (ii) $\frac{1}{2}$ is less than $\frac{2}{3}$.

27. Subtract (i) $\frac{11}{12}$ from $1\frac{1}{24}$, (ii) $\frac{3}{8}$ from $\frac{3}{5}$, (iii) $\frac{1}{2}$ from $2\frac{1}{16}$.

28. Find the difference between $3\frac{1}{10}$ cm. and $2\frac{7}{10}$ cm.

Find the value of—

29. $9\frac{1}{2} - \frac{1}{4}$. 30. $2\frac{1}{4} - 1\frac{1}{8}$. 31. $1 - \frac{1}{72}$. 32. $1\frac{1}{9} - \frac{1}{8}$.

33. $3\frac{2}{7} - 2\frac{1}{56}$. 34. $11\frac{1}{4} - 9\frac{5}{17}$. 35. $\frac{89}{90} - \frac{34}{35}$. 36. $\frac{93}{96} - \frac{5}{16}$.

37. Find the excess of $9\frac{2}{91}$ over $8\frac{1}{7}$.

38. What must be added to $2\frac{1}{5}$ to obtain $3\frac{1}{2}$?

39. Find which is greater, $\frac{2}{7} + \frac{5}{9}$ or $\frac{12}{11} - \frac{3}{13}$.

40. In 1 gram of bronze $\frac{1}{5}$ is tin and $\frac{9}{20}$ is copper. The rest is zinc. What fraction of a gram of bronze is zinc?

41. A boy measures off on a line 1 foot long, distances of $3\frac{3}{8}$ in., $4\frac{1}{4}$ in., $\frac{1}{2}$ in., and $2\frac{3}{4}$ in. How far is the last point of division from the end of the line?

42. The following shows the prices of shares in South African mines on April 18, 1913. Which shows (i) the greatest increase, (ii) the greatest decrease?

Mine.	Price on April 16.	Price on April 17.
African	$\frac{9}{16}$	$\frac{11}{16}$
Anglo-French	$\frac{7}{8}$	$\frac{15}{16}$
Apex	$\frac{13}{18}$	$\frac{11}{12}$
H. E. . . .	$\frac{7}{16}$	$\frac{1}{2}$
Nigel	$\frac{15}{16}$	$\frac{7}{8}$
Village Reef	$\frac{1}{4}$	$\frac{1}{8}$

43. A Canadian penny weighs $\frac{5}{18}$ of an ounce, and a Nova Scotian penny $\frac{4}{15}$ of an ounce. Find the difference in weight between (i) a Canadian penny and an English penny ($\frac{1}{8}$ of an ounce), (ii) a Canadian penny and a Nova Scotian penny.

44. Find how much greater an eleventh is than a twelfth, and hence find the value of $\frac{1}{132}$ of 11s.

45. A man after spending $\frac{2}{3}$ of his money has 17s. 6d. left. How much had he?

46. 8 shillings = $10\frac{8}{10}$ francs; a ducat = $10\frac{16}{375}$ francs. Find the difference in francs between 8 shillings and a ducat.

47. A boy weighs with a faulty balance, one scale-pan of which is $\frac{11}{15}$ gm. heavier than the other. He weighs a solid, putting it in the heavier scale-pan, and it appears to weigh $12\frac{1}{2}$ gm. What is its real weight?

48. In Roman times the carat was $\frac{1}{144}$ oz.; the modern carat is $\frac{33}{5000}$ oz. Which is the greater, and by how much?

49. Two samples of limestone contain respectively $\frac{29}{112}$ and $\frac{27}{100}$ of sand. How much more sand will there be in a ton of the latter than in a ton of the former?

50. Subtract $9\frac{43}{220}$ cm. from $120\frac{7}{80}$ cm., and give the answer in cm. to the nearest whole number.

51. Find the difference between $\frac{524}{917}$ and $\frac{561}{935}$.

Find the value of—

52. $8\frac{1}{4} + 2\frac{1}{8} - 1\frac{1}{3}$.

53. $1\frac{2}{11} + 3\frac{1}{22} - 1\frac{1}{2}$.

54. $8\frac{1}{4} + 9\frac{1}{8} - \frac{3}{16}$.

55. $10\frac{5}{8} - \frac{3}{8} + \frac{1}{7}$.

56. $5\frac{1}{8} - 2\frac{3}{4} + \frac{1}{3} + \frac{1}{4}$.

57. $\frac{5}{144} + \frac{61}{180} - \frac{1}{12}$.

58. $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6}$.

59. $1\frac{1}{2} + 3\frac{1}{12} - 4\frac{9}{24} + 5\frac{3}{16}$.

60. $7\frac{4}{441} - 3\frac{91}{210} + 2\frac{53}{490}$.

61. $3\frac{1}{2} - \frac{79}{120} - 5\frac{1}{4} + 4\frac{1}{3} - 7\frac{1}{8} + 6\frac{1}{5}$.

62. A tank is $\frac{5}{6}$ full, but after 80 gallons are drawn off it is found to be $\frac{2}{3}$ full. How many gallons will the tank hold?

63. A man spends $\frac{1}{3}$ of his money at one shop and $\frac{1}{4}$ of the rest at another. He has 5s. left. How much did he spend?

64. After writing out a cheque for his rates and insurance, which come to $\frac{1}{10}$ and $\frac{1}{15}$ of his income respectively, a man finds that he has £500 left. What does he pay in insurance?

65. One-third of a number added to half the same number is 30 : find the number.

66. A commercial traveller rides $\frac{3}{16}$ of his journey on a bicycle ; after walking $\frac{1}{15}$ of the remainder he rides the rest of his journey, amounting to 36 miles, by train. How far did he walk?

67. In a class of boys, $\frac{1}{5}$ are over 12 years of age, $\frac{1}{4}$ of the remainder are between 11 yr. 6 mo. and 12 yr., and the rest—15 boys—between 11 yr. and 11 yr. 6 mo. Find the number of boys in the form.

68. In a pole rising from the water, $\frac{1}{5}$ is in mud at the bottom of the river and $\frac{3}{7}$ is above the water. What fraction is in the water?

69. Three-eighths of a number subtracted from five-sixths of the same number is 121 : find the number.

70. Three men own a business. The first owns $\frac{51}{62}$, the second $\frac{11}{185}$, and the third man's share is £561. How much is the first man's share worth?

§ 16. MULTIPLICATION AND DIVISION OF FRACTIONS

Example I. Multiply $3\frac{1}{8}$ by $7\frac{1}{5}$.

[*Rough Check* :— $3 \times 7 = 21$.]

$$\begin{aligned} 3\frac{1}{8} \times 7\frac{1}{5} &= \frac{25}{8} \times \frac{36}{5} \\ &= \frac{5 \times 9}{2 \times 1} \\ &= \frac{45}{2} \\ &= 22\frac{1}{2}. \end{aligned}$$

When large numbers are to be multiplied together, instead of changing the mixed numbers to improper fractions we may use the following method :—

Example II. Multiply $291\frac{3}{4}$ by $5\frac{1}{11}$.

[*Rough Check* :— $291 \times 5 = 1455$.]

$$\begin{aligned} 291\frac{3}{4} \times 5 &= 1458\frac{3}{4} \dots\dots\dots (\alpha) \\ 291\frac{3}{4} \times \frac{1}{11} &= 26\frac{23}{4 \times 11} \dots\dots\dots (\beta) \\ \therefore 291\frac{3}{4} \times 5\frac{1}{11} &= 1458\frac{3}{4} + 26\frac{23}{4 \times 11} \\ &= 1484\frac{14}{11} \\ &= 1485\frac{3}{11}. \end{aligned}$$

The process is as follows :—Multiply $291\frac{3}{4}$ by 5 (α). Then multiply $291\frac{3}{4}$ by $\frac{1}{11}$; this is the same as dividing $291\frac{3}{4}$ by 11. To do this, first divide 291 by 11; the result will be 26 and a remainder 5. This remainder, $\frac{5}{11}$, will have to be added to the quotient of $\frac{3}{4}$ by 11. $\frac{5}{11}$ is the same as $\frac{20}{4 \times 11}$, and $\frac{3}{4} \div 11$ is $\frac{3}{4 \times 11}$; the sum of these two is therefore $\frac{23}{4 \times 11}$. This gives us line (β). To obtain the answer to the question add the results in lines (α) and (β).

EXAMPLES. XVI 16

By drawing rectangles on squared paper prove that—

1. $\frac{7}{20} \times 4 = \frac{28}{20} = 1\frac{7}{5}$.
2. $\frac{7}{16} \times 2 = \frac{7}{8}$.
3. $\frac{4}{7} \times 3 = \frac{12}{7} = 1\frac{5}{7}$.
4. $\frac{1}{3}$ of $\frac{3}{8} = \frac{1}{8}$.
5. $\frac{1}{5}$ of $\frac{4}{9} = \frac{20}{9} = 2\frac{2}{9}$.

Multiply—

6. $\frac{3}{5}$ by 2, 3, 5; $2\frac{1}{2}$ by 11, 12, 15; $3\frac{1}{3}$ by $1\frac{3}{10}$, $4\frac{9}{20}$, $9\frac{3}{5}$.
7. $2\frac{2}{3}$ by $\frac{5}{16}$ by $9\frac{1}{5}$.

Find the value of—

8. $\frac{17}{18} \times 1\frac{1}{6} \times \frac{56}{57}$. 9. $5\frac{2}{9}$ of $\frac{3}{4}$ of $3\frac{1}{47}$. 10. $2\frac{15}{16}$ of $2\frac{2}{3}$ of $2\frac{18}{19}$.
 11. $\frac{19}{25}$ of $3\frac{23}{24}$ of $\frac{360}{361}$. 12. $21\frac{1}{3} \times \frac{3}{8}$ of $\frac{9}{128}$ of $10\frac{2}{3}$.

By drawing rectangles on squared paper prove that—

13. $\frac{2}{3} \div 4 = \frac{2}{12} = \frac{1}{6}$. 14. $2\frac{1}{2} \div 10 = \frac{1}{4}$.
 15. $1\frac{1}{3} \div 9 = \frac{4}{27}$. 16. $\frac{1}{2} \div 5 = \frac{1}{10}$.

17. Write down the reciprocals of—

$$3, \frac{1}{5}, \frac{7}{8}, 1, 1\frac{1}{9}, 4\frac{1}{3}, \frac{7}{15}, \frac{8}{11}, 9\frac{1}{24}, \frac{1}{723}.$$

Divide—

18. $\frac{1}{2}$ by 2, 3, 4; $4\frac{2}{9}$ by 4, 5, 19; $1\frac{5}{11}$ by $1\frac{6}{7}$, $\frac{39}{42}$, $\frac{4}{5}$.

Find the value of—

19. $5\frac{1}{11} \div 16\frac{32}{33}$. 20. $92\frac{3}{16} \div 7\frac{3}{8}$. 21. $\frac{3\frac{1}{5}}{3\frac{3}{13}}$.
 22. $\frac{2\frac{7}{29}}{23\frac{7}{11}}$. 23. $\frac{156\frac{4}{25}}{25\frac{5}{12}}$. 24. $\frac{95\frac{7}{11}}{1\frac{54}{209}}$.

25. By what must $\frac{3}{4}$ be multiplied to obtain $\frac{3}{5}$?

26. $\frac{11}{18}$ when divided by a certain fraction becomes $1\frac{1}{54}$.

Find the fraction.

27. What is the value of $\frac{1}{7} \times \frac{1}{7} \times 7 \times \frac{7}{12}$?

28. A boy grew 3 in. in 1907, and then per year $\frac{3}{4}$ of what he grew in the previous year. Find his growth in 1910.

29. How many good pens are there in 237 boxes, each containing 144 pens, if $\frac{1}{16}$ of the pens in each box are not good?

30. Show (without working out the example) that $15\frac{1}{2} \div 1\frac{1}{2}$ cannot be $1\frac{7}{24}$.

31. A boy cuts off $\frac{1}{3}$ of a stick; he cuts off $\frac{1}{3}$ of the remainder, and then $\frac{1}{3}$ of what is left. What fraction of the original length has he now?

32. Find the value of $\frac{1}{1 - \frac{1}{3 - \frac{1}{2}}}$.

33. Measure to the nearest eighth of an inch the page you are writing on. Multiply the numbers together. What does your answer tell you about the page?

34. $\frac{19}{20}$ of bronze is copper. What fraction of an ounce is copper in a bronze coin weighing $\frac{15}{57}$ of an ounce?

35. A rupee = 1s. 4 $\frac{4}{15}$ d. Find the value in £ s. d. of 67 $\frac{1}{2}$ rupees.

36. Express £3. 11s. 2d. as pounds and fractions of a pound. Divide your result by $\frac{7}{15}$. Check your answer by multiplying it by $2\frac{1}{7}$.

37. How many pieces each $\frac{7}{8}$ inch long can be cut from a length of 8 ft. $0\frac{1}{4}$ in.?

38. The distance between London and Cambridge is $55\frac{1}{2}$ miles. Supposing that a man walked continuously from London to Cambridge at an average speed of $4\frac{5}{8}$ miles per hour, how long would the journey take him?

39. Express 5 cwt. 84 lb. and 14 cwt. 42 lb. each as cwt. and fractions of a cwt. Find the quotient of the first by the second in its simplest form.

40. $78\frac{3}{4}$ tons of a material are sold for £919. Express the cost per ton as pounds and fraction of a pound. Show that the fraction of a pound is about 13s. 6d.

41. A penny weighs $\frac{1}{3}$ oz., and a halfpenny $\frac{1}{5}$ oz. A man balances a letter weighing $2\frac{2}{3}$ oz. with an equal number of pennies and halfpennies. How many coins does he use?

42. Express $\frac{5}{8}$ of an inch as a fraction of a yard. Thence find the number of cubic yards of water on a field of an acre flooded to a depth of $\frac{5}{8}$ of an inch.

43. A pint of water weighs $1\frac{1}{4}$ lb. Find the weight in lb. of $14\frac{1}{8}$ gallons of water.

44. A gate swells in wet weather $\frac{7}{1370}$ of its length in dry weather. If it is $6\frac{4}{11}$ ft. long in dry weather, what is its length in wet weather?

45. How many pieces of string each $\frac{76}{200}$ in. long can be cut from a piece $\frac{9}{20}$ of a foot long?

46. A postage stamp is $\frac{15}{16}$ in. long and $\frac{3}{4}$ in. wide. Find its area; hence find how many will be required to cover a sheet of paper of area 15 square feet.

47. On a certain railway a second-class ticket costs half as much again as a third-class ticket, and a first-class ticket four-fifths as much again as a second-class ticket. The third-class fare between two stations is 14s. 2d.; find the first-class fare.

48. The commission charged for investing a sum of money is £ $\frac{3}{8}$ on every £100 invested. If the commission amounts to £35. 16s., how much money is invested?

49. A dollar = 4s. $1\frac{8}{5}$ d. Find the value in English money of 1000 dollars.

50. A tradesman makes up tea into 4 oz., 8 oz., and 1 lb. packages. The packages weigh with the paper $4\frac{1}{16}$ oz., $8\frac{1}{8}$ oz., 1 lb. $0\frac{1}{4}$ oz. He has the same number of each kind of package, and the total weight of the packages is 56 lb. 14 oz. How many packages are there?

51. Brass is $8\frac{1}{5}$ times as heavy as water. A cubic foot of water weighs $62\frac{1}{2}$ lb. Find the weight, in tons and fraction of a ton, of 11 cubic feet of brass.

52. Find the value of 153 times $8\frac{1}{17}$; also the value of 153 times $9\frac{1}{3}$. Hence find the value of 153 times $17\frac{2}{3}$.

53. The average weight of a shilling is $\frac{2}{11}$ oz. Find the weight of the number of shillings in £5 expressed as lb. and fraction of a lb.

54. An army loses $\frac{3}{5}$ of its number wounded in battle, $\frac{2}{11}$ of the remainder are stricken down by disease, and $\frac{1}{9}$ of the rest are killed. What fraction of the number of men remains?

55. Find the value of $\frac{1}{1\frac{1}{2}} \times \frac{1}{1\frac{1}{3}} \times \frac{1}{1\frac{1}{4}}$. Why must the result be less than 1?

§ 17. MISCELLANEOUS FRACTIONS

Example I. When considering expressions connected by different signs, *e.g.* $43 - \frac{3}{5} + \frac{7}{8}$ of $\frac{5}{6} \div \frac{1}{2} \times \frac{1}{7}$, perform the operations in the following order:—

- (i) Since $\frac{7}{8}$ of $\frac{5}{6}$ is one expression, simplify this;
- (ii) perform the division;
- then (iii) multiplication;
- and, finally,
- (iv) addition and subtraction.

Thus the successive steps in working out the above example are—

$$\begin{aligned}
 & 43 - \frac{3}{5} + \frac{7}{8} \text{ of } \frac{5}{6} \div \frac{1}{2} \times \frac{1}{7} \\
 &= 43 - \frac{3}{5} + \frac{35}{48} \div \frac{1}{2} \times \frac{1}{7} \\
 &= 43 - \frac{3}{5} + \frac{35}{24} \times \frac{1}{7} \\
 &= 43 - \frac{3}{5} + \frac{5}{4} \\
 &= 43 \frac{-72+25}{2 \times 2 \times 2 \times 3 \times 5} \\
 &= 43 \frac{-47}{120} \\
 &= 42 \frac{73}{120}.
 \end{aligned}$$

If, however, some expressions are connected by brackets, simplify those in brackets first,

Example II. Thus—

$$\begin{aligned}
 & 43 - \left(\frac{3}{5} + \frac{7}{8} \text{ of } \frac{5}{6} \right) \div \left(\frac{1}{2} \times \frac{1}{7} \right) \\
 & = 43 - \left(\frac{3}{5} + \frac{35}{48} \right) \div \left(\frac{1}{14} \right) \\
 & = 43 - \frac{319}{240} \div \frac{1}{14} \\
 & = 43 - \frac{2233}{120} \\
 & = 43 - 18\frac{73}{120} \\
 & = 25\frac{47}{120}
 \end{aligned}$$

The following examples on compound quantities are useful.

Example III. Find the value of $\frac{1}{2} \frac{1}{1}$ of 1 ton 3 cwt. 7 lb.

By the usual method—

$$\begin{aligned}
 & 1 \text{ ton } 3 \text{ cwt. } 7 \text{ lb.} \div 21 = 1 \text{ cwt. } 11 \text{ lb.} \\
 & 1 \text{ cwt. } 11 \text{ lb.} \times 11 = 12 \text{ cwt. } 9 \text{ lb.} \\
 \therefore \frac{1}{2} \frac{1}{1} \text{ of } 1 \text{ ton } 3 \text{ cwt. } 7 \text{ lb.} & = 12 \text{ cwt. } 9 \text{ lb.}
 \end{aligned}$$

If more convenient, the multiplication may be performed before the division.

Example IV. Reduce £3. 3s. $4\frac{1}{2}d.$ to the fraction of £5. 8s. $4\frac{1}{2}d.$

Reduce both sums to a convenient unit, in this case threehalfpences.

$$\begin{aligned}
 \frac{\text{£3. 3s. } 4\frac{1}{2}d.}{\text{£5. 8s. } 4\frac{1}{2}d.} &= \frac{507 \text{ threehalfpences}}{871 \text{ threehalfpences}} = \frac{507}{871} = \frac{39}{67}.
 \end{aligned}$$

Example V. Reduce 5 yd. 1 ft. 3 in. to the fraction of 7 yd. 0 ft. $1\frac{1}{2}$ in.

Reduce both lengths to **feet**.

$$\begin{aligned}
 \frac{5 \text{ yd. } 1 \text{ ft. } 3 \text{ in.}}{7 \text{ yd. } 0 \text{ ft. } 1\frac{1}{2} \text{ in.}} &= \frac{16\frac{1}{4} \text{ ft.}}{21\frac{1}{8} \text{ ft.}} = \frac{16\frac{1}{4}}{21\frac{1}{8}} \\
 &= \frac{65}{4} \times \frac{8}{169} \\
 &= \frac{10}{13}.
 \end{aligned}$$

The ratio of x to y is written $x : y$ or $\frac{x}{y}$ and is measured by the fraction $\frac{x}{y}$. Ratio is a mere number.

Example VI. Find the ratio of 5s. 6d. to 10s. 3d.

$$\begin{aligned}
 \text{The ratio} &= \frac{5s. 6d.}{10s. 3d.} \\
 &= \frac{22 \text{ threepences}}{41 \text{ threepences}} \\
 &= \frac{22}{41}.
 \end{aligned}$$

Example VII. Divide the number 110 in the ratio 6 : 5. If we divide 110 into (6 + 5) parts we have to take 6 parts and 5 parts.

$$\begin{aligned}\therefore \text{One part} &= \frac{6}{11} \text{ of } 110 \\ &= 60; \\ \text{other part} &= \frac{5}{11} \text{ of } 110 \\ &= 50.\end{aligned}$$

EXAMPLES. XVII 17

Simplify—

1. $3 - \frac{1}{2}$ of $\frac{4}{9}$; $(3 - \frac{1}{2})$ of $\frac{4}{9}$.
2. $4\frac{1}{4}$ of $\frac{3}{5} + \frac{9}{20}$; $(4\frac{1}{4} \text{ of } \frac{3}{5}) + \frac{9}{20}$.
3. $\frac{3}{4} \div \frac{1}{3} - \frac{5}{16}$; $\frac{3}{8} \div (\frac{1}{3} - \frac{5}{16})$.
4. $\frac{4}{9} + \frac{\frac{3}{5} - \frac{1}{6}}{\frac{1}{30}}$; $\frac{4}{9} + \frac{\frac{3}{5} \text{ of } \frac{1}{6}}{\frac{1}{30}}$.
5. $\frac{1}{2} + \frac{1}{3} \div \frac{1}{3} + \frac{1}{4}$; $\frac{1}{2} + \frac{1}{3} \div (\frac{1}{3} + \frac{1}{4})$.
6. $1\frac{3}{8} - \frac{5}{8}$ of $\frac{7}{12} \div \frac{3}{5}$.
7. $\frac{9}{17} + 3\frac{5}{11} \div \frac{1}{3}$ of $\frac{1}{5}$.
8. $1\frac{1}{2}$ of $(\frac{3}{5} + \frac{7}{20}) \div \frac{9}{19}$ of $1\frac{1}{50}$.
9. $\frac{1}{4} + (\frac{1}{5} \div \frac{1}{6}) - \frac{1}{7}$.
10. $(\frac{1}{4} + \frac{1}{5}) \div (\frac{1}{6} - \frac{1}{7})$.
11. $\frac{8\frac{2}{11} - 1\frac{8}{9} - 1\frac{3}{8} \text{ of } \frac{8}{15}}{1\frac{1}{3} + 2\frac{10}{11}}$.
12. $\frac{11}{17}$ of $21\frac{6}{7} - \frac{11}{19}$ of $10\frac{5}{9} + \frac{11}{21}$ of $1\frac{28}{33}$.
13. $(13\frac{1}{12} - \frac{5}{80} \text{ of } 10) \div (8\frac{1}{3} + 4\frac{3}{8} + \frac{7}{12})$.
14. $\frac{1}{4\frac{1}{2}} + \frac{5}{2\frac{1}{4}} - \frac{1}{22\frac{1}{2}}$.
15. $\frac{5\frac{3}{7} \text{ of } \frac{2}{57} + 1\frac{2}{3}}{3\frac{1}{4} \text{ of } \frac{4}{5} - 1\frac{4}{9}}$.
16. $\frac{1\frac{7}{8} + 1\frac{2}{3} + 5\frac{1}{2} - 4\frac{1}{15}}{12\frac{1}{2}} + \frac{5}{2}$.

17. The sum of two fractions is $\frac{3}{5}$; their difference is $\frac{1}{10}$. Find them.

18. Find the value of x when $17x = 1\frac{1}{8}$.

19. Simplify $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$.

20. $3\frac{1}{2}x = 1\frac{3}{11}$. Find x .

21. The product of two fractions is $\frac{1}{35}$; the smaller is $\frac{2}{17}$. What is the larger?

22. In copying a question which was set thus:—Simplify $\frac{4}{5}$ of $(\frac{3}{8} - \frac{1}{16})$, a boy omitted the brackets. By how much was his answer incorrect?

23. Square $\frac{1}{3} - \frac{1}{4}$. Show that the result is the same as simplifying $\frac{1}{9} - \frac{1}{6} + \frac{1}{16}$.

24. Prove that $(\frac{1}{84} - \frac{1}{81})$ is the same as $(\frac{1}{8} - \frac{1}{9})$ multiplied by $(\frac{1}{8} + \frac{1}{9})$.

25. $(\frac{17}{30} - \frac{1}{12})x = \frac{1}{18}$. Find x .

26. Show that $(\frac{1}{2} - \frac{1}{3})^2 = (\frac{1}{2})^2 - 2(\frac{1}{2})(\frac{1}{3}) + (\frac{1}{3})^2$.

27. The density D of a substance is given by $D = \frac{M}{V}$ where M is the mass and V the volume. Find D , given $M = \frac{3}{8}$ and $V = \frac{1}{18}$.

28. In the above question find V if $D = \frac{1}{3}$ and $M = \frac{1}{2}$.

29. Find M when $D = \frac{3}{47}$ and $V = 1\frac{1}{13}$.

30. Square $(\frac{1}{13} - \frac{1}{14})$.

31. Simplify $(\frac{1}{2} \text{ of } \frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{2}) \div \frac{1}{3} \text{ of } \frac{1}{3}$.

32. Correct the following statement :— $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ of £6 = £5.

Find the value of the following :—

33. $\frac{2}{3}$ of £4 ; $\frac{5}{6}$ of £7 ; $1\frac{7}{8}$ of £9. 10s.

34. $\frac{5}{7}$ of 3 guineas ; $2\frac{2}{3}$ of 18s. 6d. ; $\frac{1}{9}$ of £4. 10s.

35. 15s. 7d. $\div 8\frac{1}{2}$; £2. 19s. 6d. $\div 6\frac{3}{8}$.

36. $\frac{4}{5}$ of 6 cwt. 28 lb. ; $1\frac{1}{9}$ of 5 cwt. 1 qr. 1 lb. 8 oz.

37. 1 ton 8 cwt. 1 qr. $\div \frac{4}{13}$. 38. £166. 10s. $\div 1\frac{5}{12}$.

39. 9s. $1\frac{1}{2}d.$ $\div 4\frac{9}{16}$. 40. 9s. $1\frac{1}{2}d.$ $\times 4\frac{1}{2}$.

What fraction is the first of the following quantities of the second ?

41. 13s. 4d., £1.

42. 16s. 8d., £5.

43. 18s. 4d., 10s. 6d.

44. 32.5 Dm., 5.75 m.

45. £2. 15s., £1. 10s. 6d.

46. 2 ft. $7\frac{1}{2}$ in., 1 yd.

47. 3 min. 12 sec., 5 min. 20 sec.

48. 1 ton 4 cwt., 15 cwt. 1 qr. 20 lb.

49. £12. 19s. $7\frac{1}{4}d.$, £39. 17s. $5\frac{1}{2}d.$

50. 10 cwt. 14 lb., 2 tons.

51. Find the ratio of 3 cwt. 1 qr. to 5 tons.

52. Show that the ratio 7 : 25 is equal to the ratio 56 : 200.

53. Express in its simplest form the ratio of 3 Km. 5 Hm. to 17 Km. 5 Hm.

54. What decimal represents the ratio of $\frac{7}{8}$ yd. to $\frac{2}{13}$ yd. ?

55. The ratio of two sums of money is 3 : 5. The smaller sum is £10. 10s. Find the larger sum.

56. $A : B = 2.9 : 3$. Find $B : A$ correct to 3 decimal places.

57. Find the ratio of a pound avoirdupois to a pound Troy.

58. Divide £350 among two people in the ratio 4 : 3.

59. A stick of length 5 ft. 3 in. is divided into two parts. The length of one part is 3 ft. 6 in. Express in its simplest form the ratio of the lengths of the two parts.

60. On a certain railway the return fare by an excursion train is in the ratio 5 : 4 to the ordinary single fare. What is the excursion fare from London to Chorley, the ordinary single fare being 16s. 10d.?

61. There are 28.35 grams in an ounce. Show that the ratio of a pound to a kilogram is approximately 9 : 20.

62. A tradesman's receipts fall off by $\frac{1}{23}$ in 1912 from what they were in 1911. Find the ratio of his receipts in the two years.

63. The marks for an examination paper are increased so that the possible total becomes 120 instead of 100. How many should a boy get who received 55 at first?

64. The total number of employees in textile factories in the United Kingdom in the years 1896–1907 was 1087236. The ratio of males to females was 10 : 17. How many of each were there?

65. The average weekly wage of railway employees was increased by 1s. $0\frac{3}{4}$ d. in 1911 over the wage for the previous year. Find the ratio of the wages for 1910 and 1911 if the average for 1911 was 27s. $4\frac{1}{4}$ d.

66. Two lengths are in the ratio of 17 : 22. The sum of the lengths is 1 yd. 3 in. : find the shorter.

67. Standard gold contains pure gold and alloy in the ratio 11 : 1. How much pure gold is there in a sovereign, the weight of which is 123.27 grains?

68. After paying his income-tax a man finds that for every pound he earned he has now 19s. $5\frac{1}{4}$ d. Find the ratio of his income after paying income-tax to his income before paying it.

69. 18.15 gold krone = £1. Find the ratio of (i) a gold krone to a shilling and (ii) a shilling to a gold krone. (Express answer as a decimal.)

70. Two numbers are such that the first is half as big again as the second. Find the ratio of the second to the first.

71. The times of departure of two trains from London to Rugby are 8.45 p.m. and 8.50 p.m. respectively. The first train arrives at Rugby at 10.25 p.m. If their times on the journey are in the ratio 25 : 27, at what time does the second arrive?

72. The weekly wages of two men in an office are £1. 0s. 3d. and £1. 1s. 3d. Find by how much the ratio of their wages is altered by increasing their wages 1 shilling weekly.

73. A sum of money is divided among three persons in the ratio 4 : 7 : 11. Show that each will get the same amount if the money is divided in the ratio 52 : 91 : 143.

74. A ferro-concrete water-tank is 28 ft. 6 in. long by 13 ft. 3 in. broad by 2 ft. deep. Find the ratio of (i) length to breadth, (ii) depth to breadth, correct to two places of decimals.

75. In a mixture of coffee and chicory the weight of chicory is 12 cwt. 3 lb. The two substances are in the ratio 17 : 3. What is the weight of the mixture?

76. The numbers of men and women in Training Colleges in England and Wales in 1911 were 1982 and 3890 respectively. In the previous year the corresponding numbers were 2013 and 3842 respectively. Find whether the ratio of the number of men to the number of women decreased or increased in 1911.

77. 3 pennies weigh as much as 5 halfpennies, and 2 farthings as much as 1 halfpenny. Show that the ratio of the weights of a penny, halfpenny, and farthing is 10 : 6 : 3.

78. A certain brand of concrete contains cement, sand, and ballast in the ratio of 2 : 3 : 5. How much sand is there in 1 cwt. 98 lb. of concrete?

79. The profits of a company were 81000 dollars in 1898 and 108000 dollars in 1899. By what fraction was the profit in 1898 increased?

80. In an engineer's plan the scale is 1 cm. to a foot. Express this as a decimal correct to two places, given that 2.54 cm. = 1 in.

81. A ton of Dead Sea water contains 187 pounds of salt. Express the ratio of the salt to water in the form of a decimal correct to three places.

82. Genuine milk contains water, fat, and non-fatty solids in the ratio 32:1:3. Find, to the nearest pint, how much water a purchaser pays for when buying 7 gallons of milk.

83. A and B are partners in a business; they contributed £7500 and £1300 respectively to the capital. Out of the yearly profit A's share amounted to £375: what was B's share?

84. A cwt. of moist sugar cost 17s. 9d. in 1902 and 17s. in 1907. The ratio of the costs of a cwt. in 1902, 1907, 1912 is 71:68:83. Find the cost (to the nearest penny) of a pound of sugar in 1912.

85. Divide £111. 8s. 11½d. among A, B, and C in the ratio of 1:2:½.

86. Bronze contains copper, tin, and zinc in the ratio 95:4:1. How much tin is there in a sovereign's worth of pennies each weighing ½ oz.?

87. Find $(\frac{1}{3} + \frac{2}{5} + \frac{3}{10})$ of £3. Point out what is faulty in this working:— $(\frac{1}{3} + \frac{2}{5} + \frac{3}{10}) = \frac{10+12+9}{2 \times 3 \times 5} = \frac{31}{30}$ of £3 = £3. 2s.

§ 18. MISCELLANEOUS DECIMALS

Expression of decimals as vulgar fractions and vice versa.

Example I. Express 3·725 as a vulgar fraction.

$$\begin{aligned} 3\cdot725 &= 3\frac{725}{1000} \\ &= 3\frac{29}{400}. \end{aligned}$$

Example II. Express the following vulgar fractions as decimals:— $4\frac{1}{6}$, $4\frac{1}{7}$, $4\frac{1}{3}$.

$$4\frac{1}{6} = 4\cdot0625.$$

$$4\frac{1}{3} = 4\cdot3333....$$

$$4\frac{1}{7} = 4\cdot14285714285714.....$$

Sometimes it is convenient to express decimals correct to a certain number of places. For example, correct to 5 places—

$$4\frac{1}{3} = 4\cdot33333.....$$

$$4\frac{1}{7} = 4\cdot14286.....$$

Decimalisation of Money. We take as our unit 2s.

For 2s. = £1.

∴ 1s. = £05.

6d. = £025.

¼d. = £001½.

Example III. Express £5. 14s. 7½d. in pounds.

$$\begin{aligned}\text{£}5. 14s. 7\frac{1}{2}d. &= \text{£}5 + 14s. + 6d. + 5 \text{ farthings} \\ &= \text{£}5 + \text{£}7 + \text{£}0.25 + \text{£}0.05\frac{5}{24} \\ &= \text{£}5.730\frac{5}{24} \\ &= \text{£}5.730208333\ldots\end{aligned}$$

The above result is **exact**: if the answer is required **correct to 3 decimal places**, we may call each farthing £0.01, adding 1 to the third decimal place for each farthing between 12 and 23.

Example IV. Express £17. 15s. 10¼d. in pounds correct to 3 decimal places.

$$\begin{aligned}\text{£}17. 15s. 10\frac{1}{4}d. &= \text{£}17 + 14s. + 1s. + 6d. + 17 \text{ farthings} \\ &= \text{£}17 + \text{£}7 + \text{£}0.5 + \text{£}0.25 + \text{£}0.18 \\ &= \text{£}17.793.\end{aligned}$$

The above result is only **approximately correct**. We must not use the above answer to find the value of, say, 50 articles at £17. 15s. 10¼d. each: the answer would be incorrect, the error being more than a penny.

Example V. Find the cost of 500 articles at £1. 14s. 9d. each.

$$[\text{Rough Check :—} 500 \times \text{£}1\frac{3}{4} = \text{£}875.]$$

$$\begin{aligned}\text{Cost} &= 500 \times \text{£}1. 14s. 9d. \\ &= 500 \times \text{£}1.7375 \\ &= 5 \times \text{£}173.75 \\ &= \text{£}868.75 \\ &= \text{£}868. 15s. 0d.\end{aligned}$$

Example VI. Find, correct to the nearest farthing, the value of 700 articles at £1. 2s. 4¼d.

$$[\text{Rough Check :—} 700 \times \text{£}1.1 = \text{£}770.]$$

Since we require the result correct to farthings, we must have 3 figures before the decimal point and 3 after, *i.e.* 6 significant figures in all. We must have 1 figure extra for safety. Hence we must express £1. 2s. 4¼d. **correct to 7 significant figures** (see p. 129).

$$\begin{aligned}\text{Cost} &= 700 \times \text{£}1. 2s. 4\frac{1}{4}d. \\ &= 700 \times \text{£}1.1017708333\ldots \\ &= 700 \times \text{£}1.101771 \text{ (correct to 7 significant figures)} \\ &= \text{£}771.2397 \\ &= \text{£}771. 4s. 9\frac{1}{2}d.\end{aligned}$$

EXAMPLES. XVIII 18

Express as vulgar fractions—

1. $\cdot 3$, $\cdot 03$, $3\cdot 3$, $3\cdot 003$.
2. $\cdot 15$, $\cdot 015$, $3\cdot 015$, $3\cdot 0015$.
3. $5\cdot 2$, $6\cdot 5$, $6\cdot 05$, $\cdot 002$.
4. $3\cdot 0125$, $7\cdot 75$, $82\cdot 025$, $82\cdot 02$.
5. $\cdot 0016$, $\cdot 008$, $\cdot 00165$, $\cdot 0085$.
6. $\cdot 27$, $\cdot 275$, $8\cdot 0625$, $4\cdot 03125$.

Express as decimal fractions—

7. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$.
8. $2\frac{3}{10}$, $2\frac{3}{100}$, $2\frac{3}{1000000}$, $2\frac{1}{5}$, $2\frac{3}{5}$.
9. $3\frac{7}{20}$, $4\frac{51}{200}$, $5\frac{183}{300}$.
10. $1\frac{1}{8}$, $2\frac{3}{4}$, $3\frac{5}{16}$.
11. $5\frac{3}{25}$, $4\frac{3}{125}$, $5\frac{3}{625}$.
12. $2\frac{50000}{100000}$, $21\frac{37}{125}$, $8\frac{251}{2500}$.

Express as decimals correct to five decimal places—

13. $\frac{1}{3}$, $\frac{1}{9}$, $\frac{2}{3}$, $\frac{4}{9}$.
14. $\frac{2}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, $1\frac{1}{7}$.
15. $\frac{1}{13}$, $\frac{3}{13}$, $\frac{5}{13}$, $\frac{7}{13}$.
16. $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{12}$, $\frac{7}{12}$.
17. $\frac{3}{47}$, $\frac{2}{29}$, $\frac{13}{17}$.
18. $\frac{512}{611}$, $\frac{201}{202}$, $\frac{304}{761}$.

Express the following sums of money as decimals of £1 :—

19. 15s., 17s. 6d., 3d.
20. 4s. 3d., 5s. 9d., 7s. 1½d.
21. £3. 4s. 3d., £7. 18s. 4½d.
22. £2. 15s. 10½d., £17. 3s. 1½d.
23. £4. 0s. 4½d., £73. 15s. 0¾d.

Express the following sums of money as decimals of £1 correct to 3 places :—

24. £3. 4s. 5¼d., £2. 9s. 2¼d.
25. £6. 14s. 9½d., £17. 2s. 11d.
26. £1. 19s. 11¾d., 4s. 7¼d.
27. 18s. 2¼d., 19s. 3½d.
28. £20. 14s. 9¾d., £2. 15s. 10¾d.

Find correct to the nearest penny the value of—

29. £·0752; £·312; £·0085.
30. £·71531; £·9156; £·99812.
31. 1·213 of 2s. 6d.; 5·318 of £7. 10s.
32. 2·56 of £1. 1s.; 3·7129 of £10. 10s.
33. ·762 of 13s. 4d.; 127·38 of 5s. 4½d.

Find the cost of—

34. 1000 articles at £2. 5s. each.

35. 700 articles at 3s. 6d. each.

36. 500 books at 2s. 4½d. each.

37. 800 frames at 1s. 3d. each.

38. 1100 glasses at 4½d. each.

39. 300 yards of silk at 4s. 4½d. per yard.

40. 1200 boxes of matches at 1½d. per dozen boxes.

Find correct to farthings the value of—

41. 400 articles at £1. 3s. 0½d. each.

42. 700 articles at £2. 14s. 7¼d. each.

43. 1000 articles at 16s. 0¾d. each.

44. 700 articles at £14. 11s. 2¼d. each.

45. To how many significant figures must we express our money in order to obtain the value of $4000 \times £21. 11s. 2¼d.$ correct to farthings?

46. Express £4. 11s. 9½d. as the decimal of £1. Then write down the sum expressed as the decimal of (i) £100, (ii) 2s.

47. The value of a rupee is 1s. 4¾d. Express this as the decimal of a pound, and thence find correct to farthings the value of 60,000 rupees in English money.

48. The average cost of coal per ton in 1907 was 17s. 0½d. How much would a man who bought 500 tons have to pay on an average? Give your answer correct to the nearest penny.

49. Find the error in calculating the value of 700 times 3s. 4¼d. by taking a farthing = £·001 instead of its exact value.

50. The current crop estimated by a Tea Company for 1913 was 1728000 lb. at 3·79d. per lb. Find the sum to which this would amount.

51. A tradesman sells 153 articles for £59. Find the average price of each article (i) omitting farthings, (ii) to the nearest farthing.

52. Find to how many significant figures £31. 0s. 11½d. must be expressed in order to find the value of (i) 500, (ii) 50000, (iii) 5 articles at £31. 0s. 11½d. each.

53. A meeting was attended by 300 men, and the expenses were £3. Show that this amounts to £·01 each, *i.e.* to 2½d. per man (nearly). If they gave 3 cheers for the King, would this be ·01 of a cheer per man? Give reasons.

§ 19. AVERAGES

The average of a number of numbers = $\frac{\text{sum of numbers}}{\text{number of numbers}}$.

Example 1. Find the average of the numbers 273, 285, 280, 276, 282.

First Method—

$$\begin{aligned}\text{Average} &= \frac{273 + 285 + 280 + 276 + 282}{5} \\ &= \frac{1396}{5} \\ &= 279.2.\end{aligned}$$

Second Method—

The average is obviously about 280.

The numbers given differ from 280 by

$$-7, +5, 0, -4, +2.$$

The total difference is -4 .

$\therefore$ The average difference is $-4 \div 5$, *i.e.* $-.8$.

$\therefore$ The average of the numbers is $280 - .8$

$$= \underline{\underline{279.2.}}$$

Note.—It is useful to remember that the average of a set of numbers must be less than the greatest and greater than the least of the numbers.

Exercises.

1. Draw a triangle, sides 7 cm., 8 cm., and 9 cm. Measure each of the angles. The results being put on the blackboard, see how near the sum of the average values for each angle is to 180° .

2. Having drawn a triangle sides 5.9 cm., 6 cm., and 7 cm., find the area; find the average value of the results obtained by all the boys in the form.

3. Find the average age of the boys in the form.

4. Represent graphically a boy's position in class for a term, and "hit off" with the eye his average position.

EXAMPLES. XIX 19

1. Find the average of the following sets of numbers :—

- (i) 87, 93, 92, 88, 95. (ii) 100·3, 101·5, 107·8, 99·4.
 (iii) ·03, ·05, ·08, ·04, ·03.

2. There are 6 clerks in an office, and their weekly wages are £2. 10s., £2. 5s., £2. 5s., £1. 17s., £1. 10s., and £1. 1s. What is the average wage?

3. In a class of 20 boys there are 4 boys aged 12 yr. 1 mo., 3 aged 12 yr. 4 mo., 3 aged 11 yr. 10 mo., 2 aged 12 yr. 5 mo., 2 aged 11 yr. 11 mo. The ages of the others are 11 yr. 5 mo., 11 yr. 6 mo., 11 yr. 8 mo., 12 yr., 12 yr. 2 mo., 13 yr. Find the average age to the nearest month.

4. What is the average height of 4 men whose heights are 5 ft. 7 in., 5 ft. 8 in., 5 ft. 11 in., 6 ft. 1 in.? Give your answer correct to the nearest inch.

5. The average of 5 numbers is 116. Four of the numbers are 112, 120, 118, 117. Find the remaining number.

6. A train leaves Euston at 6.10 p.m. punctually every day. The times of arrival at Northampton during six days in April 1913 are 8.11 p.m., 8.10 p.m., 8.15 p.m., 8.10 p.m., 8.10 p.m., 8.16 p.m. Find the average time taken on the journey.

7. During his holidays a boy cycles on an average 18 miles a day. Find the total distance he rides during 3 weeks (excluding Sundays).

8. The barometer readings on the seven days commencing April 8, 1913, were 30·11, 30·18, 30, 29·63, 30·05, 30·14, 30·07. Find correct to two places of decimals the average reading.

9. The average age of four men is 55 : the average of three of them is 52. What is the age of the fourth man?

10. The contents of each box in a packet of a dozen boxes of matches are counted, and the numbers of matches are found to be 62, 68, 65, 63, 63, 65, 60, 68, 67, 62, 70, 75. How many boxes are there containing more than the average?

11. A man purchased 15 books at 2s. 6d. each and 8 at 2s. 4½d. each. What is the average cost of a book? Give your answer to the nearest penny.

12. In a cricket match the average score of a batsman was 23. The total score was 271. Find the number of extras.

13. The yearly price of silver per standard troy ounce in the London market during the years 1902–1911 may be taken as $24\frac{1}{8}$, $24\frac{3}{4}$, $26\frac{3}{8}$, $27\frac{13}{16}$, $30\frac{7}{8}$, $30\frac{3}{16}$, $24\frac{3}{8}$, $23\frac{11}{16}$, $24\frac{11}{16}$, $24\frac{9}{16}$, all the amounts being reckoned in pence. What is the average price?

14. Find by how much the average length of 9.75 cm., 9.82 cm., 9.8 cm., 9.72 cm., 9.06 cm. is increased by adding 1 mm. to each length.

15. The dividends paid by a certain company during the years 1899–1904 are 108000, 127623.07, 166750, 213739.22, 269277.98, 284890, the amounts being given in dollars. Find the average dividend to the nearest cent.

16. The quantity of tobacco consumed in France in 1910 amounted to 42000 tons, an average of 2.35 lb. for each person. What is the population of France?

17. The following different values for the density of the Earth have been found:—5.45, 5.53, 5.49, 5.53, 4.7. Omitting the last value as differing too much from the others, find the average value for the density.

18. Three men start a business in partnership and put in respectively amounts of £550, £525, £485. A fourth man wishes to join, and is allowed to do so, on condition that his contribution is the average of the others. What is now the amount of money invested in the business?

19. A man owns a row of houses. He lets 8 of them at a yearly rental of £38, 10 at £42, and 18 at £45. What is the average yearly rental of each house?

20. Ten boys weigh a flask, and their results give an average of 16.05 gm. Nine of the boys give up their results as follows: 15.95 gm., 16.2 gm., 16.1 gm., 16.05 gm., 15.9 gm., 15.9 gm., 16 gm., 16 gm., 16.35 gm. What was the other boy's result?

21. There are 17 forms in a school. Of these, 7 contain 30 boys, 4 contain 29 boys, 3 contain 28 boys, and there are 3 forms with 27, 25, and 19 boys respectively. Find the average number of boys in a form.

22. The value of the general trade of the United Kingdom (merchandise only) in 1906 was £1068566318, and in 1907 £1163785109. The total value from 1908–1911 was £4593349931. By how much is the value in 1907 above the average for the years given?

23. The following is an extract from a time-table showing the times of the trains on the London and North-Western Railway between Birmingham and Manchester:—

Birmingham.	Manchester.	Birmingham.	Manchester.
6 0 a.m.	9 28 a.m.	12 5 p.m.	3 13 p.m.
7 15 „	10 0 „	2 20 „	4 50 „
8 30 „	10 30 „	3 30 „	6 5 „
9 10 „	11 22 „	4 55 „	7 10 „
9 15 „	12 10 p.m.	5 50 „	8 5 „
9 55 „	12 30 „	8 0 „	10 46 „
11 15 „	1 35 „		

Find the average time taken on the journey.

24. The value of the coal produced in the five years 1907–1911 is as follows:—

1907 . .	£12053000	1910 . .	£10840000
1908 . .	£11660000	1911 . .	£11079000
1909 . .	£10628000		

Find the average value. By how much must the value in 1911 be increased so that the average for the five years may be £11300000?

25. A boy's scores during a cricket season are 17, 0, 25, 3, 35, 19, 20, 0, 5, 28, 39, 51, 50, 0, 1, 27. What is his average? Show that to increase his average by 1 he should have scored one run more in each innings during the season.

26. The number of marks current for a sovereign during six days has the following values: 20·42, 20·42, 20·42, 20·40, 20·48, 20·44. A merchant exchanges £500 at the average rate for the week. Find how much he would have gained or lost by exchanging his money at the first rate given.

27. Find the average excess in the value of imports into China over the exports, during the years 1906–1909.

	1906	1907	1908	1909
Imports (£)	67523618	67664222	52600730	54477665
Exports (£)	38916838	42961863	36888050	44139689

28. The following are the amounts of rain in inches which fell during 1898 in Glasgow: 2.90, 1.48, 2.27, .79, 2.13, 1.97, 2.83, 2.56, 2.99, 1.86, 2.62, 4.57. What was the average monthly rainfall?

29. From the adjoining table find the extent in acres and the population of Glamorganshire:—

Counties.	Extent in Acres.	Population.
Anglesey	120417	35359
Brecknockshire	473080	56370
Cardiganshire	595412	80769
Carmarthenshire	465226	151050
Carnarvonshire	322742	141767
Denbighshire	378309	136810
Flintshire	106878	69722
Glamorganshire	...	...
Merionethshire	524035	60280
Montgomeryshire	591999	62201
Pembrokeshire	357419	84874
Radnorshire	238663	17505
Average	395893	168946

30. A has three plantations of trees which B wishes to buy. A considers that the trees in the first plantation are worth 2s. each, those in the second 1s. 6d., and those in the third 1s.; he charges B the average of these three prices. When, however, the trees are counted there are found to be 200 in the first plantation, 100 in the second, and 300 in the third. Taking the 200 at 2s. each, the 100 at 1s. 6d., and the 300 at 1s., B finds that he has paid too much. What should the average price have been?

31. A train leaves Paddington at 8.45 a.m. for Cardiff. At 9.25 it arrives at Reading, a distance of 37 miles: Swindon, 77 miles from London, is passed at 10.15. Find the average speed during the run to Swindon, and if the train running at this speed reaches Cardiff at 11.35, find how far Cardiff is from Paddington.

32. The prices of flour per sack during the twelve months in 1804 were 49s. 1½d., 44s. 9½d., 44s. 10¾d., 44s. 9¼d., 44s. 9¼d., 44s. 10¼d., 49s. 4¼d., 63s. 2d., 63s. 11¾d., 71s. 4¾d., 87s. 8½d., 98s. 3½d. Find the average price per month.

§ 20. UNITARY METHOD

Example I. The cost of 11 m. 5 cm. of material is 38 fr. 40 c. Find correct to the nearest cm. the length of material that can be bought for 100 fr.

$$\begin{aligned}
 &38\cdot4 \text{ fr. is the cost of } 11\cdot05 \text{ m.} \\
 &\text{Then 1 fr. } \left(\frac{1}{38\cdot4} \text{ of } 11\cdot05\right) \text{ m.} \\
 &\therefore 100 \text{ fr. } \left\{100 \times \left(\frac{1}{38\cdot4} \text{ of } 11\cdot05\right)\right\} \text{ m.} \\
 &\therefore \text{Length of material} = \frac{100 \times 11\cdot05}{38\cdot4} \text{ m.} \\
 &= \frac{1105}{38\cdot4} \text{ m.} \\
 &= 28\cdot776\dots \text{m.} \\
 &= 28 \text{ m. } 78 \text{ cm.} \\
 &\quad \text{(correct to the nearest cm.)}
 \end{aligned}$$

It is advisable to keep the brackets in the first two steps as a clear indication of the method.

Example II. The speed of a train is 36 miles per hour. How many feet per second is this?

$$\begin{aligned}
 &\text{In 1 hr. the train covers 36 miles.} \\
 &\text{i.e. in 3600 sec. } 36 \times 5280 \text{ ft.} \\
 &\text{Then in 1 sec. } \left(\frac{1}{3600} \text{ of } 36 \times 5280\right) \text{ ft.} \\
 &\therefore \text{Speed} = \frac{36 \times 5280}{3600} \text{ feet per sec.} \\
 &= 52\cdot8 \text{ ft. per sec.}
 \end{aligned}$$

Example III. If 4 men can do a piece of work in 52 days, how long should 13 men take to do the same?

$$\begin{aligned}
 &4 \text{ men do the work in 52 days.} \\
 &\text{Then 1 man does } (4 \times 52) \text{ days.} \\
 &\therefore 13 \text{ men do } \left\{\frac{1}{13} \text{ of } (4 \times 52)\right\} \text{ days.} \\
 &\therefore \text{Time} = \frac{4 \times 52}{13} \text{ days} \\
 &= 16 \text{ days.}
 \end{aligned}$$

EXAMPLES. XX 20

1. A man walks at the rate of 4 miles per hour. How long will it take him to walk 15 miles?

2. Eggs cost 5 for 3*d.* How much should be paid for 100?

3. Eggs cost 5 for 3*d.* How many can be bought for 8*s.* 3*d.*?

4. A piece of wire 4 in. long weighs 55 grains. Find the weight of 3 ft. of the wire.

5. A bankrupt pays 3s. 5d. in the £. How much will a creditor receive to whom he owes £56. 10s.?

6. On a certain day 25 fr. are of the same value as 20s. What is the value, in fr. and c., of £7. 10s.?

7. Sound travels at the rate of 1140 ft. per sec. How long after the shot is fired will the report be heard by a man standing 1000 yd. away from the gun?

8. If 10 metres = 393·7 in., find the number of feet and inches in a kilometre.

9. How much ribbon at 2 fr. 5 c. per m. can be bought for 18 fr.?

10. On a map a square mile is represented by ·01 sq. in. What will be the real area of a district represented on the map by 8·35 sq. in.?

11. A train travels at the rate of 60 miles per hour. Express its speed in ft. per sec.

12. 1·86 sq. in. = 12 sq. cm. Find the number of sq. in. in 54 sq. cm.

13. A man works 63 hr. per week, for which he is paid £2. 2s. He is away one week for 27 hr. How much ought he to be paid?

14. On a certain railway the fare for a distance of 360 miles is 29s. 3d. What should be the cost for a journey of 400 miles?

15. Matches cost $1\frac{3}{4}$ d. per dozen boxes. Find the cost of 100 gross of boxes.

16. Zeppelin airships carry passengers from Berlin to Düsseldorf, a distance of 400 miles, for £10. What is the cost per mile?

17. In 1910 Continental countries produced 52 millions of tons of beetroot; from this 8 millions of tons of sugar were made. How many pounds of sugar (to the nearest whole number) does a ton of beetroot give on an average?

18. 100 litres of oxygen, at standard temperature and pressure, weigh 142·9 gm. Find (correct to two places of decimals) how much space a Kg. of oxygen will occupy.

19. A glacier was observed to flow 4.9 in. in 24 hr. Express the average velocity (i) in ft. per hr., (ii) in ft. per yr. (Take a year = 365 days.)

20. Look on your rule to see the nearest number of mm. to a foot. Then give the number of mm. in 5 ft. $7\frac{1}{2}$ in.

21. One cwt. of sugar costs a tradesman £1. 2s. 2d. He spends £25. 9s. 10d. : how much sugar did he buy?

22. A watch loses $3\frac{1}{2}$ min. in a week. It is put right at midday on Wednesday ; what will be the time by it on Friday at midday?

23. 22 lb. = 10000 gm. How many Kg. are there in a ton?

24. A tradesman buys two kinds of wine, one at 1s. 8d. per qt. and the other at 2s. 1d. per qt. He sells the first at 2s. 6d. per qt. : at what price should he sell the second so as to gain at the same rate?

25. At 9d. in the £ a man pays £50 income-tax. What is his income?

26. Bronze contains 95 parts of copper to every 4 parts of tin. How much copper is there in a bronze medal containing 17 oz. of tin?

27. 20s. = 4.867 Canadian dollars. What is the value, to the nearest penny, of 100 dollars?

28. It is said that the coast of Sweden rises 52 in. in a century. How many feet is this per year?

29. A man employs 16 men to build a wall ; they take $4\frac{1}{2}$ days to do it. How long would it have taken if he had employed 2 more men?

30. On a duplicating machine a boy can roll off 100 copies in half an hour. How long will it take him to do 144 copies? If two other boys had two similar machines, how long would it have taken the three to roll off the same number of copies?

31. On a house, the rent of which is £42, the occupier pays £6. 15s. rates. He moves into a larger house the rent of which is £49 ; how much more does he pay in rates?

32. The pumping capacity of a motor fire-engine is 570 gall. per min. Working at this rate it took $2\frac{1}{2}$ hrs. to put out a fire. How much water was used?

33. A man flying an aeroplane left Dover at 12.40 p.m. and landed at Cologne at 4.58 p.m. The distance between the two places is 245 miles. What was his average speed in miles per hr.?

34. 3 tons 8 cwt. of coal cost £3. 8s. 10d. Find the cost per cwt. to the nearest penny.

35. A certain Urban Council received a loan of £10663 for the purpose of erecting 50 houses. If the loan be paid back in rent at 7s. 6d. per week, how long will it take before the Council is out of debt?

36. Iron slabs $\frac{5}{8}$ in. thick cost 2½d. per sq. ft. What should be the cost, at the same rate per sq. ft., of slabs $\frac{3}{4}$ in. thick? The published price for the latter is 5d.; is this greater or less than the price should be at the above rate?

37. The standard weight of a sovereign is 123.27 grains. A cashier at a bank receives a number of new sovereigns and finds that they weigh, to the nearest grain, 12204 grains. How many sovereigns were there?

38. A contractor employs 30 men on a building operation; they take 58 days in which to do it. How many more men should he have had to employ in order that the work might have been completed within 49 days?

39. 5 gall. occupy .803 cu. ft. A cubical tank has a side of length 10 ft. Find, to the nearest whole number, the number of galls. it will contain.

40. A chemist buys 28 lb. of a certain powder for £9. 10s. and sells it at 6d. per oz. packet. How much does he gain?

41. How much would a man pay for 13 articles catalogued at £7. 18s. 2d. per gross?

42. A block of wood 15 cm. by 20 cm. by 5 cm. weighs 1620 gm. What is the weight of wood left, after a piece 7.5 cm. by 8.5 cm. by 5 cm. is cut from the block?

43. 25.22 Greek drachmas are equivalent in value to 20.43 German marks. What is the value, to the nearest drachma, of 1000 marks?

44. In a certain examination a candidate received 2038 marks out of a possible total of 2800. In the published list it was stated that his marks, reckoning 100 as a maximum, were 73. Show that this is correct to the nearest whole number.

45. The internal dimensions of a water tank are 28 ft. 6 in. by 32 ft. 6 in. by 2 ft. It has a capacity of 8700 gallons. How many galls. are there in a cu. ft.?

46. In the quarterly report of a certain company it was stated that during March 8015 barrels were sold, realising 76560 dollars, as compared with 6529 barrels and 60641 dollars in February. Find whether the average selling price was the same in both months.

47. In April 1912 the gold output of New Zealand was 35590 oz. of the value of £140552. At this rate what is the value of 1000 oz. of gold?

48. The assets of a bankrupt amount to £752. 8s. ; he pays 4s. $8\frac{1}{2}d.$ in the £. Find to the nearest penny the amount of his debts.

49. In working a mine the expenses per ton of ore treated amounted to £1617, being 3s. $6\frac{3}{4}d.$ per ton. What weight of ore was treated?

50. In Cumberland the total expenditure for six schools amounted to £11766, of which £4390 was provided by the county rate. What sum does the county provide per £100 expenditure?

51. If 16·375 tons of a certain chemical cost £8. 2s. 6d., what will 582 tons cost?

52. 32 knots are equivalent to 36·85 miles per hour. How many knots is a ship sailing if it covers a distance of 12·25 miles per hour?

53. In a pigeon race the winning bird travelled at the rate of 1260 yd. per min. What was its speed in miles per hour?

54. An electric fan 14 in. in diameter revolves at the rate of 800 revolutions per min. Half a pint of methylated spirit is used up in 10 hr. in the working of the fan. What decimal fraction of a pint is used per revolution? What is the length of the circumference of the circle traced out by the fan? (Circumference of a circle = $2\pi \times$ radius.)

55. The price of crude oil at Baku is 40 copecks per pood. 288740 poods = 4657 tons, and 100 copecks = 2s. 1d. Find the price per ton in English money.

56. A South American company proposed to lay a railway between two towns 22 Km. apart at a cost of £95500. How much will this cost them per mile? (1609 m. = 1 mile.)

§ 21. AREAS AND VOLUMES.—II

We shall treat here of more difficult examples than on pp. 36-40.

In the English system of measurement, areas and volumes are generally given in square feet and inches, and cubic feet and inches: occasionally examples may arise in which other units are needed.

Example I. Find the area to be papered in a room 16 ft. 3 in. long, 14 ft. 6 in. wide, and 12 ft. high.

$$\begin{aligned}\text{Area of walls} &= \{2(16\frac{1}{4} \times 12) + 2(14\frac{1}{2} \times 12)\} \text{ sq. ft.} \\ &= 2(16\frac{1}{4} + 14\frac{1}{2})12 \text{ sq. ft.} \\ &= 2 \times 30\frac{3}{4} \times 12 \text{ sq. ft.} \\ &= 738 \text{ sq. ft.}\end{aligned}$$

Example II. The inside diameter of a cylindrical pipe is 2 in.: how many cu. in. of water will $\frac{1}{4}$ mile of the pipe hold?

$$\begin{aligned}\text{Vol. of water} &= \text{vol. of cylinder, radius of base 1 in., length} \\ &\quad (\frac{1}{4} \times 5280 \times 12) \text{ in.} \\ &= \pi \times 1^2 \times \frac{1}{4} \times 5280 \times 12 \text{ cu. in.} \\ &= 49739\cdot6\dots \text{ cu. in.}\end{aligned}$$

Example III. The section of an iron tube is a circle of 6 in. diameter from which a square, side 2 in., has been cut. How much metal will be required for 18 in. of tubing?

$$\begin{aligned}\text{Vol. of tubing} &= \text{vol. of cylinder} - \text{vol. of rectangular prism} \\ &= \{(\pi \times 3^2 \times 18) - (2^2 \times 18)\} \text{ cu. in.} \\ &= 18(9\pi - 4) \text{ cu. in.} \\ &= 436\cdot68\dots \text{ cu. in.}\end{aligned}$$

In addition to the formulæ on page 36, the following will be useful for the examples on this chapter:—

$$\begin{array}{ll}\text{Surface area of cylinder (radius } r, \text{ height } h) &= 2\pi r(h + r). \\ \text{Surface area of cone (radius } r, \text{ slant height } l) &= \pi r(l + r). \\ \text{Surface area of sphere (radius } r) &= 4\pi r^2. \\ \text{Volume of sphere} &= \frac{4}{3}\pi r^3.\end{array}$$

EXAMPLES. XXI 21

1. Find the area of a rectangular field, length $121\frac{1}{2}$ ft., breadth $53\frac{1}{3}$ ft.

2. How many square ft. of paper will be required for the walls of a room 17 ft. long, 15 ft. wide, and 11 ft. high?

3. Find the area of a sheet of paper 1 ft. 3 in. by 8 in. Give answer in (i) sq. ft., (ii.) sq. in.

4. How many tiles each 6 in. square will be required for a courtyard 108 ft. by 66 ft.?

5. What would be the cost of painting a wall 12 ft. by 15 ft. at 7*d.* per sq. yd.?

6. Find the area of a border 3 ft. wide round the outside of a grass plot in the shape of a rectangle 25 ft. by 18 ft.

7. A sheet of brown paper, Imperial size, measures 29 in. by 22 in. How many sq. ft. are there in a ream (480 sheets)?

8. A picture frame is 1 ft. long and 8 in. wide. The wood of which the frame is made is $\frac{1}{4}$ in. wide. What is the area inside the frame?

9. A rectangular grass plot measuring 100 yd. by 126 yd. is surrounded by a path 2 yd. broad; find the cost of repairing the path at $1\frac{3}{4}$ *d.* per sq. yd.

10. How much will it cost to paint the walls of a room 19 ft. $10\frac{1}{4}$ in. long, 16 ft. $1\frac{3}{4}$ in. wide, and 10 ft. 3 in. high at $9\frac{1}{2}$ *d.* per sq. yd.?

11. What is the cost of 200 tiles each 6 in. by 4 in. at $\frac{1}{2}$ *d.* per sq. in.? What would they cost if they measured 3 in. by 4 in.?

12. The cost of carpeting a room 16 ft. 6 in. long at 4*s.* per sq. yd. is £3. 15*s.* 2*d.* Find the breadth of the room.

13. What length of paper 3 ft. wide will be required for the walls of a room 18 ft. 4 in. long, 17 ft. 8 in. broad, and 12 ft. high?

14. A box without a lid is 2 ft. 1 in. long, 10 in. broad, and 11 in. deep. The wood of which it is made is $\frac{1}{2}$ in. thick. The inside is lined with tin-foil: how much will be required?

15. On a map drawn to a scale of 1 : 1000 a field is represented by a rectangle $2\frac{1}{4}$ in. by $1\frac{1}{4}$ in. What is the real area in sq. yd. of the field?

16. How many postage stamps each $\frac{15}{16}$ in. by $\frac{3}{4}$ in. would make a frieze a foot wide all round a room 16 ft. 6 in. by 14 ft. 9 in.?

17. Find the cost of papering the walls of a room 20 ft. 6 in. long, 17 ft. wide, and 12 ft. high, there being a door 9 ft. by 3 ft. and a window $4\frac{1}{2}$ ft. square. The paper, 2 ft. 3 in. wide, costs $4\frac{1}{2}$ d. per yd.

18. The area of the surface of the wood in a box with a lid is 664 sq. in. The box is 1 ft. 2 in. long and 8 in. wide: find the height.

19. The outer rim of a chess-board is $\frac{3}{10}$ in. wide; the side of each of the squares is $2\frac{1}{10}$ in. Find the area of the board.

20. Find the area of the given figure in sq. cms.



21. The following is an extract from a catalogue:—

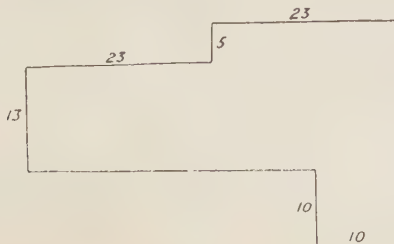
Asbestos Slabs.

$\frac{5}{32}$ in. (A)	$2\frac{1}{4}$ d. per sq. ft.
$\frac{3}{16}$ in. (B)	$2\frac{3}{4}$ d.
$\frac{3}{8}$ in. (C)	5d.

A crate contains 10 “B” slabs 8 ft. by 4 ft.; 15 “B” slabs 4 ft. square; 13 “A” slabs 8 ft. by 4 ft.; 20 “C” slabs 4 ft. square. Find the cost of a crate.

22. A strip the same width all the way round is cut off from the four sides of a rectangular sheet of zinc 15 cm. by 10 cm. It leaves a rectangle of area 104 sq. cm.: what is the width of the strip? [Let x cm. = width of strip.]

23. The diagram below represents a field. Find the area of the field, the lengths given being yards.

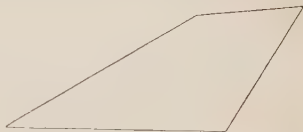


24. The sides of a triangular piece of aluminium are 6, 8, 10 in. respectively. Find its area.

25. Express the area of a circular plot 10 feet radius in sq. yd.

26. A piece of glass tubing is .76 cm. in diameter. Find the cross-sectional area. (The tube is a circular cylinder.)

27. Find the area of the figure, measuring any lines and angles you require.



28. A kite is in the shape of a semicircle and an equilateral triangle described on opposite sides of a straight line 2 ft. long. How many square feet of paper will be required to make it?

29. A circular photograph-frame has an outside radius of 10 in. and is made of wood an inch wide. How much space is there inside the frame?

30. A halfpenny is an inch across: a shilling is $\frac{1}{16}$ in. across. By how much does the surface area of one face of a halfpenny exceed that of a shilling?

31. Each page of a book measures 7.2 in. by 4.8 in. The area of the total number of leaves in the book is 74.16 sq. ft. How many pages are there in the book?

32. Find the volume of a log of wood 15 ft. long, 4 ft. wide, 2 ft. thick.

33. How many cubic feet are there in a plank 7 ft. long, 6 ft. wide, 2 in. thick?

34. The volume of a block of iron is 1080 cu. in. It is 15 in. long and 12 in. wide. How thick is it?

35. A glass rod is 3 ft. long and has a volume of 2.5 cu. in. Find its cross-sectional area.

36. A cistern holds 450 cu. ft. of water. It is 15 ft. long by 10 ft. wide: how deep is it?

37. A cu. ft. of water weighs 1000 oz. How many tons, etc., will a tank 18 ft. by 14 ft. by 5 ft. hold?

38. The dimensions of a brick with the mortar attached to it average 9.1 in. long, 4.6 in. broad, and 3.2 in. thick. How many will be required for a wall 65 yd. long, 9 ft. 7 in. high, and 14 in. thick?

39. The volume of a tree trunk is generally found by multiplying the length by ($\frac{1}{4}$ of the girth)². Find the volume of a trunk 10 ft. long, of girth 8·64 ft.

40. Water flows through a pipe of cross-section 3 sq. in. at the rate of 6·4 miles per hour. Find the volume of water which flows through the pipe in two hours. Answer in cu. ft.

41. A beam is 16 ft. long and its volume is $26\frac{2}{3}$ cu. ft. What is the area of cross-section?

42. Find the cost of painting the inside of an open tank $2\frac{1}{2}$ yd. long, 4 ft. wide, $4\frac{1}{2}$ ft. deep, at $3\frac{1}{2}$ ¢. per sq. ft.

43. A gallon = 277·25 cu. in. How many gallons of water will a reservoir 30 ft. 6 in. long, 16 ft. 6 in. broad, and 20 ft. deep hold?

44. A box containing chalks is 16 cm. by 10 cm. by 9 cm. The volume of a piece of chalk is 6·28 cu. cm. How many pieces will a box hold?

45. A vessel has a cross-sectional area of 128 sq. in. 10 gall. of water are poured in: find the depth of the water in inches. (1 gall. = $277\frac{1}{4}$ cu. in.)

46. Find the volume of a column of mercury, cross-sectional area 1·05 sq. cm., 758 mm. high. Give answer in cu. cm.

47. Gravel is laid down to a depth of 3 in. on the surface of a straight road 400 yd. long and 28 ft. wide. How many cu. ft. of gravel will be required?

48. A hole whose section is a square of side $\frac{1}{2}$ in. is bored through a cube of side $8\frac{1}{2}$ in. Find the volume of the solid thus formed.

49. A brass cylinder is 3·8 cm. long. The diameter of the base is 1·4 cm. What is its volume?

50. Find the volume of a glass cylinder 1 ft. long, radius of base 3 in. Give answer in cu. in.

51. A cylindrical jar is graduated in cu. cm.; its cross-sectional area is 7·07 sq. cm. Find the distance between two consecutive cu. cm. marks on the jar correct to two places of decimals.

52. Rubber tubing has an outside diameter of 1 cm. and is ·25 cm. thick. Find the number of cu. cm. of rubber required for a length of a metre of tubing.

53. The volume of a glass prism is 13·284 cu. cm. It is 10·8 cm. long. Find its cross-sectional area. If the cross-section be a triangle of base ·9 cm., find correct to two decimal places the altitude of the triangle.

54. Water flows through a pipe of cross-section 3 sq. in. at the rate of 5 ft. per sec. How long will it take to fill a tank 15 ft. by 8 ft. by 8 ft.?

55. The cost of carpeting a room twice as long as it is broad at 5s. per sq. yd. is £6. 2s. 6d. Find the length of the room.

56. At the point where a stream passes a bridge it is 18 m. wide and has a uniform depth of 3·6 m. If the stream flows at the rate of 1·75 Km. per hr., how many cu. m. of water pass through the bridge in a minute?

57. Find the weight of water that a tank 15 ft. 4 in. by 16 ft. 6 in. by 8 ft. will hold, given that a cu. ft. of water weighs 997·21 oz. Give answer correct to the nearest cwt.

58. Find the volume of a ball of radius 2 in.

59. What volume of air is contained in a hollow sphere, outside diameter 16 cm., 2 cm. thick?

60. A sphere just fits into a cubical box, side 10 cm. What volume of the box is not occupied by the sphere?

61. How many sq. ft. of gold leaf will be required to cover a ball, radius 27 inches?

62. Strips of paper are used completely to cover a wooden cone the radius of whose base is 18 in. and whose height is 2 ft. Find how much paper will be needed.

§ 22. DENSITY

The **Density** of a substance is the mass per unit volume. Represented by symbols, $D = \frac{M}{V}$, where M is the number of units of mass and V the number of units of volume in the substance.

The density of a substance depends upon the units chosen. For example, we may have the density of iron expressed as **grams per cubic centimetre**, or as **pounds per cubic foot**, or again as **ounces per cubic inch**.

Example I. Find the density of brass, given that a cube of side 5 cm. weighs 1025 gm.

The mass (M) of the brass = 1025 gm.

The volume (V) = 5^3 cu. cm.
= 125 cu. cm.

$\therefore$ The density (D) = $\frac{M}{V}$
= $\frac{1025}{125}$ gm. per cu. cm.
= 8.2 gm. per cu. cm.

Example II. The weight of a brick 10 in. by 5 in. by 3 in. is 10 lb. What is its density?

The mass (M) of the brick = 10 lb.

The volume (V) = $(10 \times 5 \times 3)$ cu. in.

The density (D) = $\frac{M}{V}$
= $\frac{10}{10 \times 5 \times 3}$ lb. per cu. in.
= $\frac{1}{15}$ lb. per cu. in.

The **Specific Gravity** of a substance is the ratio of the weight of a given volume of the substance to the weight of an equal volume of a standard substance. Or,

Specific Gravity = $\frac{\text{weight of given volume of substance}}{\text{weight of equal volume of standard substance}}$

If we choose our standard substance as water, and our given volume as 1 cu. cm., we obtain the fact that

Specific Gravity = $\frac{\text{weight of 1 cu. cm. of substance}}{\text{weight of 1 cu. cm. of water}}$
= $\frac{\text{weight of 1 cu. cm. of substance}}{1 \text{ gm.}}$
= weight (in gm.) of 1 cu. cm. of substance.

The Specific Gravity of a substance differs from the density in that the first is a ratio, and is therefore independent of the units employed, while the second must be expressed in definite units.

Example III. The S.G. of marble is 2.6. Find the weight of a block 10 cm. by 2 cm. by 2 cm.

1 cu. cm. weighs 2.6 gm.

$\therefore (10 \times 2 \times 2)$ cu. cm. weigh $2.6 \times (10 \times 2 \times 2)$ gm.

$\therefore$ Weight = $2.6 \times 10 \times 2 \times 2$ gm.
= 104 gm.

Example IV. 2 litres of salt solution, S.G. 1·15, are mixed with 1 litre of water. Find the S.G. of the resulting mixture.

1 cu. cm. of salt solution weighs	1·15	gm.
∴ 2000 cu. cm. weigh	2230	gm.
1 cu. cm. of water weighs	1	gm.
∴ 1000 cu. cm. weigh	1000	gm.
3000 cu. cm. of the mixture weigh	(2230 + 1000)	gm.
∴ S.G. of mixture = weight in gm. of 1 cu. cm.		
	$= \frac{2230 + 1000}{3000}$	gm.
	$= \frac{3230}{3000}$	gm.
	$= 1·077... \text{ gm.}$	

Roomage is the amount of space taken up by a unit mass of a substance.

Roomage is the inverse of density.

For example, if the density of wood be given as ·8 gm. per cu. cm., we know that

$$\begin{aligned} & \cdot 8 \text{ gm. takes up } 1 \text{ cu. cm.} \\ \therefore & 1 \text{ gm. } \frac{1}{\cdot 8} \text{ cu. cm.} \\ \therefore & \text{Roomage} = \frac{1}{\cdot 8} \text{ cu. cm. per gm.} \\ & = 1·25 \text{ cu. cm. per gm.} \end{aligned}$$

EXAMPLES. XXII 22

[Note.—1 cu. ft. of water weighs 1000 oz.]

1. Find the weight of a cu. cm. of a cube of glass, side 2 cm., weight 21 gm. Express this in the language of density.

2. The density of copper is 540 lb. per cu. ft. Express this as oz. per cu. in.

3. The dimensions of a block of wood are 3 cm. by 2 cm. by 1·5 cm. It weighs 7·31 gm. Find its density in (i) gm. per cu. cm., (ii) Kg. per cu. m.

4. A glass tube is filled with mercury to a height of 10 cm. The density of mercury is 13·6 gm. per cu. cm. If the cross-sectional area of the tube be ·2 sq. cm., find the weight of the mercury.

5. 47·3 gm. of a liquid just fill a 50 cu. cm. flask. What is the density of the liquid?

6. Find the volume of 1 lb. of granite, density 2·5 gm. per cu. cm. (1 lb. = 453·6 gm.)

7. Two cubes of different metals have the same weight. The length of a side of one is twice the length of a side of the other. Compare their densities.

8. The density of a marble is 2.7 gm. per cu. cm. Its radius is .5 cm. Find its weight.

9. In an experiment to find the density of oil, the following readings were taken :—

Weight of flask, empty	= 16.08 gm.
..... full of water	= 42.19 gm.
..... oil	= 39.58 gm.

What is the density of oil?

10. What is the length of the side of a cube weighing 18.4 gm., density 2.3 gm. per cu. cm.?

11. If 25 cu. cm. of milk weigh 25.75 gm., what is (i) the density of milk, (ii) the S.G. of milk?

12. The S.G. of olive oil is .9; find the weight of 45 cu. cm. of the liquid.

13. A brass cube, side 7 cm., weighs 2.8126 Kg. Find its S.G.

14. What is the volume of 53 gm. of sea water, S.G. 1.03?

15. A flask weighs 12.9 gm. when empty and 43.1 gm. when full of water. Find the capacity of the flask.

16. The S.G. of zinc is 7.1. Find the weight of 150 cu. in. of zinc.

17. Find the weight of a bar of iron 1 ft. long, 1 in. wide, and .3 in. thick, given the S.G. of iron to be 7.2.

18. What volume will 50 lb. of ice, S.G. .92, occupy?

19. The density of copper is 556 lb. per cu. ft. A length of 72 ft. of copper wire weighs 8 lb. Find the cross-sectional area of the wire.

20. Gravel of density 112 lb. per cu. ft. is laid down on a road 24 ft. long and 3 ft. wide. 1 ton 16 cwt. of gravel is used; to what depth is the road covered?

21. The S.G. of air is .0013. What is the weight of air in a globe, internal diameter 20 cm.?

22. How much coal will fill a scuttle of capacity .8 cu. ft.? S.G. of coal = 1.21.

23. A vessel weighs 51.3 gm. empty and 92.5 gm. when full of water. What will it weigh when filled with sea water, S.G. 1.03?

24. A flask which will contain 50 cu. cm. when filled to a mark is weighed empty. A liquid is poured in up to the mark and the flask is weighed again. The difference in weight is 50 gm. What is the S.G. of the liquid?

25. A tumbler is filled to the brim with water. A piece of zinc, S.G. 7 and weight 30.8 gm., is put in. What volume of water will overflow?

26. Hydrochloric acid of S.G. 1.1 is poured into a flask up to a 25 cu. cm. mark. The weight of the flask is now 43 gm. Find how much the flask would weigh if water were substituted for the acid.

27. 10 gm. of milk, S.G. 1.03, is adulterated by the addition of an equal weight of water. Find the S.G. of the mixture.

28. If in the above question an equal volume of water instead of an equal weight were added, what would be the S.G. of the mixture?

29. To a quart of milk a tradesman adds $\frac{1}{2}$ pint of water. Find the weight of the mixture. (1 pint of water weighs $1\frac{1}{4}$ lb.)

30. Find the S.G. of a mixture of—

(i) Equal volumes of liquids, S.G. 1.2 and 1.3.

(ii) Equal weights of liquids, S.G. 1.2 and 1.3.

31. A chemist mixes 19.8 gm. of a liquid, S.G. 1.8, with 15.4 gm. of a liquid, S.G. 1.1. Find the S.G. of the resulting mixture.

32. The weight of a flask filled to a certain mark with water is 63 gm. Some shot is put in, and the weight is now 97 gm. The water above the mark is taken out, and the weight is now 94 gm. Find the density of the shot.

[Note.—If W = weight of a substance in air and w = its weight in water, the S.G. of the substance = $\frac{W}{W-w}$.]

33. A brass cylinder weighs 50 gm. in air and 43.75 gm. in water. Find its S.G.

34. What is the S.G. of a glass prism which weighs 10 gm. in air and 6 gm. in water?

35. A piece of lead of S.G. 11.4 weighs 100 gm. in air. Find correct to 2 decimal places its weight in water.

$$[\text{Let } x = \text{weight in water: then } 11.4 = \frac{100}{100 - x}.]$$

36. What is the volume of a solid which weighs 19 gm. in air and 16 gm. in water?

37. An iron weight of S.G. 7.5 weighs 2 lb. Find how much it weighs in water. (1 lb. = 453.6 gm.)

38. A stick of sulphur 10 cm. long weighs 248.4 gm. in air and 128.4 gm. in water. What is its cross-sectional area?

39. A halfpenny is an inch across and is $\frac{1}{16}$ in. thick. It weighs $\frac{1}{8}$ oz. Find its density in oz. per cu. in.

40. Find from the data given in the above question the S.G. of bronze.

41. A flask weighs 70.2 gm. when full of water. A piece of tin is put in, and the weight is now 90.5 gm. If the S.G. of tin is 7.5, how much water overflows?

42. The capacity of a bottle of weight 15 gm. is 25 cu. cm.; it is filled with water. What would it weigh if 10 gm. of marble, S.G. 2.8, were inserted, and the liquid which overflowed removed?

43. Find the roomage of pine wood, its density being .5 gm. per cu. cm.

44. How much space would a gm. of nickel, density 8.9 gm., per cu. cm. occupy?

45. The roomage of granite is .675 cu. ft. per cwt. Find the density of granite in lb. per cu. ft.

46. .04 lb. of a liquid occupies a cu. in. Express its roomage in cu. cm. per gm.

47. Find the volume taken up by a Kg. of steel, density 4 cwt. per cu. ft. Give answer in cu. cm. (453.6 gm. = 1 lb.; 16.4 cu. cm. = 1 cu. in.)

48. Find the weight of a sphere, radius 3.7 cm., roomage 50 cu. cm. per oz.

49. A cubic foot of copper weighing 560 lb. is rolled into a square bar 40 ft. long. An exact cube is cut from the bar; what is its weight?

50. A spherical ball 3.75 in. in diameter weighs 5.75 oz. Find correct to 2 places of decimals the volume taken up by a cwt. of the material of which the ball is made.

§ 23. GRAPHS

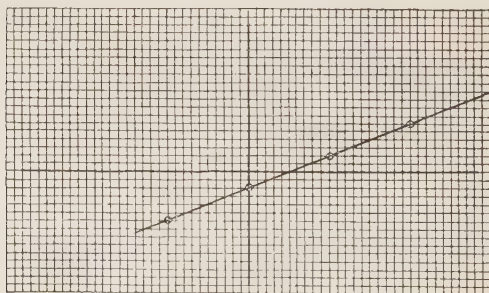
Example I. Graph $\frac{2x-1}{5}$.

The values of x and the corresponding values of $\frac{2x-1}{5}$ should always be written down in a table thus:—

x	-1	0	1	2	3
$\frac{2x-1}{5}$	-.6	-.2	.2	.6	1

A suitable unit for x having been chosen, the scale should always be stated.

Scale: $x = 1$ inch.

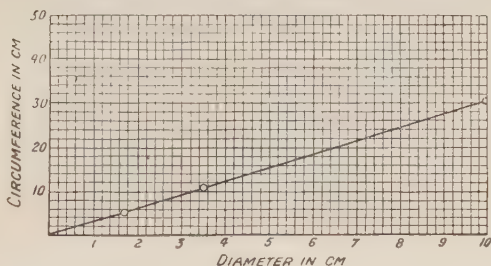


APPLICATION OF GRAPHS

Example II. The circumference of a circle is 3.14 times its diameter: draw a graph showing the connection between the circumference of a circle and its diameter. Thence find the circumference of circle, diameter 3.5 cm., and the diameter of a circle, circumference 5.4 cm.

Diam. . .	0	10	20	30
Circ. . .	0	31.4	62.8	94.2

Scale of diameters: $\frac{1}{2}$ in. represents 1 cm.
 ,, ,, circumferences: $\frac{1}{2}$ in. ,, 10 cm.



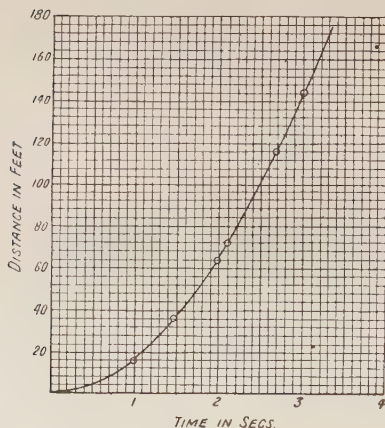
From the graph, the circumference of a circle, diameter 3.5 cm., is 11 cm., and the diameter of a circle circumference 5.4 cm., is 1.7 cm.

By taking one large division as the unit—as for the diameter above—we can measure to one place of decimals: if we take two of these divisions as the unit, we can measure to two places of decimals. An instance of this occurs in the next example, where it is possible to find the time to two places of decimals.

Example III. When a body is let fall, the time it takes to reach the ground is given by $s = 16t^2$, where s is the distance through which it falls and t is the time taken, s being measured in feet and t in seconds. Find the time taken to fall distances of 16, 28, 72, 116 feet.

t	0	1	2	3
t^2	0	1	4	9
$16t^2$	0	16	64	144

Scale of times : 1 in. represents 1 sec.
 „ „ distances : 1 in. „ 40 ft.



From the graph

Distances.	Time.
Feet.	Seconds.
16	1
28	1.47
72	2.12
116	2.67

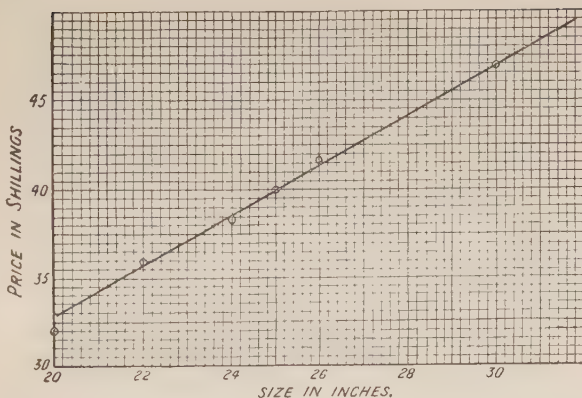
Example IV. From a catalogue of suit-cases the following gives the prices for different lengths:—

20 in.	22 in.	24 in.	26 in.
32s.	35s. 3d.	38s. 3d.	41s. 6d.

Estimate the probable cost of suit-cases 25 in. long and 30 in. long.

Here we have no definite connection between the lengths and prices. We must draw a graph connecting the given values, and find the probable values from the curve.

Scale of sizes : 1 in. represents 2 in.
 „ „ prices : 1 in. „ 5s.



The points lie very nearly on a straight line : draw a straight line having the points distributed as equally as possible on both sides of this line.

From the graph, the probable prices of 25-in. and 30-in. cases are 39s. 10d. and 47s.

Example V. A piece of sheet zinc, of unlimited length and 4 ft. wide, has to be bent into the form of a duct for water. Find how it must be bent so as to let most water through.



Let x ft. be the length of one of the sides of the duct.

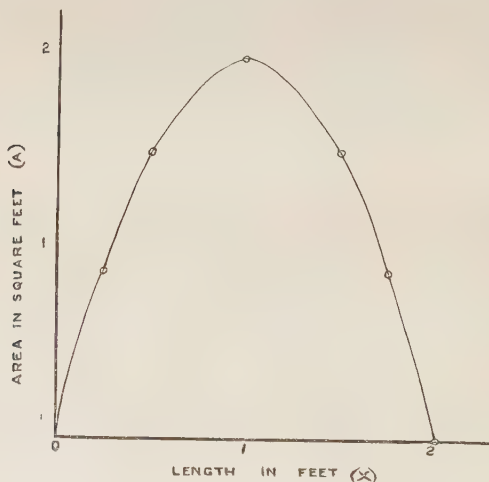
Then the area of cross-section of the duct will be $x(4 - 2x)$ sq. ft.

Most water will flow through when this area is greatest.

Graph the area (A) against the length (x), and find which value of x gives the greatest value of A .

x	0	$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	$1\frac{3}{4}$	2
$4 - 2x$	4	$3\frac{1}{2}$	3	2	1	$\frac{1}{2}$	0
A	0	$\frac{7}{8}$	$1\frac{1}{2}$	2	$1\frac{1}{2}$	$\frac{7}{8}$	0

Scale of lengths: 2 in. represents 1 ft.
 „ areas: 2 in. „ 1 sq. ft.



A is greatest when $x = 1$.

$\therefore$ Most water will flow through when the depth of the duct is 1 ft.; in that case the width will be 2 ft.

EXAMPLES. XXIII 23

1. There are 2.54 cm. in an inch. Draw a graph for converting inches to cm., and from the graph find the number of inches in (i) 6 cm., (ii) 7.8 cm.

2. 1 Kg. = 2.2 lb. Determine graphically the number of Kg. in 8 lb., and the number of lb. in 7.5 Kg.

3. The area of a circle is πr^2 , where r is the radius of the circle. Show by means of a graph how to find the area, given the radius; and hence find (i) the area of a circle, radius 7.5 cm., and (ii) the radius of a circle, area 8 sq. cm.

4. 20.4 marks = £1. Draw a graph connecting marks and pounds; find the number of pounds in (i) 35 marks, (ii) 14.5 marks.

5. Graph (i) $\frac{3x-5}{8}$; (ii) $\frac{1.7-3x}{6}$.

6. The maximum marks for an examination are 150. This maximum is afterwards reduced to 100. Find, by drawing a graph, the number of marks a boy will obtain if his marks originally were 66. With the new maximum a boy's marks were 51; how many marks did he obtain originally?

[Note.—Take the graph to be a straight line.]

7. There are 453 gm. in 1 lb. Express graphically the connection between gm. and lb., and find the number of lb. in (i) 1000 gm., (ii) 270 gm.

8. The speed of a body starting from rest is given by $s = 12t$, where t is the time in seconds from the beginning of the motion, and s the speed measured in feet per second. Find the speed after intervals of .5, 4, 7.5, 11.25 seconds, by drawing a graph connecting the speed and time.

9. A manufacturer sells paint according to the following scale:—

Quantity.	Price.
1-gallon drum . .	3s. 6d.
5-gallon drum . .	10s. 6d.
10-gallon drum . .	19s. 6d.
20-gallon keg . .	35s. 0d.
40-gallon cask . .	70s. 0d.

Find the probable cost of a 15-gallon drum and a 25-gallon keg.

10. The surface of a sphere is $4\pi r^2$, where r is its radius. By drawing a graph, find the radius of the sphere when its surface area is (i) 10 sq. cm., (ii) 16 sq. cm., (iii) 22 sq. cm.

11. 25.2 francs = 20 shillings. Determine graphically the number of francs in 12s. 6d., and the number of shillings in 12 fr. 35 c.

12. The length of a pendulum which beats seconds varies at different places according to the relation $l = \frac{1}{\pi^2} g$, where l is measured in cm. and g is a number known when the place is known. Draw a graph connecting l and g , and find (i) l when $g = 981$, (ii) g when $l = 99$.

13. In an experiment with a water-heating apparatus the following gives the temperatures at different times :—

Time.	Temperature (F.).
6.30	44°
6.35	94°
6.40	122°
6.45	133°
6.50	139°
6.55	142°
7.0	146°

Draw a graph connecting time and temperature: thence estimate the probable temperature at 7.5, and at what time the temperature was 100°.

14. The volume of a cube, side a cm., is a^3 cu. cm. Show by means of a diagram the connection between V (the volume in cu. cm.) and a (the length of a side in cm.), and hence find the length of the side of a cube of volume (i) 8 cu. cm., (ii) 15 cu. cm., (iii) 75 cu. cm.

15. The number of letters and lines of different kinds of type in a column 13 sq. in. in area is given in the table below :—

Type.	Lines.	Letters.
Double Pica . . .	22	280
Great Primer . . .	27	430
English	34	680
Pica	37	890
Small Pica	43	1120

Find whether a regular graph can be drawn connecting the number of letters with the number of lines. Estimate the number of lines taken up by the 1360 letters long primer type which would fill the column.

16. Graph x^2 : hence find the square roots of 19, 38, 75, 109.

17. The electrical resistance of a metre of silver wire is given by $R = \frac{149}{a}$, where a is the cross-sectional area of the wire in sq. cm. Find, by drawing a diagram, the cross-sectional area of a length of a metre of wire for which $R =$ (i) 100, (ii) 356.

18. Shelves are priced in a catalogue according to their depth thus:—

Depth in Inches.	Price.
$7\frac{1}{2}$	4s. 0d.
$8\frac{1}{2}$	4s. 3d.
10	4s. 6d.
$11\frac{1}{2}$	5s. 0d.
$12\frac{1}{2}$	5s. 6d.
$13\frac{3}{4}$	5s. 10d.
$15\frac{1}{2}$	6s. 6d.
$19\frac{3}{4}$	7s. 8d.

Find the probable cost of a shelf 17 in. deep by means of a diagram.

19. The area of the surface of a circular cylinder is given by $A = 2\pi r(h + r)$, where r and h are the radius of the base and the height respectively. Draw a graph connecting the surface area and radius of base of cylinders of constant height 12 cm. Hence find the radius of the base of one of these cylinders whose surface area is 250 sq. cm.

20. A reading of 0° on a centigrade thermometer corresponds to 32° on a Fahrenheit thermometer. 100° C. correspond to 212° F. Draw a graph showing the connection between the two scales, and find readings on the other scale corresponding to 0° F., 16° C., -40° C., 100° F.

21. The volume of a sphere = $\frac{4}{3}\pi r^3$, r being the radius. Find by means of a graph the radius of the sphere whose volume is (i) 100 cu. cm., (ii) 390 cu. cm., (iii) 1000 cu. cm.

22. The price of galvanised hot-water cylinders varies with their capacities according to the adjoining table.

Capacity in Gallons.	Price.
20	£2. 2s. 6d.
25	£2. 7s. 6d.
28	£2. 10s. 0d.
31	£2. 12s. 6d.
34	£2. 15s. 0d.
40	£3. 2s. 6d.
45	£3. 7s. 6d.
50	£3. 12s. 6d.
65	£4. 7s. 6d.
75	£4. 17s. 6d.

Find the probable price of a 38-gall. cylinder and a 70-gall. cylinder.

23. The load required to stretch a brass wire through a distance d cm. is given by $L = \frac{948}{\pi} \cdot \frac{1}{d}$. Draw a graph connecting L and d , and find the load necessary to stretch the wire through 5 cm.

24. In an experiment with a screw-jack a power P lb. lifted a weight W lb. in accordance with the formula $W = 11 \cdot 24P - 41 \cdot 6$. Draw a graph connecting W and P ; find the power necessary to lift a load of (i) 100 lb., (ii) 180 lb.

25. For copper wires the electrical resistance varies with the diameter according to the following table:—

Diameter of cross-section in mm.	·38	·32	·27	·23	·19	·15	·12
Resistance in ohms per m.	·15	·21	·28	·38	·56	·89	1·40

Plot a curve connecting these values, and estimate the diameter of cross-section of a wire whose resistance is $\cdot 72$ ohm per m.

26. The pressure (p) and volume (v) of a gas as shown on a gas-engine indicator diagram were found to vary so that $v = \frac{2\cdot 18}{p} - 2\cdot 1$. Find by means of a graph the value of p when $v = 6\cdot 4$, and the value of v when $p = \cdot 538$.

27. Draw a graph illustrating the variation in the male population of Scotland from 1821–1911.

Year	1821	'31	'41	'51	'61	'71	'81	'91	1901	1911
Population in millions	$\cdot 98$	$1\cdot 11$	$1\cdot 24$	$1\cdot 38$	$1\cdot 45$	$1\cdot 60$	$1\cdot 80$	$1\cdot 94$	$2\cdot 17$	$2\cdot 31$

Estimate the probable population in 1876 and 1906.

28. The following table gives the weight of potassium nitrate (nitre) soluble in water at different temperatures :—

Weight in gm. dissolved in 100 gm. of water	12	30	62	83	108	136
Temperature in degrees centigrade	0	20	40	50	60	70

Draw a graph, and find at what temperature 100 gm. of nitre will be dissolved.

29. The present value of a lease of the net annual value of £100 is given below, compound interest being reckoned at 4 per cent.

Years the lease has to run	10	12	17	20	25	32
Present value (£) . . .	811	939	1217	1359	1562	1787

Draw a diagram connecting the above values: estimate the present value of a lease having 28 years to run.

30. In a system of n pulleys a power of E lb. will lift a load of W lb. according to the formula $W = (2^n - 1)E$. If $E = 26.5$ lb., plot a curve connecting W and n and find the load lifted by 26.5 lb. if there be 5 pulleys. (Take $n = 1, 2, 3, \dots$)

31. Glass measures are sold according to the number of oz. they contain, thus:—

Number of oz.	$\frac{1}{2}$	1	2	4	6	8	10	20	40
Price per doz.	7s. 9d.	8s. 6d.	10s.	12s.	14s.	15s.	18s.	28s.	38s.

Find the probable cost per dozen of glass measures holding 13 oz. and 25 oz.

32. N lb. of water per sec. are discharged through a circular orifice, h ft. being the head of water above the orifice. N is given by the equation $N^2 = (14.6)^2 h$. Plot a graph connecting N and h ; determine the discharge in lb. per sec. under a head of 4 ft. of water.

33. The weight of powder to fill a spherical shell is given by $P = .017D^3$, where D is the internal diameter in inches and P is expressed in pounds. By drawing a graph find the diameter of the shell that 29 lb. of powder will fill.

34. An Englishman is travelling in France. He wishes to find out how many miles he has covered and how much money he has spent in pounds. Knowing that 1.6 Km. = 1 mile and 25.25 francs = £1, construct convenient graphs for expressing French units in English ones. Hence find how many miles he has travelled if he covered 500 Km., and how much the 750 francs he spent represent in English money. He finds that his travelling expenses averaged $2\frac{1}{4}$ l. per mile: express this in centimes per kilometre.

35. Graph x^3 . Hence find the cube root of 19.

36. Find the greatest value of $x(10 - x)$ by drawing the graph of $y = x(10 - x)$.

37. The perimeter of a rectangle is 36 feet. Find what must be the lengths of the sides in order that the rectangle may have the greatest possible area.

38. A man has 1000 yards of fencing wire, and he wishes to fence off as large an area of prairie land as he can in the form of a rectangular field.

(i.) If x yards be the length of one side of the field, find the other side and the area.

(ii.) Draw a graph by plotting the area (A) against x , and hence read off when the area is greatest.

39. If in the above question a river had flowed through his estate which he could take as one side of his field, calculate the side parallel to the river, if the sides perpendicular to the river be each x yards. Draw the graph of the area (A) against x , and hence find when the area is greatest.

40. Use squared paper to find how to divide a line 17 cm. long into two parts so that the sum of their squares is a maximum.

41. The perimeter of a window is 12 ft. It has the shape of a rectangle surmounted by a semi-circle. Find the dimensions which will admit most light, by using squared paper.

42. A man orders a plumber to make a cistern with a circular base to hold 1000 cu. ft. of water. It is to be open at the top. Sheet lead at $4\frac{1}{2}$ d. per sq. ft. has to be used to line the inside. Find the least that the cost of the lining will be.

43. A carpenter has a piece of wood, 27 sq. ft. in area. He wishes to make from this a lidless box, which must be on a square base. What must be the dimensions of the box, in order that it may hold the greatest possible amount?

44. When a vessel is steaming at x knots, the total cost of the journey is $\pounds(4 + .001 x^3)$ per hr. At what speed ought the vessel to go on a journey of 1000 miles, in order that the expenses should be the least possible?

45. A cylindrical tree-trunk 20 ft. long is 12 ft. in diameter. A rectangular beam is to be cut from this. What is the greatest volume the beam can have?

46. A man has a strip of wood 2 in. wide and 3 ft. 6 in. long. Assuming that there is no waste of material, what is the area of the largest picture he can frame with the wood, the frame to be 2 in. wide?

47. A hedge runs along the side of a man's estate. He wishes to fence off a field 1000 sq. yd. in area, its shape to be a right-angled triangle of which the hedge is to be the hypotenuse. What is the least amount of fencing he can use?

48. The Post Office regulations state that for a parcel to be sent by parcel post the sum of its length and girth must not exceed 6 ft. Find the dimensions of the largest parcel that may be sent by parcel post.

§ 24. SQUARE ROOT

Method I. By Factors.

Example I. Find the square root of 75625.

$$\begin{aligned} 75625 &= 5 \times 5 \times 5 \times 5 \times 11 \times 11. \\ &= 5^2 \times 5^2 \times 11^2. \end{aligned}$$

$$\begin{aligned} \therefore \sqrt{75625} &= 5 \times 5 \times 11. \\ &= 275. \end{aligned}$$

Method II.

Example II. Find the square root of 758641.

[*Rough Check:* $\sqrt{810000} = 900.$]

$$\begin{array}{r} 75,86,41 \overline{)871} \\ 167 \overline{)1186} \\ 1741 \overline{)1741} \end{array}$$

Mark off the digits in pairs, commencing from the decimal point (which in the case of a whole number is understood at the end, thus:—758641 $\cdot$). Choose the largest number whose square is less than the number in the first group: in this case the number will be 8. This will be the first number in the square root. Subtract 8^2 from 75 and bring down the next two digits. Double the 8, insert the 16 in the tens' place on the left, and find a figure 7 to insert in the units' place so that

the new number, 167, when multiplied by this 7 shall be as nearly as possible 1186. Subtract 167×7 from 1186, double the 87, and proceed as before. We obtain the answer thus: 871.

Example III. Find correct to two places of decimals the value of $\sqrt{348.3}$.

[*Rough Check:* $\sqrt{324} = 18.$]

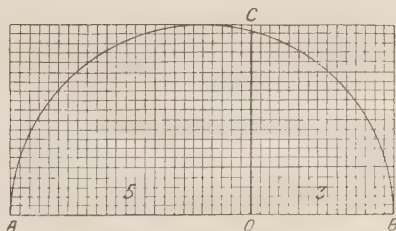
	3,48.30,00,00	18.662
28	2 48	
366	24 30	
3726	2 34 00	
37322	10 40 00	

$$\therefore \sqrt{348.3} = 18.66 \dots$$

Mark off the figures in pairs from the decimal point, on both sides of it, and proceed as usual with whole numbers.

For small numbers the following graphical method is useful.

Example IV. Find geometrically the square root of 15.



Choose any two factors of 15, e.g. 5 and 3: draw a straight line $AB = 5 + 3$ units in length. On AB as diameter describe a semicircle. O is the point of AB 5 units distant from A . Draw OC at right angles to AB , cutting the semicircle at C .

$$\text{Then } AO \cdot OB = OC^2$$

$$\therefore 5 \times 3 = OC^2$$

$$\therefore OC = \sqrt{15}.$$

By measurement $OC = 3.87$ units.

$$\therefore \sqrt{15} = 3.87.$$

When the number is a prime, e.g. 11, choose AB so that $AO = 11$ and $OB = 1$.

EXAMPLES. XXIV 24

Write down the square roots of—

1. 64, 121, 3600, 361.
2. 169, 289, 10000, 1440000.
3. $\frac{4}{9}$, $\frac{9}{64}$, $\frac{49}{121}$, $\frac{100}{256}$.
4. .04, .0009, 2.89.
5. $2\frac{1}{4}$, $3\frac{1}{6}$, $2\frac{4}{9}$, $1\frac{41}{400}$. [*First convert to improper fractions.*]
6. Find by factors the square roots of—
 (a) 441, 625; (b) 576, 22500; (c) 14641, 12250000;
 (d) 676, 4356; (e) 99225, 12446784.
7. Find the square roots of—
 (a) 53361, 208849; (b) 5715.36, 25.3009;
 (c) 48.1636, .00491401; (d) 600.25, 5053.7881.
8. Given that the square root of 124609 is 353, write down the square roots of 12.4609, 1246.09, .124609, .0000124609.
9. Find graphically the square roots of—
 (i) 3; (ii) 8; (iii) 14; (iv) 18; (v) 20; (vi) 7;
 (vii) 11; (viii) 24; (ix) 2; (x) 17.
10. Show that $\pi = \sqrt{9.8694}$ approximately.
11. The area of a square field is 15876 sq. metres. Find the length of a side.
12. How many yards are there in the side of a square field, area 10 acres?
13. Show that $\sqrt{10}$ lies between 3.15 and 3.16.
14. How many metres of fencing will be required to enclose a square field of area 3199.0336 square metres?
15. A square is to be cut from a piece of wood so that its diagonal should measure 9 in. Draw the square to scale and measure the length of its side.
16. A ladder placed against a wall reaches 12 ft. up the wall. Its foot is 5 ft. away from the bottom of the wall. Find the length of the ladder.

17. A plank 6 ft. long reaches to the top of a stool: the foot of the plank is $5\frac{1}{2}$ ft. from the bottom of the stool. What is the height of the stool?

18. A town A is 64 miles south of another town B, and is 100 miles east of a third town C. Find the distance between B and C.

19. When a boy is flying a kite he finds he has paid out 289 feet; how high is the kite above the ground, if it is vertically above a point 64 feet from the boy?

20. Two trains start from the same point at the same time. One goes east at 48 miles per hour, and the other south at 40 miles per hour. How far apart will they be at the end of half an hour?

21. A man digs two stakes into the ground at a distance apart of 2 ft. He joins the tops of the stakes by a piece of string 2 ft. 2 in. long. If the smaller stake is a foot high, what is the height of the larger one?

22. In a square courtyard 549081 tiles, each a square of area 9 sq. cm., are required to pave it. What is the length of the side of the courtyard?

23. A man in an aeroplane is vertically above a town A at a height of a mile. He flies for 20 minutes, keeping parallel to the ground. If his speed is 36 miles per hour, how far is he from A measured in a straight line?

24. A field ABCD is in the form of a rectangle, sides 35 metres and 28 metres. A man walks along AB and then along BC. How much less would he have had to walk if he had walked from A to C by going straight across the field?

25. How far away from the foot of the wall must a fire-escape, 90 feet long, be placed so as just to reach the top of a house 50 feet high?

26. Three streets AB, BC, CA form a triangle, right-angled at C. The lengths of AB, BC are 153 yd. and 72 yd. respectively. Two men start from A at the same time; one walks along AB, and the other along AC and then along CB. If they both walk at the same rate, find how far the second one will be from B when the first arrives there.

27. Two points 2 feet apart are joined by a loose wire. If when a weight is suspended from the middle point of the wire, the two pieces of wire on either side of the weight are at right angles, find the length of the wire.

28. A pole 7 feet high is stuck in the ground, and to keep it upright a wire rope 12 feet long is fastened to the top of the pole and to the ground. How far away from the foot of the pole is the rope fastened?

29. Find to the nearest inch the side of a square pane of glass of area 330 sq. in.

30. What is the length of the longest straight line that can be drawn on a sheet of paper 11 in. by 8 in.?

31. The area of a circle is 1000 sq. cm. Find its radius correct to the first decimal place.

32. Two circular discs of the same thickness and of radii 5 in. and 1 ft. respectively are melted down, and one circular disc of the same thickness made from the metal without any waste of material. What is the radius of this disc?

33. What length of string will just go round a circle of area 28 sq. in.?

34. A manufacturer has a stock of balls for gilding, the radii of the balls ranging from 15 cm. to 25 cm., and the radius of each of the 11 kinds being a whole number of centimetres. Find the radius of the ball which can be covered with least waste of material from a piece of gold-leaf of surface area 4500 sq. cm., and find how much will be left after gilding.

35. Find the internal diameter of the cylindrical vessel of height h which will just contain as much liquid as two vessels each of height h and of internal diameters 13.1 cm. and 28.2 cm. respectively.

36. A square piece of iron is of side 15 in. and is an inch thick. Find the radius of the greatest circular disc an inch thick that can be made from this (i) without melting it down, (ii) by melting it down and re-casting it.

37. A step-ladder rests symmetrically on the ground with its feet $3\frac{1}{2}$ ft. apart. If each half of the ladder is 6 ft. long, find the height of the top of the ladder above the ground.

38. Find the length of the side of the largest square carpet that can be placed on the floor of a circular room of radius 12 ft.

39. Hanley and Montgomery are due N. and due W. of Wolverhampton respectively. The distance between Hanley and Wolverhampton is 32 miles, and that between Hanley and Montgomery is 56 miles. If Cardiff is 64 miles due S. of Montgomery, how far is Cardiff from Wolverhampton?

40. The volume of a lead pencil is $\cdot 424$ cu. in.; the length is 6 in. Find the radius of the base.

41. The area of a semicircular lamina is 30 sq. in. What length of string would just go round it?

42. A and B are two towns 15 miles apart. C is a town due east of A and due north of B. A man starts from A and walks towards C at the rate of 3 miles per hour; at the same time another man starts from B and walks towards C at the rate of 4 miles per hour. The men arrive at C at the same instant. How far is C from A?

43. Find correct to three places of decimals the square root of $1\frac{1}{2}$. (*Multiply numerator and denominator by 2.*)

44. What is the value of $\sqrt{.00093}$ correct to the third decimal place?

45. Show that $\frac{1}{\sqrt{3} + \sqrt{2}} = 1.318\ldots$ (*Multiply numerator and denominator by $\sqrt{3} - \sqrt{2}$.*)

46. Find the fourth roots of (i) 810000, (ii) 2401.

47. Show that $\frac{3 - \sqrt{5}}{3 + \sqrt{5}} = \frac{1}{2}(7 - 3\sqrt{5})$.

48. The time t in seconds that a body takes to reach the ground when dropped from a height h feet is given by $h = \frac{1}{2}gt^2$, where $g = 32.2$. Find how long a body would take to fall from the Eiffel Tower, which is 985 feet above the ground.

49. Find to the fifth place of decimals by how much $\sqrt[4]{7}$ differs from 2.6455.

50. What is the value of $\frac{1.452}{\sqrt{1+ax}}$ correct to the third decimal place, where $a = 2.73$ and $x = .9205$?

51. Find the length of the longest ruler which will just go in a rectangular drawer 8 in. by 6 in. by 24 in.

§ 25. USE OF FORMULÆ

If we wish to find the depth of a quarry-hole, we can obtain a result which is very nearly correct by allowing a stone to drop from the edge of the quarry and noting the time between the instant it is let fall and the instant that it is heard to strike the bottom. Then, by knowing the fact that the depth in feet is sixteen times the square of the time in seconds, we can obtain the required result. We can state the last fact in the formula $S = 16t^2$. Several formulæ have already been used; such are $D = \frac{M}{V}$, for the density of a substance; $V = abc$, for the volume of a rectangular prism.

We will take some illustrative examples.

Example I. $A = HP$ gives the area of the walls of a room, where H is the height and P the perimeter. Find the area of the walls of a room 12 ft. high, 15 ft. long, and 13 ft. wide.

Here

$$H = 12 \text{ ft.}$$

$$P = 2(15 + 13) \text{ ft.}$$

$$\begin{aligned} \therefore A &= HP \\ &= \{12 \times 2(15 + 13)\} \text{ sq. ft.} \\ &= (12 \times 56) \text{ sq. ft.} \\ &= 672 \text{ sq. ft.} \end{aligned}$$

Example II. The simple interest ($\mathcal{L}I$) on a sum of money ($\mathcal{L}P$) for T years at R per cent. per annum is found from the formula $100I = PRT$. Find the simple interest on $\mathcal{L}700$ for 3 years at $3\frac{1}{3}$ per cent.

$$\begin{aligned} 100I &= PRT \\ &= \mathcal{L}(700 \times 3\frac{1}{3} \times 3). \\ \therefore I &= \mathcal{L} \frac{700 \times 3\frac{1}{3} \times 3}{100} \\ &= \mathcal{L}70. \end{aligned}$$

Example III. If s be the semi-perimeter of a triangle whose sides are a , b , c , then the area of the triangle S is $\sqrt{s(s-a)(s-b)(s-c)}$. What is the area of a triangle of sides 10, 12, 14 cm.?

$$s = \frac{1}{2}(10 + 12 + 14) \\ = 18.$$

$$s - a = 8.$$

$$s - b = 6.$$

$$s - c = 4.$$

$$\begin{aligned} \therefore S &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{18 \times 8 \times 6 \times 4} \\ &= \sqrt{3^2 \times 2 \times 2^3 \times 2 \times 3 \times 2^2} \\ &= 3 \times 2^3 \sqrt{2 \times 3} \\ &= 24 \sqrt{6} \\ &= 58.8 \text{ sq. cm.} \end{aligned}$$

Further examples of a more difficult nature will be found under Examples XXXII. on Logarithms.

EXAMPLES. XXV 25

1. The time, T , taken by a railway train travelling at V miles per hour to cover a distance of S miles, is given in hours by the formula $S = VT$. How long will it take a train travelling at 35 miles per hour to cover the distance between London and Cambridge (56 miles)?

2. Find the area to be papered in a room 13 ft. high, 20 ft. long, and 17 ft. wide, from the formula $A = HP$.

3. $s = 16t^2$. Using this fact, find how long a stone let fall from a balloon 961 ft. above the earth will take to reach the ground.

4. Find the volume of a rectangular prism, sides 8.1 cm., 7 cm., 5.3 cm. $V = abc$.

5. The density of a substance is given by $D = \frac{M}{V}$. What is the density in gm. per cu. cm. of a metal ball of mass 100 gm. and of volume 14 cu. cm.?

6. Using the formula in Question 1, find the speed necessary in order that a train may cover the distance between Bristol and Paddington ($108\frac{1}{2}$ miles) in 2 hours.

7. The total surface area of a cone, radius r and slant height l , is $\pi r(l + r)$. Find the area of paper required completely to cover a cone of radius 3 in. and slant height 17 in.

8. $D = \frac{M}{V}$. What volume of zinc, density 7.1 gm. per cu. cm., will weigh 100 gm.?

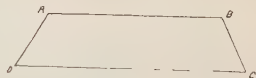
9. Find the simple interest on £500 for 4 years at 3 % per annum, given that $100I = PRT$.

10. The cost price, £C, of an article sold for £S at a profit of $r\%$ is found from the formula $C = \frac{100S}{100 + r}$. What would be the cost price of an article sold for £2. 15s. at a profit of 10 %?

11. What percentage of profit is made by selling cloth costing 2s. 6d. per yard at 3s. 3d. per yard?

12. The time of swing of the pendulum of a clock is $\pi\sqrt{\frac{l}{32}}$, where l feet is the length of the pendulum, and the time is reckoned in seconds. Find the time of swing of a pendulum 3.2 ft. long.

13. The area of a trapezium $= \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the distance between them. Find the area of the trapezium ABCD.



14. What is the radius of a circle of area 1600 sq. cm.?

$$r = \sqrt{\frac{A}{\pi}} \quad [\text{Take } \sqrt{\pi} = 1.772.]$$

15. The amount realised when £P is put out at compound interest for n years at $r\%$ per annum is given by $A = P\left(1 + \frac{r}{100}\right)^n$. Find the amount when £700 is invested for 2 years at 3 %.

16. Using the formula in Question 10, find what must be the selling price of an article which cost originally £2. 10s., in order to gain $12\frac{1}{2}\%$.

17. If W and w be the weight of a body in air and water respectively, then the specific gravity of the body $= \frac{W}{W - w}$. Apply this to find the S.G. of a body which weighs 100 gm. in air and 91 gm. in water.

18. $100I = PRT$. In how many years will the simple interest on £750 amount to £135 at 4 % per annum?

19. The area of a circular disc an inch wide is $\pi(r_1 + r_2)$ sq. in., where r_1 and r_2 are the external and internal radii respectively. Find the area of a disc whose radii are 11.4 in. and 10.4 in. respectively.

20. The length of the hypotenuse of a right-angled triangle is $\sqrt{a^2 + b^2}$, where a and b are the lengths of the sides. By how much does the hypotenuse exceed the greater of the other two sides of a right-angled triangle, whose sides containing the right angle are 15 cm. and 17 cm. respectively?

21. The volume of a segment of a sphere, radius r , of height h , is $\pi h^2 \left(r - \frac{h}{3} \right)$. Find the volume of a segment of height 9 cm. cut from a sphere of radius 12 cm.

22. A glass flask which would hold V cu. cm. of water at 0°C . will hold $\frac{.267Vt}{10^4} + V$ when heated to a temperature of $t^\circ \text{C}$. How much will a flask hold when heated to 100°C . if it holds 500 cu. cm. at 0°C .?

23. In order to change from Fahrenheit readings to centigrade readings we use the formula $C = \frac{5}{9}(F - 32)$. Find the centigrade readings corresponding to 60°F ., 0°F ., 32°F .

24. Deduce from the above a formula for F in terms of C , and thence find Fahrenheit readings corresponding to 60°C ., 0°C ., 32°C .

25. The area of a triangle is $\frac{1}{2}bh$, where b is the base and h the perpendicular height. Denoting this by S , the volume of a pyramid of height H on a triangular base is $\frac{1}{3}SH$. Find the volume of a pyramid of height 15 cm. on a base in the form of a triangle for which $b = 10$ cm. and $h = 8.2$ cm.

26. The weight, W tons, of a gun is connected with its calibre, d inches, by the formula $W = .027d^3$. What is the weight of a 9-inch gun?

27. If b , d , l be the breadth, depth, and length in inches of a beam of oak, the load that can be supported at the middle of the beam (in lb.) is $840 \frac{b d^2}{l}$. Find the load supported by a beam 12 ft. long, 8 inches broad, and 12 inches deep.

28. The weight of coal consumed per hour in a steamer is $\left\{ .2 + \left(\frac{v}{10} \right)^3 \right\}$ tons, where v is the speed in knots. How much coal is consumed in a day of 24 hours by a steamer whose average speed is 8.5 knots?

29. $F = 2.4 + .117 W$ gives the friction in pounds weight on a pulley wheel on which is a load of W lb. Find the friction for a load of 98 lb.

30. In a certain pension scheme the amount receivable each year is given by $P = 30 - .36I$, where I is the yearly income (not to exceed £70). What pension should be drawn by persons whose incomes are (i) £30, (ii) £52?

§ 26. PROPORTION

Consider the following table giving the circumferences of circles and their radii :—

Circumferences in cm.	6.28	12.56	18.84	31.40
Radii in cm.	1	2	3	5

The ratio of any two corresponding numbers, *e.g.* $12.56 : 2$, is the same, *viz.* 6.28.

The circumference of a circle is said to be **directly proportional** to its radius.

Similarly the number of francs equivalent to pounds in English money may be tabulated thus :—

Francs	25	50	125	62.5	1000
Pounds	1	2	5	2.5	40

These numbers form a series, each pair of which gives the same ratio as any other pair: they are therefore a series in direct proportion.

If we are given any three out of four numbers which are proportional, we can easily find the fourth.

Example I. 19 books cost £2. 7s. 6d.; what would be the cost of 15?

If $£x$ be the cost of the 15 books, we know that

$$\begin{aligned}\frac{19}{15} &= \frac{2\frac{7}{8}}{x} \\ \therefore 19x &= 15 \times \frac{19}{8} \\ \therefore x &= \frac{15}{8} \\ \therefore \text{cost of 15 books} &= \pounds \frac{15}{8} \\ &= \pounds 1. 17s. 6d.\end{aligned}$$

Such examples as the above may be more conveniently worked by the unitary method (§ 20).

Suppose we consider the time taken by trains running at

different speeds to cover the same distance, say 12 miles. Then we have—

Speed in miles per hr.	60	48	36	40	54	20
Time in min.	12	15	20	18	$13\frac{1}{3}$	36

These numbers are said to be in **inverse proportion**, and are such that the product of any pair, *e.g.* 54 and $13\frac{1}{3}$, is the same, viz. 720.

Example II. A train travelling at 60 miles per hr. takes 36 min. to cover the distance between two towns. How long would it take a train travelling at 48 miles per hr. to traverse the same distance?

Since the distance covered in both cases is the same, the product of the rate and the time must be the same.

∴ if x min. be the time taken by the second train,

$$\begin{aligned} 60 \times 36 &= 48 \times x. \\ \therefore x &= \frac{60 \times 36}{48} \\ &= 45 \text{ min.} \end{aligned}$$

More complicated cases of proportion will now be dealt with.

Example III. 50 men dig 130 yards of a trench in 12 days; how long would it take 20 men to dig 39 yards?

130 yards are dug in 12 days.

Then 1 yard could be dug in $(\frac{1}{130} \text{ of } 12)$ days.

∴ 39 yards $\{39 \times (\frac{1}{130} \text{ of } 12)\}$ days.

50 men could dig 39 yards in $\{39 \times (\frac{1}{130} \text{ of } 12)\}$ days.

Then 1 man $50 \times \{39 \times (\frac{1}{130} \text{ of } 12)\}$ days;

∴ 20 men $\frac{50}{20} \times \{39 \times (\frac{1}{130} \text{ of } 12)\}$ days.

$$\begin{aligned} \therefore \text{time} &= \frac{50}{20} \times \frac{39}{1} \times \frac{1}{130} \times \frac{12}{1} \text{ days} \\ &= 9 \text{ days.} \end{aligned}$$

We may have other laws than direct or inverse proportion. For example, the area of a circle is proportional to the square of the radius, and the volume of a sphere is proportional to the cube of the radius.

Example IV. A stone falls so that the distance dropped is proportional to the square of the time. When dropped from a height of 100 ft. it takes 2.5 sec. to fall; how long will it take to fall through a height of 4900 ft.?

If we translate the words of the question into algebraic language we get the fact that $\frac{d}{t^2}$ has always the same value.

Then in the first case $\frac{d}{t^2} = \frac{100}{(2.5)^2},$

and in the second case $\frac{d}{t^2} = \frac{4900}{t^2}.$

$$\therefore \frac{100}{(2.5)^2} = \frac{4900}{t^2}.$$

$$\therefore t^2 = \frac{4900 \times (2.5)^2}{100}$$

$$= 49 \times (2.5)^2.$$

$$\therefore t = 7 \times 2.5.$$

$$\therefore \text{time} = 17.5 \text{ sec.}$$

EXAMPLES. XXVI 26

1. Show that the numbers $\frac{8}{15}, \frac{3.2}{6}, \frac{16}{30}, \frac{2.4}{4.5}$ are directly proportional.

2. The numbers $\frac{5}{20}, \frac{2}{8}, \frac{3}{12}, \frac{x}{4}, \frac{.8}{3.2}, \frac{.7}{y}$ are in direct proportion. Find x and y .

3. Prove that $\frac{8}{15}, \frac{3.2}{37.5}, \frac{2}{60}, \frac{40}{3}$ are inversely proportional.

4. $\frac{5}{20}, \frac{2}{50}, \frac{3}{33\frac{1}{2}}, \frac{4}{x}, \frac{8}{12\frac{1}{2}}, \frac{z}{5}$ are in inverse proportion. Find x and z .

5. 3 men can do a piece of work in 8 days. How long would it take 2 men to do the same piece of work?

6. 5 taps turned on together fill a tank in 4 hours. If 6 taps had been turned on, how long would it have taken to fill the tank?

7. Two customers spend the same amount in a shop. One buys 5 lb. of tea at 1s. 8d. per lb.; the other buys sugar at $2\frac{1}{2}$ d. per lb. How much sugar did the second customer buy?

8. A train whose rate is 45 miles per hour completes a journey in 2 hr. 15 min. Another train completes the same journey in 4 hr.; what is its speed?

9. It takes 13 horses 24 days to plough a field; how many less would have been needed if 39 days had been allowed for the work?

10. With 8 gas burners in use, 1000 cu. ft. of gas are used in 32 hr. How long would the same quantity last 15 burners?

11. The sectional area of a pipe emptying a tank is 13.45 sq. cm.; it can empty the tank in 69 min. An extra pipe of sectional area 9.55 sq. cm. is used, and the two pipes empty the tank. How long will it take?

12. 35 men earn £105 in 14 days; what will 25 men earn in 21 days at the same rate?

13. The cost of a railway ticket is proportional to the distance; 12 tickets to a town 39 miles distant cost 32s. 6d. What would be the cost of 13 tickets to a town 48 miles distant?

14. A vessel of 6400 horse-power consumes 384 tons of coal in 3 days; find the amount of coal used per hour per horse-power.

15. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days, by 2 women and 4 boys in 10 days. At this rate how long would a woman take to do the work single-handed?

16. If 2 tons 14 cwt. of biscuit had to serve a ship's crew of 182 men and 15 boys for a fortnight, and if each man's allowance is half as large again as each boy's, find how much each man and each boy would get per day.

17. A pint of water weighs $1\frac{1}{4}$ lb.; a cu. ft. of water weighs 1000 oz. Find the number of cu. in. in 7 gall.

18. 56 men build 448 yd. of a wall 1000 yd. long in 27 days. How many extra men must be employed in order that the wall may be finished in 23 days more?

19. £1 = 25.2 fr., and 1 metre = 39.37 in. Find the cost per yard, in shillings and pence, of a metre of silk costing 19 fr. 25 c.

20. An airman flew from Lyons to Villacoublais, a distance of 310 miles, in 3 hr. 10 min. Find his speed in Km. per hr. if 1.6 Km. = 1 mile.

21. 6 compositors in 16 days of $10\frac{1}{2}$ hours each set in type 720 pages, each of 2400 letters. In how many days of 7 hours each will 9 compositors set 960 pages, each of 2250 letters?

22. A beam of wood 18 ft. long, 3 ft. broad, and 9 in. thick weighs 1440 lb. What is the length of a beam of the same kind of wood $2\frac{1}{2}$ ft. broad, 6 in. thick, weight 1200 lb.?

23. The area of a circle is proportional to the square of its radius. The area of a circle, radius 5 cm., is 78·25 sq. cm. Find the area of a circle, radius 11 cm.

24. A cubical tank, side 4 ft., contains when full 400 gallons. What is the capacity of a tank, side 2 ft. 6 in. ? Give your answer correct to the first decimal place.

25. Two plates are cut from a piece of copper. The first plate is 17·1 cm. long, 12·2 cm. broad, and 2 cm. thick ; the second is 15·3 cm. long, 10·1 cm. broad. Find the thickness of the second plate, the weights of the plates being 2920·68 gm. and 2704·275 gm. respectively.

26. The volume of a certain quantity of gas is inversely proportional to the pressure. When the pressure is 14·7 lb. per sq. in. the volume is 386 cu. cm. Find to the nearest whole number the volume when the pressure is 14·9 lb. per sq. in.

27. The current flowing through a wire is inversely proportional to the electrical resistance of the wire. A wire through which a current of ·9 amperes is flowing has a resistance of 4 ohms. If the wire be replaced by one of resistance 2·7 ohms, what will be the increase in the current ? Draw a graph connecting current and resistance.

28. A sphere of radius 2 cm. has a volume of 33·52 cu. cm. Find the volume of a sphere of half the radius.

29. Two pieces of iron wire are of the same volume. The length of one is 123 cm. and its diameter is ·1 cm. ; the diameter of the second is ·08 cm. Find its length.

30. When a man stands so that the height of his eye above sea-level is h feet, the distance of his horizon may be found from the formula $D^2 = 2\cdot0164h$, where D is measured in miles. Plot a curve connecting D and h , and hence find D when $h = 125$ ft.

31. A penny and a halfpenny are made of the same thickness of bronze. A halfpenny weighs $\frac{1}{5}$ oz. and is an inch across. Find the diameter of a penny given that its weight is $\frac{1}{3}$ oz.

32. On every £1 above £160 of his income a man pays income-tax. At $7\frac{1}{2}\text{d.}$ in the £ he has to pay £10. What should he pay if the tax were increased to 9d. in the £ ?

33. The pressure of the air in lb. on a given surface is proportional to the square of the velocity in ft. per sec. If the pressure is ·17 lb. when the velocity is 18 ft. per sec., find the pressure when the velocity is 9 ft. per sec.

34. A current of 10 amperes will plate .4 gm. of silver in 10 hours. How long will it take to plate a gm. using a current of 45 amperes?

35. The gear of a bicycle is inversely proportional to the number of teeth on the front chain wheel. The gear is 57 when there are 10 teeth. What will be the gear when the wheel has 7 teeth?

36. In the United States the price of silver is quoted in cents per troy ounce fine. In order to convert an English quotation into cents per ounce fine, the price must be expressed in pence and multiplied by a certain constant. When the English price is 24.375*d.* the United States price is 53.43 cents: find the United States price when silver is quoted at 24.75*d.*

37. The area of an ellipse is proportional to the product of the lengths of its two axes. Two ellipses of equal area have their longer axes 20 cm. and 16 cm. respectively. The length of the shorter axis of one is 6.36 cm. What is the length of the shorter axis of the second?

38. The effort E required to lift a weight W is given by a law $E = aW + b$: find a and b , knowing that $E = 3$ when $W = 7$, and $E = 14$ when $W = 42$. Hence draw a graph connecting E and W , and find W when $E = 16$.

39. To stretch wires of given length so that they may emit the same musical note, weights must be hung at the end: these weights are proportional to the square of the length of the wire. If there be two wires of lengths 40 and 50 cm., and a weight of 15 Kg. is attached to the first, find the weight that must be attached to the second.

40. The velocity of sound in a gas is inversely proportional to the square root of the density of the gas. In air of density .001293 the velocity is 33180 cm. per sec. Find the velocity of sound in hydrogen (density .00009).

§ 27. PERCENTAGES

Example I. What is the value of 15 % of £28?

$$\begin{aligned} 15 \% \text{ of } £28 &= £\left(\frac{15}{100} \text{ of } 28\right) \\ &= £\frac{420}{100} \\ &= £4.2 \\ &= £4. 4s. 0d. \end{aligned}$$

Example II. Express $\frac{25}{32}$ as a rate per cent.

We may use the fact that 7 % of anything is $\frac{7}{100}$ of that thing.

$$\text{Let } \frac{25}{32} = x \%.$$

$$\text{Then } \frac{25}{32} = \frac{x}{100}.$$

$$\therefore x = \frac{25 \times 100}{32}$$

$$= 78\frac{1}{8}.$$

$$\therefore \frac{25}{32} = 78\frac{1}{8} \%.$$

Example III. 35 is 7 % of a number. What is the number?

$$35 \text{ is } 7 \% \text{ of } x,$$

$$i.e. \quad 35 \text{ is } \frac{7}{100} \text{ of } x.$$

$$\therefore 35 = \frac{7}{100}x.$$

$$\therefore x = \frac{35 \times 100}{7}.$$

$$\therefore x = 500,$$

$$i.e. \text{ the number is } 500.$$

Note that to divide by 100 it is only necessary to move the decimal point two places to the left, and to multiply by 100 to move it two places to the right.

Example IV. In a class of 28 boys, 11 wear spectacles. What percentage of the number of boys is this?

$$\frac{11}{28} \text{ of the number wear spectacles,}$$

$$i.e. \cdot 39285... \text{ of the number wear spectacles.}$$

$$\therefore \text{ required percentage} = 39\cdot285...$$

EXAMPLES. XXVII 27

1. Write down the following rates per cent. as fractions in their lowest terms:—2, 5, 10, 15, 25, $33\frac{1}{3}$, $12\frac{1}{2}$, $66\frac{2}{3}$, 75, 85.

2. Express the following fractions as rates per cent.:— $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{1}{10}$, $\frac{1}{25}$, $\frac{1}{16}$, $\frac{1}{6}$, $\frac{1}{30}$.

3. Find (i) 2 %, (ii) 10 %, (iii) 25 %, (iv) $33\frac{1}{3}$ % of—

(a) £750.

(β) 15 Km.

(γ) 8s. 4d.

(δ) A ton.

(ε) A guinea.

4. What percentage is—

(i) 3 of 24?

(ii) 8d. of 3s.?

(iii) A pint of a quart?

(iv) 11 yd. of a mile?

(v) 50 pages of a book of 250 pages?

5. 25 % of a number is 72 ; find the number.
6. 10 % of a class of boys leave ; what percentage remains ?
7. 35 % of a number is 56 ; what is the number ?
8. If $12\frac{1}{2}$ % of a man's income is £35, what does he earn ?
9. 9 is 15 % of what number ?
10. 50 is x % of a certain number. Show that the number is $\frac{5000}{x}$.
11. A man has £20. By investing it he increases the sum by 10 %. What does it become ?
12. If a tradesman buys goods for 18s. and sells them for 22s. 6d., find his gain per cent.
13. In a school of 500 boys, 29 are in the Sixth Form. What percentage of the number of boys is this ?
14. A newsagent generally charges $4\frac{1}{2}$ d. for a magazine priced at 6d. What percentage does a customer pay of the marked price ?
15. Duty is charged at the rate of 15 %. A man pays 9s. on an article ; what is its value ?
16. What percentage is 3d. of £1 ?
17. In a box of 150 oranges a tradesman finds that 27 are bad. What percentage is good ?
18. If 10 % of a certain number is 32, what is 18 % of the same number ?
19. Prove that 11 % of £2. 5s. is the same as the sum of 8 % of £2. 5s. and 3 % of £2. 5s.
20. The population of a town increases from 5000 to 5750. By how much per cent. has it increased ?
21. Find to the nearest whole number—
 (i) 17 % of £204. (ii) 33 % of 17d.
 (iii) 8 % of 7351. (iv) What percentage 5 is of 11.
22. Find correct to two places of decimals—
 (i) $4\frac{1}{4}$ % of a mile. (Answer in yards.)
 (ii) $2\frac{1}{8}$ % of 17 guineas. (Answer in £ s. d.)
 (iii) What percentage 3.72 is of 11.08.
 (iv) What percentage 23 is of 29.
23. Express as rates per cent. correct to two places of decimals—(i) $\frac{3}{7}$, (ii) $\frac{1}{3}$, (iii) $\frac{1}{8}$, (iv) $\frac{2}{11}$, (v) $\frac{9}{17}$.

24. What percentage is 1s. in the pound? Hence write down what percentage the following represent:—

- | | |
|------------------------|----------------------------|
| (i) 6d. in the pound. | (ii) 2s. 6d. in the pound. |
| (iii) 2s. 6d. in £100. | (iv) 5s. in £100. |
| (v) 3d. in the pound. | |

25. A man charges commission at the rate of $\frac{1}{2}\%$. What should be paid on £2560?

26. Show that a profit of £6 on £144 gives a return of $4\frac{1}{6}\%$.

27. A boy in drawing a right-angled triangle makes the right angle a degree too small. What is the percentage error?

28. A penny is $\frac{1}{20}$ copper, $\frac{1}{25}$ tin, and the rest zinc. Express as percentages the amounts of each metal in the coin.

29. The price of silver per oz. averaged $24\frac{9}{16}$ d. in 1911 as against $24\frac{1}{16}$ d. in 1910. Find the percentage decrease in price correct to three places of decimals.

30. A tradesman gives $2\frac{1}{2}\%$ discount for cash. By this means a customer saves 5s. What was the amount he paid?

31. A bankrupt who owed £1575 paid his creditors £225. What is this expressed as a rate per cent.? How much did he pay in the pound? (To nearest penny.)

32. In a litre of dilute sulphuric acid 25 cu. cm. is water. What percentage is pure acid?

33. A coal merchant defrauds his customers to the extent of 1% by taking some coal out of each sack before selling it. Assuming that he takes the same amount from each sack, how much do 20 sacks, each supposed to weigh a cwt., really weigh? (Answer to the nearest lb.)

34. The death-rate in a certain town for April 1913 was 2.01%. If the population of the town was 35728, find how many people died during the month.

35. The following statement compares the religious professions of the inhabitants of Ireland in 1911:—

Roman Catholics . . .	73.9 %
Protestants . . .	24.5 %
Others . . .	1.6 %

If the total population was 4390219, find how many people there were who were (i) Roman Catholics, (ii) neither Roman Catholics nor Protestants.

36. A ton of Atlantic water yields 8 pounds of salt. Express the weight of salt in the water as a rate per cent. correct to four places of decimals.

37. The number of members of a society increased from 3100 in 1911 to 3794 in 1912. Find the percentage increase.

38. A man saws wood, and in doing so wastes $2\frac{1}{4}\%$. What volume does he waste if he saws a plank 15 ft. long, 10 ft. wide, and 9 in. thick?

39. The report of a mining company stated that in a certain week 6050 tons of crude ore was treated, yielding 16% lead. What weight of lead was obtained?

40. If $x\%$ of a certain number is y , find the number. What is $z\%$ of the number?

41. In 1910 the French Government received 21.4 millions of pounds for tobacco. Expenses amounted to 5.4 millions of pounds. Find the profit per cent. expressed as a decimal correct to two decimal places.

42. In 1886 the total value of merchandise imported into and exported from the United Kingdom amounted to £618823000. In 1905 this figure was increased by 50% . Find the value in 1905. It is reckoned that the receipts for 1913 will be £1340000000. By how much per cent. will the value have increased over that for 1905?

43. Of the world's submarine vessels, Great Britain owns 65. The total number in use is 155. Express the number owned by Great Britain as a rate per cent. correct to two places of decimals.

44. The price per cwt. of sugar has increased by 31.2% over the price in 1907. Find to the nearest penny the price per cwt. at present, given that the price in 1907 was 19s.

45. A man's income decreases from £110 to £100. What is the decrease per cent.?

46. From a cask containing a gallon of wine a pint is drawn off. Water is poured in so that the amount of liquid remains the same. What percentage of the mixture is water?

47. The total output of pins for Europe each day is 84 millions; of these Birmingham produces 37 millions. Find, correct to two places of decimals, the output of Birmingham expressed as a percentage.

48. 5 tons of concrete consist of 1 ton of cement and the rest sand and ballast in the proportion 3 : 5. What percentage of concrete is sand?

49. In a 1912 catalogue the following prices were quoted for different sizes of machines: £11. 5s., £15. 10s., £17. 5s., £25. In the catalogue for the year 1913 the same figures were given with the note "10 % advance" against them. Find the prices for 1913.

50. A mine produced 1015 tons of ore containing 23345 oz. silver. What percentage of the ore was silver? Answer correct to two places of decimals.

51. The total original amount of a Brazilian loan was £4599600. 63·5 % of this was quoted in London: to how much does this amount?

52. The numbers of people visiting the Tate Gallery during 1912 were roughly 260250 on Free Days, 62000 on Sundays, and 39000 on Students' Days. What percentage of the total number went on Sundays? Give your answer to the nearest whole number.

53. The temperature of water in a flask decreased from 50° C. to 40° C. Show that this is a different percentage decrease than from 55° C. to 45° C. If the temperature had increased from 40° C. to 50° C., what would have been the percentage increase?

54. During the months January to March 1913, 70·25 million pounds of tea were consumed in the British Isles. 60 % of this came from India: what is this in millions of pounds?

55. Genuine milk contains 2·75 % of fat, 8·5 % of non-fatty solids, and the rest water. Find to the nearest pint the amount of water in 9 gallons of milk.

56. Of a regiment after 1·5 % have been killed, 8·75 % wounded, and 12 % invalided there still remain 933 men available; what was the original strength of the regiment?

57. The rateable value of a certain town is £104316. Find to the nearest penny the smallest rate per £100 which must be levied in order to raise a sum of not less than £4836.

58. In a certain country the birth-rate per annum is 4·1 % and the death-rate 2·5 % at the beginning of the year. If the

population at the beginning of 1880 was 4000000, find to the nearest thousand what it was at the beginning of 1877, assuming no immigration or emigration.

59. Britannia metal contains 84 % tin, 10 % antimony, 4 % copper, and the rest bismuth. Find by how much the amount of tin exceeds the amount of bismuth in a teapot weighing 3 lb.

60. Coal gas contains 32.87 % marsh gas, 12.89 % carbon monoxide, the rest being a mixture of hydrogen and other gases. The percentage of hydrogen in this mixture is 93.2 %. How much hydrogen is there in 100 litres of coal gas?

61. A sum of money put out at interest increases 5 % in a year; it increases 5 % in the next year. Why is the total increase not 10 %?

62. The floor of a room is 15 ft. long and $12\frac{1}{2}$ ft. wide. It is covered by a carpet so as to leave an 18-in. border all the way round. What percentage of the area of the floor is uncovered?

63. Three lists are prepared after an examination, namely of those who passed in the first class, those who passed in the second class, and those who failed. 1275 candidates presented themselves; 40 % failed, and 35 % of those successful were placed in the first class. How many passed in the second class?

64. 150 gm. of English brass, containing $33\frac{1}{3}$ % zinc and the rest copper, are melted down with the same weight of Dutch brass (which contains the same two metals as constituents but with half the amount of zinc). In the resulting alloy, what is the weight of copper?

65. A ton of ore taken from a certain mine showed 3.07 % of copper and 2.12 % of lead. How many more pounds of copper were there than of lead?

66. In an election 63 % of those who voted voted for the successful candidate. His majority was 1352. How many people voted?

67. x gallons of a mixture containing y % water are mixed with a gallons of another mixture containing b % water. What percentage of the resulting mixture is water?

§ 28. PROFIT AND LOSS

The gain or loss per cent. is, unless stated otherwise, reckoned on the **Cost Price**.

Example I. A man buys an article for £16, and sells it at a gain of 8%. Find the selling price.

If the cost price is £100, the selling price is £108.

$$\therefore \dots\dots\dots \text{£16,} \dots\dots\dots \text{£} \left(\frac{108 \times 16}{100} \right).$$

$$\therefore \text{selling price} = \text{£} \frac{108 \times 16}{100}$$

$$= \text{£17.28}$$

$$= \text{£17. 5s. 7d.}$$

(to the nearest penny)

Example II. A man sells an article for £16, at a gain of 8%. Find the cost price.

If the selling price is £108, the cost price is £100.

$$\therefore \dots\dots\dots \text{£16,} \dots\dots\dots \text{£} \left(\frac{100 \times 16}{108} \right).$$

$$\therefore \text{cost price} = \text{£} \frac{100 \times 16}{108}$$

$$= \text{£14} \frac{22}{27}$$

$$= \text{£14. 16s. 4d.}$$

(to the nearest penny).

Example III. A man sells an article for £16 at a gain of 8%. At what price must he sell it to gain 17%?

If the cost price is £100, the two selling prices are £108 and £117.

£108 is the selling price corresponding to £16.

$$\therefore \text{£117} \dots\dots\dots \text{£} \left(\frac{16 \times 117}{108} \right).$$

$$\therefore \text{required selling price} = \text{£} \frac{16 \times 117}{108}$$

$$= \text{£17} \frac{1}{3}$$

$$= \text{£17. 6s. 8d.}$$

We may employ the method illustrated in Examples II. and III. on page 112 thus:—

Example IV. A man buys an article for £16, and sells it at a gain of 8%. Find the cost price.

Let $\text{£}x$ be cost price.

Then £16 is $\frac{108}{100}$ of $\text{£}x$,

$$\text{i.e., } \text{£16} = \text{£} \frac{108}{100} x.$$

$$\therefore \text{£}x = \text{£} \frac{16 \times 100}{108}$$

$$= \text{£17.28}$$

$$= \text{£17. 5s. 7d.}$$

Examples II. and III. above may be done in a similar manner.

EXAMPLES. XXVIII 28

1. Find the gain or loss per cent. in the following cases:—

(i) Cost price, £2.	Selling price, £3.
(ii) £5.	 £6. 10s.
(iii) 3s.	 2s. 9d.
(iv) $4\frac{1}{2}d.$	 6d.
(v) £110.	 £100.

2. What is the gain or loss per cent. when—

(i) Cost price is £275.	Selling price, £286.
(ii) £18.	 £16.
(iii) £12.	 £16.
(iv) £12.	 £8.
(v) £15. 2s. 6d.	 £12. 17s. 6d.?

3. Find the selling price in the following cases:—

(i) Cost price, £25.	Gain per cent., 8.
(ii) £2.	Gain 25.
(iii) £14.	Gain $7\frac{1}{2}\%$.
(iv) 6d.	Loss 60.
(v) $4\frac{1}{2}d.$	Loss $33\frac{1}{3}\%$.

4. What is the cost price when—

(i) Selling price £252	represents a Gain of 8%
(ii) £2	 Gain ... 25%
(iii) 14s. 4d.	 Gain ... $7\frac{1}{2}\%$
(iv) 6d.	 Loss ... 60%
(v) $4\frac{1}{2}d.$	 Loss ... $33\frac{1}{3}\%$?

5. A tradesman buys sugar at 14s. per cwt. and sells it at $2\frac{1}{4}d.$ per lb. Find his gain per cent.

6. A book is sold for 6s., at a gain of 20% . What did it cost?

7. Cigars are priced at £2. 10s. per 100. A tobacconist buys them at this rate and sells them at 8d. each. Find his gain per cent.

8. A man sold a carriage for £29. 18s. 9d., at a loss of $16\frac{2}{3}\%$. What did it cost him?

9. A grocer buys a cheese weighing 53 lb. for £2. 4s. 2d. He sells it at 1s. per lb. Find his gain per cent.

10. In 1907 the average price of a cwt. of rice was 11s. 6d. This increased to 12s. 5d. in 1912. Find the percentage increase in the price of a cwt.

11. A gas engine is bought by a wholesale dealer for £17. 5s. He sells it so as to gain 10%. Find the price at which he sells it.

12. Electric energy costs a company $\cdot 15$ of a penny per unit. They retail it at a selling price of $\frac{3}{4}$ d. per unit. How much do they gain per cent?

13. Suit-cases are sold for 32s. each, representing a gain of 28%. What is the cost price?

14. In their annual report a tea company stated that the cost per pound of tea was 4.53d. The average price realised for the crop was 7.59d. Find correct to 2 decimal places the gain per cent.

15. A man buys some articles for £200. He sells half of them at a gain of 18% and the other half at a gain of 21%. Find his total gain per cent.

16. A grocer buys a cwt. of tea for £7. 9s. 4d.; he sells 56 lb. at the rate of 1s. 3d. per lb., and the remainder at 1s. 6d. per lb. What does he gain or lose per cent.?

17. A greengrocer buys 100 oranges for 5s. 25 go bad, and he sells the rest at 1d. each. How much does he gain per cent.?

18. By selling a book for 6s. a tradesman gains 50%. At what price should he sell it so as to gain $62\frac{1}{2}\%$?

19. A man sells a horse for £96. 10s. Find his gain or loss per cent., knowing that if he had sold it for £85 he would have lost $7\frac{1}{2}\%$.

20. The value of an article decreases, and a tradesman sells it for 2s. 1d., at a loss of 25%. At what price should he have sold it so as only to lose $12\frac{1}{2}\%$?

21. A manufacturer sells an article to a wholesale dealer so as to gain $12\frac{1}{2}\%$. The dealer retails it to a tradesman for £35. 8s. 9d. and thus gains $12\frac{1}{2}\%$ on his outlay. How much did the article cost the manufacturer?

22. Goods sold for a shilling each represent a loss of 4%. What would be the gain or loss per cent. if they were sold for 1s. 0 $\frac{3}{4}$ d. each?

23. A tradesman allows a customer 10% discount off his bill and still makes a profit of 8%. What would have been his gain per cent. if he had allowed no discount?

24. By selling tea at 2s. 3d. per lb. a grocer gains $12\frac{1}{2}\%$. What price does another grocer charge who sells the same tea so as to gain 20%?

25. The difference between the amounts obtained by selling an article so as to gain $33\frac{1}{3}\%$ and selling the same article so as to gain $12\frac{1}{2}\%$ is $2\frac{1}{2}d.$ Find the cost price of the article.

26. A manufacturer sells an article to one dealer so as to gain 14% and to another, who buys in large quantities, so as to gain 13%. The first dealer sells the article at a profit of 13% on his outlay, and the second at a profit of 14%. Is there any difference in the two selling prices if the cost price to the manufacturer is £20?

27. A grocer mixes 1 lb. of tea at 1s. 6d. with 1 lb. at 1s. 8d., and sells his tea at 1s. 10d. per lb. Find his gain per cent.

28. Milk costs a tradesman 2d. per quart. He adds a pint of water to every quart, and sells the mixture at 2d. per pint. Find his gain per cent.

29. Sugar costing 2d. per lb. is mixed in equal ratios with a better kind costing 3d. per lb., and the mixture is sold for $2\frac{3}{4}d.$ per lb. What is the gain or loss per cent.?

30. Coffee at 1s. 3d. per lb. is mixed in equal proportions with chicory at 6d. per lb. The mixture is retailed at 1s. 1d. per lb. What is the gain per cent.?

31. 10 galls. of spirit at 12s. per gall., 2 galls. at 10s. per gall., and 8 galls. at 5s. are mixed with 20 galls. of water. The mixture is sold at 8s. 4d. per gall. Find the gain per cent.

32. A merchant buys two kinds of tea, one at 1s. 6d. per lb. and the other at 1s. $7\frac{1}{2}d.$ per lb. He sells the mixture at 1s. 9d. per lb., thereby gaining $12\frac{1}{2}\%$; in what proportion did he mix them?

33. A grocer has two kinds of tea which cost him 2s. 8d. per lb. and 2s. 1d. per lb. respectively. Find how many lb. of the former kind he must mix with 56 lb. of the latter so that by selling the mixture at 2s. 11d. per lb. he may gain 25%.

34. How much coffee costing 1s. 1d. per lb. must be mixed with 1 cwt. at $10\frac{1}{2}d.$ per lb. so that 25% may be gained by selling the mixture at 1s. 3d. per lb.?

35. A milkman dilutes a number of cans of milk by withdrawing some milk from each can and adding water. What percentage must he withdraw from each can so that he may gain 25 % by selling the mixture at the same price per pint as he paid for the milk?

36. Goods are sold so that when 10 % is allowed off the selling price a profit of 10 % is made. Find how much per cent. the selling price is greater than the cost price.

37. A tradesman allows a discount of 1s. 3d. in the £ off the marked price, which is such that if sold at that price 20 % would be gained. What percentage of profit does the tradesman make?

38. A profit of 20 % is made on goods when a discount of 10 % is allowed off the marked price. What profit per cent. would be made if 15 % discount were allowed?

39. By selling an article for £ x a merchant makes a profit of 15 %. At what price should he sell it so as to gain 20 %?

40. Goods are sold by a manufacturer to a dealer so that the manufacturer makes a profit of 10 %. The dealer sells the goods to a tradesman for £ x , so as to gain 10 %. Find the cost price of the goods to the manufacturer.

41. Find the gain or loss per cent. by selling material costing 80 c. per m. for 7d. per yard. (25 fr. = £1 ; 39·37 in. = 1 m.)

§ 29. PRACTICE

Example I. Find the cost of 1181 articles at £3. 14s. 2½d. each.

	£	s.	d.
Cost of 1181 articles at £1 each is	1181	0	0
£3 = 3 × £1, ∴	3543	0	0
12s. = $\frac{1}{5}$ of £3, ∴	708	12	0
2s. = $\frac{1}{6}$ of 12s., ∴	118	2	0
2d. = $\frac{1}{12}$ of 2s., ∴	9	16	10
$\frac{1}{2}$ d. = $\frac{1}{4}$ of 2d., ∴	2	9	2½
∴ cost of 1181 articles at £3. 14s. 2½d. each is	£4382	0	0½

The method of Practice may be conveniently used when working with decimals thus :—

Example II. What is the value of 313 times £7. 9s. 2d.?

	313 × £1	is	£313	
£7 = 7 × £1,	∴ 313 × £7	...	2191	
10s. = $\frac{1}{2}$ of £1,	∴ 313 × 10s.	...	156·5	
	∴ 313 × £7. 10s.	...	2347·5	(α)
10d. = $\frac{1}{12}$ of 10s.,	∴ 313 × 10d.	...	13·0417..	(β)
	∴ 313 × £7. 9s. 2d.	...	2334·4583	
	∴ 313 times £7. 9s. 2d. = £2334. 9s. 2d.			

Subtract the line (β) from the line (α); work to four places of decimals. Line (α) may be omitted and the addition and subtraction performed in one step.

Rough checks should be used wherever possible.

It should be noted that Practice is merely the practical application to arithmetic of the Distributive Law—

$$x \times (a + b + c) = (x \times a) + (x \times b) + (x \times c).$$

Example III. Find the value $9\frac{5}{6}\%$ of £22. 11s. 4d.

$$£22. 11s. 4d. = £22·5667.$$

	£22·5667	
At 9 %	2·0310	(α)
$\frac{1}{2}\%$	·1128	(β)
$\frac{1}{3}\%$	·0752	(γ)
	2·2190	

$$\therefore 9\frac{5}{6}\% \text{ of } £22. 11s. 4d. = £2. 4s. 5d.$$

At line (α) we multiply the sum of money, £22·5667, by 9 and move the decimal point two places to the left; lines (β) and (γ) are similarly obtained by dividing the top line by 2 and 3 respectively and moving the decimal point as before.

This method is very useful in working examples on Simple Interest (§ 33).

Practice is useful in finding the areas of rectangles and triangles.

Example IV. Find the area of a rectangle, sides 9 ft. 3 in. and 2 ft. 7 in.

Area of rectangle 1 ft. by 9.25 ft. is			sq. ft.
			9.2500
2 ft. = 2×1 ft., $\therefore$	2 ft.		18.5000
6 in. = $\frac{1}{2}$ of 1 ft., $\therefore$	6 in.		4.6250
1 in. = $\frac{1}{6}$ of 6 in., $\therefore$	1 in.		.7708
$\therefore$ area of rectangle 2 ft. 7 in. by 9.25 ft. is			23.8958

$\therefore$ area of rectangle = 23.8958 sq. ft.

= 23 sq. ft. 129 sq. in.

Note that .8958 must be multiplied by 144 to obtain the decimal of a square foot as square inches.

One example on Compound Practice will suffice as being typical of the method.

Example V. Find the cost of 12 oz. 12 dwt. 15 gr. of gold at £4. 10s. $2\frac{1}{2}d.$ per oz.

Cost of 1 oz. is		£	s.	d.
		4	10	2.5
12 oz. = 12×1 oz., $\therefore$	12 oz. ...	54	2	6
10 dwt. = $\frac{1}{2}$ of 1 oz., $\therefore$	10 dwt. ...	2	5	1.25
2 dwt. = $\frac{1}{5}$ of 10 dwt., $\therefore$	2 dwt. ...		9	0.25
12 gr. = $\frac{1}{4}$ of 2 dwt., $\therefore$	12 gr. ...		2	3.06
3 gr. = $\frac{1}{4}$ of 12 gr., $\therefore$	3 gr. ...			6.77
Total cost		£56	19	5.33

Result = £56. 19s. 5d.

EXAMPLES. XXIX 29

The answers should be given to the nearest penny, unless otherwise stated.

Find the value of—

1. $153 \times £1. 15s.$

2. $291 \times 1s. 3d.$

3. $1491 \times 4s.$

4. $98 \times £1. 2s. 8d.$

5. $2111 \times £1. 4s. 2d.$

6. $1314 \times 4\frac{1}{2}d.$

7. $6196 \times £5. 10s. 4\frac{1}{2}d.$

8. $3121 \times £2. 5s. 7d.$

9. $1889 \times 15s. 7\frac{1}{2}d.$

10. $1799 \times 13s. 10d.$

Find the cost of—

11. 133 articles at £1. 6s. 4d. each.

12. 159 4s. 2d. ...

13. 887 11s. $4\frac{1}{2}d.$...
14. 973 9s. 11d. ...
15. 1000 £2. 5s. 7d. ...
16. 765 £2. 17s. 8d. ...
17. What is the total weight in 29 sacks each containing 13 cwt. 2 qr. 5 lb.?
18. Find the cost of 4 tons 15 cwt. 56 lb. at £30 per ton.
19. Find the value of 3·75 times £2. 7s. 10d.
20. Multiply £9. 4s. $3\frac{1}{2}d.$ by 7·08.
21. The value of gold is £3. 19s. 1d. per oz. How much will 14·68 oz. of gold be worth?
22. To what does $3\frac{1}{2}\%$ of £953. 10s. amount?
23. Find the value of $7\frac{1}{4}\%$ of £171. 5s. 6d.
24. Use the method of practice to find the value of $71\frac{1}{8}$ times £7. 15s. 8d.
25. By how much does 17·53 times £51. 8s. differ from 51·8 times £17. 5s. 3d.?
26. What is the value of $17\frac{1}{3}\%$ of £150. 4s.?
27. Find the area of a rectangle 3 ft. 4 in. by 7 ft. 7 in.
28. The altitude of a triangle is 1 ft. 8 in.; the length of the base is 2 ft. 1 in. Find the area of the triangle.
29. How many square feet of carpet (to the nearest whole number) will be required for the floor of a room 15 ft. 7 in. by 9 ft. 9 in.?
30. Find the surface area of a cube, side 8 ft. $7\frac{1}{2}$ in.
31. A bankrupt owes £531. 12s. 3d. How much will he pay at 10s. 4d. in the £? (*Reduce 10s. 4d. to the decimal of £1.*)
32. A Portuguese milreis = 3s. $10\frac{1}{2}d.$ Find the value of 1579 milreis in English money.
33. The average cost per head of educating children in a certain county is £14. 15s. 8d. There are 621 children in the schools in the county: find the total cost.
34. Find the price realised for 78115 lb. of rubber at 4s. 11d. per lb.

35. In 1901 the average weekly wage of the 43710 railway servants employed in Scotland was £1. 3s. $1\frac{1}{2}d$. Find by practice the total amount paid out in wages per week. How much was paid out during the year?

36. Find the cost of 72900 cu. ft. of gas at 2s. 10d. per 1000 cu. ft.

Find the value of—

37. 5 tons 16 cwt. at £1. 2s. per ton.

38. 4 oz. 10 dwt. at 2s. $1\frac{1}{2}d$. per oz.

39. 71 ac. 1 ro. 15 po. at £2. 12s. 6d. per acre.

40. 1 ton 3 cwt. 8 lb. at £1. 2s. 2d. per cwt.

41. 53 tons 16 cwt. 96 lb. at £1. 3s. 4d. per ton.

42. The dividend on £475. 12s. 6d. at 12s. in the £.

43. The rate on £15315. 10s. at 2s. 4d. in the £.

44. 76 ac. 3 ro. 8 po. at £3. 0s. $7\frac{1}{2}d$. per ac.

45. Find the cost of 17 cwt. 2 qr. 23 lb. at £18. 13s. 4d. per ton.

46. The average price of wheat in May 1913 was £1. 12s. 10d. per quarter. Find how much was realised by the 38719 qr. 2 bush. sold.

47. A bankrupt pays 7s. $3\frac{1}{2}d$. in the £. How much will a creditor to whom he owes £71. 14s. 8d. receive?

48. A train travels at the rate of 49 miles per hr. How far does it go in 3 hr. 45 $\frac{1}{2}$ min.? Give your answer to the nearest mile.

49. Find the value of 12.69 times 14s. 3d.

50. Find, by practice, the amount of tin-foil required to line a box without a lid, internal dimensions 1 ft. 10. in. long, 1 ft. 2 in. wide, and 10 in. deep.

51. A contractor charges £38. 2s. 8d. per mile for making a road. How much would he charge to make a road 3 fur. 19 yd. long?

52. Find the cost of carpeting a room 19 ft. 4 in. long and 14 ft. 3 in. broad with carpet 2 ft. 9 in. wide at 4s. $1\frac{1}{2}d$. per yd.

53. A truck is loaded with 100 sacks; the sack itself in each case weighs 6 lb. 11 oz., and each sack contains 85 lb. of grain. What is the weight of the whole load in tons, cwt., and lb.?

54. Every time the pointer on a circular dial moves it traces out a small arc of $15' 45''$. Find the angle turned through in 49 movements of the pointer. If the pointer moves once a second, find the number of degrees it has traced out in 35 min. 35 secs.

55. Find the value of 3.667 times 18s. 6d.

56. The cost of warming a school is £3. 14s. 2d. per day. How much does it cost to warm the school for 9 days 4 hr. 30 min.? (The school is warmed for 9 hours a day.)

57. An American dollar = 4s. 13d. Find the value in English money of 7513 dollars 35 cents.

§ 30. APPROXIMATION AND CONTRACTED WORK

Suppose that when measuring the length of a line with an ordinary ruler a boy sees that the length lies between (say) 3.6 and 3.7 cm. He imagines the interval between 3.6 and 3.7 to be divided into 10 equal parts, and then "hits off" with his eye to how many of these subdivisions the length of his line corresponds; *e.g.* 3.68 cm.

Example I. If he were called upon to find the area of a rectangle and measured the sides in this way to be 3.68 cm. and 7.92 cm., he would have to multiply these two numbers together.

The first step, as in ordinary multiplication of decimals, would be to use a rough check to determine the position of the decimal point in the product.

[*Rough Check:* (4×8) sq. cm. = 32 sq. cm.]

Suppose he multiplies 368 by 792 right out thus: (omitting all decimal points):—

$$\begin{array}{r} 792 \\ 368 \\ \hline 2376 \\ 4752 \\ 6336 \\ \hline 291456 \end{array}$$

All these figures are not necessarily reliable. In order to find how many figures he can rely on, let him put "*x*" (unknown) in the above thus:—

$$\begin{array}{r}
 792 \\
 368 \\
 \hline
 2376xx \\
 4752x \\
 6336 \\
 \hline
 2914xx
 \end{array}$$

Hence the area is $29\cdot14$ sq. cm., where the 4 is only approximately correct. To test its reliability take (i) $x=0$, (ii) $x=9$. The "carried figure" in either case is 1, and hence the result is reliable.

The work can be set down thus:—

$$\begin{array}{r}
 79\cancel{2} \\
 368 \\
 \hline
 2376 \\
 475 \\
 63 \\
 \hline
 2914
 \end{array}$$

$$\therefore \text{area} = 29\cdot14 \text{ sq. cm.}$$

The principle of division is the same. Here again the position of the decimal point ought to be determined by a rough check.

Example II. Divide 8798 by $31\cdot21$ correct to the third decimal place.

$$[\text{Rough Check: } \frac{9000}{30} = 300.]$$

Six figures will be required in the answer. Keep one more figure in the dividend than is required in the answer, adding noughts if necessary.

$$\begin{array}{r}
 2818969 \\
 31\cancel{2}\cancel{1})8798000 \\
 \underline{6242} \\
 25560 \\
 \underline{24968} \\
 5920 \\
 \underline{3121} \\
 27990 \\
 \underline{24968} \\
 3022 \\
 \underline{2808} \\
 214 \\
 \underline{186} \\
 28
 \end{array}$$

$$\therefore \text{result} = 281\cdot897.$$

With practice the second line in the subtraction may be omitted.

Note.—In the above example 8798 is known exactly, and we may insert as many zeros as we want in the division.

Significant Figures

The earnings of the Canadian Pacific Railway for March 1913 amounted to £2222378. This sum of money correct to **three significant figures** is £2220000; correct to **five significant figures** it is £2222400.

Let us take another example. Express $\frac{1}{343}$ as a decimal correct to four significant figures.

$$\frac{1}{343} = \cdot 0029154 \dots$$

The result correct to **four significant figures** is $\cdot 002915$. Note that a zero may be a significant figure, *e.g.* $\cdot 020408$ correct to **four significant figures** is $\cdot 02041$.

Relative Error

If a person measuring a yard makes a mistake of an inch, it is a bad error. If he measures the distance between London and Manchester and makes a mistake of 20 yards, it is not bad. The relative error is different in the two cases. Hence we say that an error of an inch in a yard (*i.e.* in 36 inches) is a relative error of 1 in 36—

i.e. a **relative error** of $\frac{1}{36}$ or $\cdot 03 \dots$

i.e. a **percentage error** of $\frac{100}{36}$ or 3 %

The error actually made in measurement is called the **absolute error**.

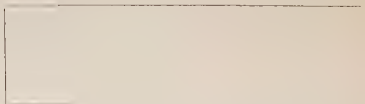
EXAMPLES. XXX

30

1. The sides of a rectangle are approximately 3.52 cm. and 2.18 cm. Find its area as accurately as the data allow.
2. A sheet of paper measures 11.11 in. by 8.23 in. Calculate its area to the nearest tenth of a sq. in.
3. If the side of a square be given as 3.02 in. correct to two places of decimals, find the area of the square correct to two places of decimals.
4. A rectangular board is 20.3 cm. long and 10.2 cm. wide correct to one place of decimals. Find the area of the board as accurately as possible from the data.

5. A cu. cm. of oak weighs $\cdot 91$ gm. correct to two places of decimals. What is the volume of a piece weighing 200 gm.?

6. Find the area of the rectangle as accurately as you can. Measure the sides in inches.



7. The volume of a glass sphere is $2\cdot 74$ cu. cm. correct to the nearest cu. mm., and its S.G. is $2\cdot 51$. Find the approximate weight of the sphere.

8. The length and breadth of a rectangular hall are $3\cdot 35$ m. and $2\cdot 24$ m., each length being given to the nearest cm. Find the area. What is the greatest possible error in your result?

9. A field in the form of a trapezium has the two parallel sides 120 yd. and $65\frac{1}{2}$ yd. in length, these lengths being known accurately. The distance between the parallel sides is found to be approximately $29\frac{1}{4}$ yd. Find the area of the field correct to the first decimal place.

10. A boy constructs an equilateral triangle on a base of 2·1 inches. He finds the length of the perpendicular from a vertex on to the opposite side to be 1·83 in., but he cannot depend upon the second place. What is the probable area of the triangle?

11. To find the circumference of a circular disc, a boy very carefully places a piece of cotton round the disc. In measuring the cotton he uses a ruler which marks cm. and mm., and approximates the length of the cotton at $33\cdot 55$ cm. Taking π to be 3·14, find the radius of the disc as accurately as possible.

12. Find the volume of a cube of side $2\cdot 05$ in. correct to the nearest tenth of a cubic inch.

13. There are $39\cdot 3701\dots$ in. in a metre. If a metre be taken as equal to $39\cdot 4$ in., to how many significant figures is this correct?

14. The density of air is $1\cdot 293$ gm. per litre. State this result correct to 2 significant figures.

15. State the following lengths correct to 4 significant figures:—

(i) $\cdot 00051679$ cm.

(ii) $\cdot 050505$ yd.

(iii) $\cdot 5167781$ cm.

(iv) $1\cdot 00000818181$ Km.

16. If the value of $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \frac{1}{5^4}$ be taken as $\cdot 25$, to how many significant figures is this correct? (*Express each fraction as a decimal.*)

17. To how many significant figures is $\pi = \frac{22}{7}$ a correct assumption?

18. Four measurements are taken, and the volume of a piece of copper wire is found to vary with the temperature, so that the results are $\cdot 0003492$ cu. cm., $\cdot 0003501$ cu. cm., $\cdot 0003488$ cu. cm., and $\cdot 00035$ cu. cm. To how many significant figures does the volume remain the same?

19. The report of a certain company stated that the total crop of tea for 1912 was 1689285 lb. as compared with an estimate of 1728000 lb. Find to how many significant figures the estimate agreed with the actual amount.

20. During 1911 the total merchandise carried by the railways of the United Kingdom amounted to 5235177178 tons. The returns for 1912 showed a decrease of 3500000 tons (correct to two significant figures). Find the amount carried during 1912 as accurately as the figures will allow.

21. The circumference of the earth is 4×10^7 m. If the diameter of the earth be taken as 7900 miles, find to how many significant figures this value of the diameter is correct, given that 1 m. = 39·37 in. and $\pi = 3\cdot 14$.

22. A kilometre is roughly $\frac{1}{2\frac{1}{8}}$ of a mile. To how many significant figures is this the same as taking a yard as $\frac{1}{1100}$ of a kilometre?

23. Construct a triangle with sides 5, 6, 7 cm. respectively. Measure the angles. You are told that the angles are $44^\circ 25'$, $57^\circ 7'$, $78^\circ 28'$. Write down your absolute and relative errors.

24. Which is the worse relative error, to make a mistake of $\frac{1}{4}$ mile in measuring the distance from London to Brighton (51 miles), or $2\frac{1}{4}$ miles in measuring from London to Aberdeen (480 miles)?

25. The length of a line is known to be 3·82 cm. Find the absolute and relative errors if it is taken as 3·8 cm.

26. What is the relative error in taking a mile to be $1760 \times \cdot 91$ metres when a yard is equal to $\cdot 9144$ metre?

27. The distance between Cambridge and London is $55\frac{3}{4}$ miles. A surveyor measures the distance to be 56 miles. Find his relative error.

28. The pressure of the atmosphere under certain conditions is equivalent to a height of 29.9 cu. in. of mercury. If the value is taken as 30 cu. in., what is the percentage error?

29. The electro-chemical equivalent of copper is .000328. It is often taken to be .00033. Find the percentage error.

30. In two experiments a man made a mistake in weighing. In the first, he was .7 gm. too much in weighing a solid whose true weight was 53.9 gm.; and in the second, he erred by finding that a gram of a powder weighed .99 gm. Which was the worse relative error?

31. A watch gains 15 seconds a day. Find the maximum relative error.

32. The side of a triangle is known to be x units in length. A boy measures it and finds it to be $x - a$ units, where a is small. Show that his percentage error is $\frac{100a}{x}$.

Find **roughly** the value of the following (the amount of working should be as little as possible):—

33. $.00056 \times 83.2$.

34. $\sqrt{4896}$.

35. $14s. 2d. \times 239$.

36. πr^2 , where $r = 24.9$.

37. $\frac{1}{.0511}$.

38. Show that 10 yd. a sec. is about 20 miles per hr.

39. The weight of 1 cu. cm. of water is 1 gm. Given that 4.5 litres = 1 gall. and 454 gm. = 1 lb., show that a pint of water weighs $1\frac{1}{4}$ lb.

40. In a certain mine 1547000 cu. yd. were dredged for a yield of 9340 oz. of gold of total value £36087. Show that this is equivalent to an average recovery of between 5d. and 6d. per cu. yd.

41. 7283 lb. of rubber were produced on the plantations of a certain company in 1912: the output in 1913 was 10937 lb. Find roughly the percentage increase.

42. It costs 1s. per sec. to coal a fleet of steamers. Show that this is the same as £30000 a week, roughly.

43. Lengths of 37.122 m., 19.871 m., 13.005 m. are correct to the nearest mm. Find the total length and the greatest possible error.

44. The wine exported from Ecuador to various countries is as follows:—France, 19658148 Kg.; United States, 8161578 Kg.; Germany, 3944964 Kg.; England, 1702628 Kg.; Spain, 1605371 Kg. Find the total export to the nearest thousand.

45. The profit made by a certain company was £1069000 in 1912 as against £773993 in 1911. Find the difference to the nearest thousand.

46. Correct to a milligram, the weights of three different amounts of a liquid are 1·719 gm., 1·688 gm., and 1·775 gm. Find the total weight and the greatest possible error.

47. Express $\frac{1}{7}$, $\frac{1}{13}$, $\frac{4}{11}$ as decimals, and find their sum correct to four decimal places.

48. The numbers of Bulgarians in three different towns in Turkey are 92805, 118047, 5194. How many are there in the three together to the nearest thousand?

49. Multiply ·000081 by ·00073, and give the result correct to two significant figures. Do this by writing ·000081 as $8\cdot1 \times 10^{-5}$ and ·00073 as $7\cdot3 \times 10^{-4}$.

50. Use the above method to find the square of ·00096 correct to two significant figures.

Find the value of—

51. $1\cdot83 \times 3\cdot14$ correct to one decimal place.
52. $\cdot81 \times \cdot82$ two significant figures.
53. 819×829 three significant figures.
54. $\cdot00912 \times 95296$ the nearest whole number.
55. $\cdot0021 \times 523809$ the nearest whole number.
56. $7\cdot25 \times 810\cdot281$ two decimal places.
57. $57\cdot8234 \times \cdot00045$ three significant figures.
58. 71986×81976 the nearest million.
59. $51\frac{1}{13} \times 79\frac{2}{37}$ the nearest whole number.
60. $\cdot013057 \times 23\cdot7814$ five significant figures.
61. Square 1·0099 correct to three decimal places.

Find the value of—

62. $55\cdot09976 \div 3\cdot2$ correct to three decimal places.
63. $3\cdot758 \div 72\cdot358$ four significant figures.
64. $7\cdot04 \div \cdot0847$ two decimal places.

65. $3\cdot14159 \div 3600$ correct to six decimal places.
66. $2\cdot873 \div 48\cdot607$ two decimal places.
67. $64\cdot375 \div 9\cdot573$ five significant figures.
68. $3\cdot743 \div 52\cdot804$ four significant figures.
69. $111000 \div 8\cdot0908$ the nearest integer.
70. $47362 \div \cdot0435$ the nearest thousand.
71. $1 \div \cdot0095$ two decimal places.
72. Work out to the nearest integer $\frac{185\cdot32 \times \cdot157}{\cdot0081}$.
73. How many complete kilometres are there in 3040 miles, taking a centimetre to be $\cdot3937$ in.?
74. If $Q = \cdot00545 \times d^2 \times 250$, find Q to the nearest hundred when $d = 59\cdot39$.
75. $y = a + bx$. What value of y corresponds to a value of $8\cdot7$ for x when $a = 119$ and $b = 45\cdot7$? Give your answer to the nearest integer.
76. Prove by actual multiplication that $\sqrt[3]{50}$ lies between $3\frac{1}{2}$ and $3\frac{3}{4}$.
77. The volume V of water in cu. ft. displaced by a vessel of length l ft., breadth b ft., and draught h ft. is given by $V = \cdot45lbh$. Find V correct to four significant figures when $l = 265$, $b = 27\cdot3$, $h = 6\cdot3$.
78. In the formula (connecting the power and load of a certain machine) $P = \cdot0144W - 24$, the value of P is known to be $20\cdot59$. Find W to the nearest integer.
79. Find the value of $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots$ correct to three decimal places.
80. Find the value of $\frac{1}{3} + \frac{1}{3 \times 5} + \frac{1}{3 \times 5 \times 7} + \dots$ correct to three decimal places.
81. The expression $1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \dots$ is denoted by " e ." Find its value correct to four significant figures.
82. Find to within one-hundredth of the whole the value of $2\cdot034 \times 3\cdot125 \div 5\cdot236$.

83. A bankrupt's assets amount to 403163 dollars, and his liabilities are 430143 dollars. Find to the nearest cent how much he can pay in the dollar.

84. In 1907 there were 138697 telephone stations in London; in 1913 there were 227313. Find the increase per cent. to the nearest integer.

85. A ton of Pacific water yields 79 pounds of salt. Express this as a percentage correct to one decimal place.

86. The value of San Paulo Treasury Bonds was given on a certain date as £3057080. The number of holders was 58393. How much was a bond worth to each of the holders on an average? Give your answer to the nearest 10s.

87. The report of an Indian railway company stated that the gross earnings increased by 4004062 rupees, and the net earnings by 2973526 rupees, or 73 % of the increase of gross earnings. Find to how many significant figures the percentage is correct.

88. The greatest area of land owned by any landowner in England is 198572 acres, of an annual value of £180750. Find the value of an acre to the nearest penny.

89. In 1912, 635728 persons visited the National Gallery and 361477 the Tate Gallery. Show that for every 4 persons visiting the latter 7 visited the former.

90. A liner can cross the Atlantic from Liverpool to New York (3050 miles) in 4 days 12 hours. Find the average speed in knots to the nearest integer. (A knot = 6080 ft. per hr.)

91. Tobacco duties for 1888 and 1913 amounted to £8858781 and £17342360 respectively. Find the increase per cent. correct to one decimal place.

92. In the Balkan War there were 86734 casualties on the Bulgarian side, made up of officers and men killed, wounded, and missing. 330 officers and 29711 men were killed; 950 officers and 52550 men were wounded. Find to two significant figures the percentage of the casualties represented by (i) officers killed, (ii) men killed, (iii) officers wounded, (iv) men wounded.

93. In 1912, 9107 vessels from foreign ports entered the Thames. They carried 13924 passengers and 53740 seamen. Find how many passengers and how many seamen each vessel carried on the average.

94. In the above question, show that the ratio of passengers to seamen may be represented approximately by the fraction $\frac{34}{131}$.

95. The population of Salonika, a division of Turkey in Europe, is 626533, of whom 228989 are Turks and 194739 are Greeks. How many (i) Turks, (ii) Greeks, are there per thousand to the nearest whole number?

96. In a certain diamond mine 5002 loads were washed during April 1913, producing 2465.75 carats. Find to three significant figures how many carats this represents per load.

97. The census of 1911 showed that the population of Ireland was 4390219, less by 68556 persons than at the census of 1901. Show that the decrease per cent. was 1.54 correct to two places of decimals.

98. Of the 85941 motor cars of all sorts registered in London, 22110 are motor cycles. Express the ratio of motor cycles to motor cars as a percentage correct to two places of decimals.

99. The S.G. of mercury is approximately 13.596; find to three decimal places the weight of mercury in a flask holding 49.974 cu. cm. Show that your result may have an error of as much as .03 gm.

100. The radius of a sphere is 2.01 cm. correct to two decimal places. Within what limits does the volume lie, given that $\pi = 3.14$ correct to two decimal places?

101. The weight of copper deposited in an electro-plating experiment is given by $W = ECt$. E is known to be .00033 correct to two significant figures, C is 1.22 amperes to the second decimal place, and t is the time in seconds—1200 exactly. Find as accurately as the data will allow the weight (W) in gm. of copper plated.

§ 31. GEOMETRICAL APPLICATIONS

If we wish to find the areas or perimeters of certain figures such as squares, rectangles, or circles, all we need know are particular dimensions and then we can calculate the required result without drawing a figure. On the other hand, if we wish to find the area of a triangle or the perimeter of an irregular figure it is advantageous to draw the figure accurately and measure

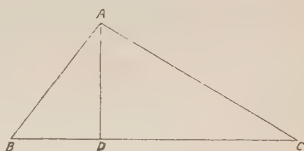
the lengths we require, in order to avoid the use of long and sometimes difficult formulæ.

Example I. Find the area of a triangle whose sides are 1·5 in., 1·2 in., ·75 in. respectively.

Draw the triangle ABC having sides of the required lengths.

Let fall a perpendicular AD from A on to BC.

Then area of $\Delta = \frac{1}{2}(\text{base} \times \text{height})$
 $= \frac{1}{2}(1\cdot5 \times \cdot6)$ sq. in.
 $= \cdot45$ sq. in.



Example II. By making your own measurements, find the area of the shaded part of the given figure.

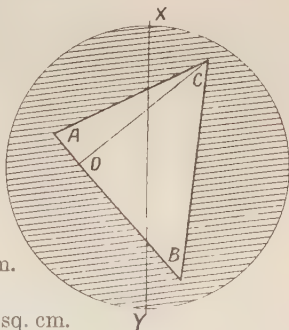
Draw a diameter XY.

Let fall a perpendicular CD on to AB.

Then area of circle $= \left(\frac{\pi XY}{2}\right)^2$
 $= \pi \times (2\cdot5)^2$ sq. cm.
 $= 19\cdot625$ sq. cm.

Area of $\Delta = \frac{1}{2}(AB \times CD)$
 $= \frac{1}{2}(3\cdot5 \times 3)$ sq. cm.
 $= 5\cdot25$ sq. cm.

$\therefore$ Area of figure $= (19\cdot625 - 5\cdot25)$ sq. cm.
 $= 14\cdot375$ sq. cm.



Example III. In the following map it is known that the distance from London to Brighton is 51 miles. Find the distance of the other towns indicated from London.

Measure the distance on the map from London to Brighton. It is 42 mm.

$\therefore$ 42 mm. represents 51 miles,
i.e. 1 mm. $\frac{51}{42}$ miles,
i.e. 1 mm. 1·21 miles.

The distance from London to Eastbourne on the map is 48 mm.

$\therefore$ Actual distance $= (48 \times 1\cdot21)$ miles
 $= 58\cdot1$ miles.

The other distances may be found similarly.

□ LONDON

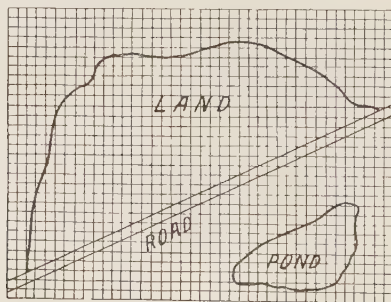
○ GUILDFORD

○ TUNBRIDGE WELLS



In finding the area of an irregular figure it is very often advisable (as mentioned on page 37) to draw the figure on squared paper and then count the squares contained in the figure.

Example IV. A tract of land is of the following shape. Find its area in acres. (Scale: 1 square represents 100 sq. yd.) Find also the area of the pond on the other side of the road.



Number of squares in diagram of tract of land = 417.

$$\begin{aligned}\therefore \text{Area of land} &= (417 \times 100) \text{ sq. yd.} \\ &= \frac{417 \times 100}{4840} \text{ acres} \\ &= 8.6 \text{ acres.}\end{aligned}$$

Number of squares in diagram of pond = 70.

$$\begin{aligned}\therefore \text{Area of pond} &= (70 \times 100) \text{ sq. yd.} \\ &= 7000 \text{ sq. yd.}\end{aligned}$$

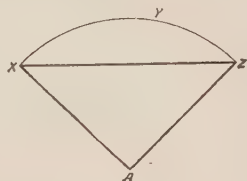
EXAMPLES. XXXI 31

1. Find the area of the triangle whose sides are 8 cm., 9 cm., and 10 cm. respectively.

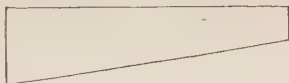
2. Draw a circle of radius 2.5 in. Choose two points A and B 1 in. apart on the circumference. Take a point C also on the circumference, 2 in. from A on the side remote from B. Find the difference between the areas of the circle and the triangle ABC.

3. Construct a triangle ABC with base AB 8.9 cm. and base angles 30° and 45° respectively. Find its perimeter.

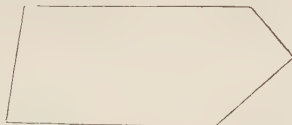
4. In the accompanying diagram the curved part is a quadrant of a circle. Find the area of the figure AZYX, and hence that of the segment XYZ.



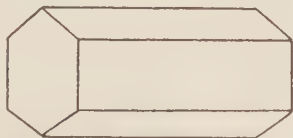
5. What is the volume of 10 feet of the bar of which the accompanying diagram is an accurate end-on view? Scale 1 foot to 1 cm.



6. Calculate the area of the accompanying diagram. Give your answer in (i) sq. cm., (ii) sq. in.



7. Find the volume of the solid; the base is a regular hexagon. If it were made of brass of density 8.6 gm. per cu. cm., what would be its weight?



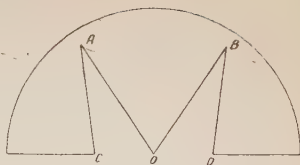
8. The relative positions of three of the largest towns in Russia are marked here. The map from which the positions have been taken was drawn on a scale of 100 miles to 8 mm. If a triangle were drawn through the three points, what area would be marked out between the three towns?

⊙ *St. Petersburg*

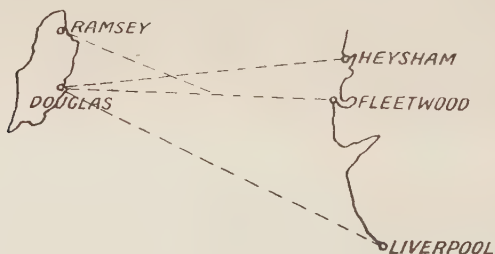
⊙ *Moscow*

⊙ *Vilna*

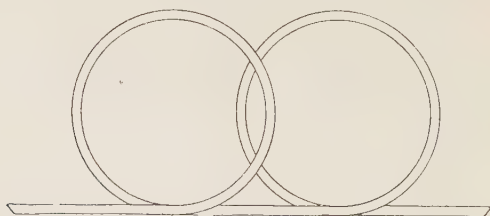
9. Two triangular pieces have been cut from a semicircular piece of zinc. Find the volume of the remainder, the thickness of the zinc being 5 mm.



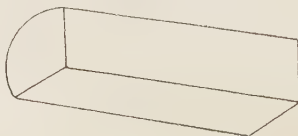
10. The distance from Douglas to Liverpool is 70 miles. Find the distance from Douglas to Heysham, and from Ramsey to Fleetwood.



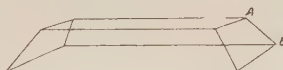
11. A piece of string is coiled into two loops. Find its length. You may neglect the thickness of the string.



12. Find the volume of the solid; the curved part is one-third of a circle. Scale: 1 foot to 1 cm.



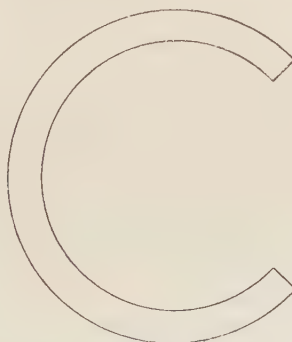
13. In the subjoined picture of a cardboard box 5 inches is represented by 1 cm. Both the base and the top are square, and the slant height AB is 8 inches. How much cardboard will be required for the box if there is to be no lid and 3 % is to be allowed for overlapping?



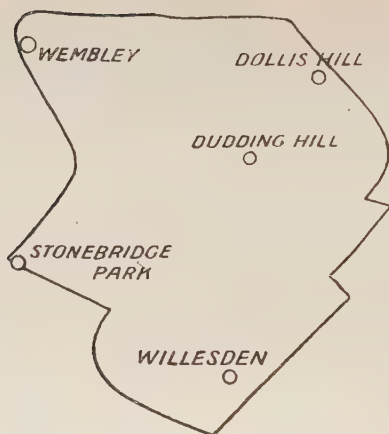
14. Copy the following figure on squared paper, and thus find its area in any convenient units. If it represent a field drawn to a scale of 1 sq. yd. to $\cdot 01$ sq. cm., what would be the area of the field in acres?



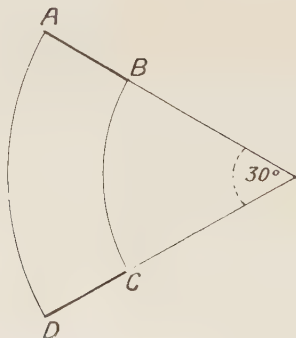
15. The letter in the following diagram is to be made out of wood 1.75 in. thick: it is drawn in the figure to a scale of a foot to 2.5 cm. If the shape is three-quarters of a circle, find how much wood will be required.



16. Luggage is delivered from Willesden Junction by the London and North-Western Railway within the district shown. If the scale is 1 mile to $\cdot 9$ in., find as nearly as you can the area represented.



17. Find the area of the part ABCD in (i) sq. cm.,
(ii) sq. in.

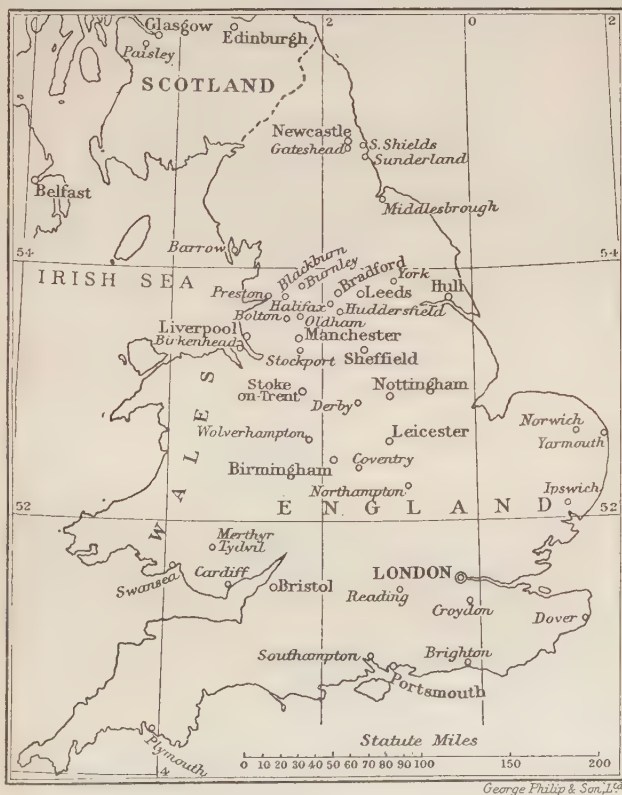


18. The stool whose end-on view is indicated in the diagram has four legs, each of which is on a square base. The top is also a square. Find the volume of wood required for the stool. Scale : 1 foot to 5 cm.



19. A table is inlaid with alternate black and red pieces, each in the shape of an equilateral triangle of side 2 in. The table is completely covered by these pieces and is thus in the shape of a rhombus. If along the shorter diagonal 8 triangles are placed

thus :— , find the number of pieces required for the table, and their total area.



20. Find the distances between the principal towns in the above map.

21. Draw the following figure four times the size, and then find its area.



22. A man starts from a town A and walks 3 miles north to B; he then turns and walks 2 miles east to C. From here he goes north-west to D, a distance of 4 miles; and finally he rides 10 miles south-west to X. How far is he from A in a direct line? Draw a diagram to scale.

§ 32. LOGARITHMS

We know that—

$$\begin{aligned} 100 &= 10^2 \\ 10000 &= 10^4 \end{aligned}$$

Also by extracting the square root—

$$\begin{aligned} 3.162 &= 10^{\frac{1}{2}} \\ &= 10^{.5} \\ 1.778 &= 10^{.25} \\ &\text{and so on.} \end{aligned}$$

It may be shown that—

$$\begin{aligned} 2 &= 10^{.3010} \\ 3 &= 10^{.4771} \\ &\text{etc.} \end{aligned}$$

If a number be expressed as a power of 10, the power is said to be the **logarithm** of the number to base 10.

We may write any one of the above equalities therefore in the following manner:—

·25 is the logarithm of 1.778 to base 10.

This is shortened to the form

$$\log_{10} 1.778 = .25.$$

In numerical examples, since 10 is the base generally adopted, the “10” is omitted and we simply write

$$\log 1.778 = .25.$$

The decimal part of a logarithm is called the **mantissa**, and the integral part the **characteristic**.

We have the following rules :—

$$(\alpha) \log (xyz \dots) = \log x + \log y + \log z + \dots$$

$$(\beta) \log \frac{x}{y} = \log x - \log y.$$

$$(\gamma) \log x^y = y \log x.$$

$$(\delta) \log \sqrt[y]{x} = \frac{1}{y} \log x.$$

Example I. Given that $\log 2 = \cdot 3010$ and $\log 3 = \cdot 4771$, find (i) $\log 6$, (ii) $\log 1\cdot 5$, (iii) $\log 200$, (iv) $\log 2 \times 10^5$, (v) $\log \cdot 002$.

$$\begin{aligned} \text{(i) } \log 6 &= \log (2 \times 3) \\ &= \log 2 + \log 3 \dots \dots \dots (\alpha) \\ &= \cdot 3010 + \cdot 4771 \\ &= \cdot 7781 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \log 1\cdot 5 &= \log \frac{3}{2} \\ &= \log 3 - \log 2 \dots \dots \dots (\beta) \\ &= \cdot 4771 - \cdot 3010 \\ &= \cdot 1761 \end{aligned}$$

$$\begin{aligned} \text{(iii) } \log 200 &= \log (2 \times 100) \\ &= \log (2 \times 10^2) \\ &= \log 2 + \log 10^2 \\ &= \log 2 + 2 \log 10 \dots \dots \dots (\gamma) \\ &= \cdot 3010 + 2 \\ &= 2\cdot 3010 \end{aligned}$$

$$\begin{aligned} \text{(iv) } \log 2 \times 10^5 &= \log 2 + \log 10^5 \\ &= \log 2 + 5 \log 10 \dots \dots \dots (\gamma) \\ &= \cdot 3010 + 5 \\ &= 5\cdot 3010 \end{aligned}$$

$$\begin{aligned} \text{(v) } \log \cdot 002 &= \log \frac{2}{1000} \\ &= \log 2 - \log 10^3 \\ &= \log 2 - 3 \log 10 \dots \dots \dots (\gamma) \\ &= \cdot 3010 - 3 \\ &= -3 + \cdot 3010 \end{aligned}$$

This is written as

$$\bar{3}\cdot 3010$$

so that the mantissa may always be positive.

Since

$$\log 2 = \cdot 3010,$$

$$\log 200 = 2\cdot 3010,$$

$$\log 2 \times 10^5,$$

$$i.e. \log 200000 = 5\cdot 3010,$$

and

$$\log \cdot 002 = \bar{3}\cdot 3010,$$

we see that the mantissæ of logarithms of numbers having the same significant figures are the same.

The characteristic is determined by the following rule :—

For a number **greater** than unity the characteristic is **positive** and is one **less** than the number of figures to the left of the decimal point. For a number **less** than unity the characteristic is **negative** and is one **more** than the number of noughts to the right of the decimal point.

In practical work we determine the characteristic by a *rough check*, thus :—

log 10 = 1	∴ log 23	lies between 1 and 2, <i>i.e.</i> is 1...
log 100 = 2		
log 1000 = 3	log 485	 2 and 3, <i>i.e.</i> is 2...
log 10000 = 4	log 7961	 3 and 4, <i>i.e.</i> is 3...
.....		and so on.

The logarithm tables give the decimal part.

The characteristic of the logarithm of a number such as $\cdot 03$ may be determined by a rough check, thus :—

$$\begin{aligned} \cdot 01 &< \cdot 03 < \cdot 1 \\ \therefore \log \cdot 01 &< \log \cdot 03 < \log \cdot 1 \\ \text{i.e. } \log 10^{-2} &< \log \cdot 03 < \log 10^{-1} \\ \text{i.e. } -2 &< \log \cdot 03 < -1 \\ \therefore \log \cdot 03 &= 2 \dots \end{aligned}$$

Example II. Given that $\log 8 \cdot 61 = \cdot 9350$, write down $\log \cdot 0861$, $\log 861$, $\log 86100$, $\log \cdot 861$.

$$\begin{aligned} \log \cdot 0861 &= 2 \cdot 9350. \\ \log 861 &= 2 \cdot 9350. \\ \log 86100 &= 4 \cdot 9350. \\ \log \cdot 861 &= 1 \cdot 9350. \end{aligned}$$

Example III. Find the value of $\sqrt[5]{\cdot 03}$.

$$\begin{aligned} \text{Now } \log \sqrt[5]{\cdot 03} &= \frac{1}{5} \log \cdot 03 \dots \dots \dots (\delta) \\ &= \frac{1}{5} \times 2 \cdot 4771 \\ &= \frac{1}{5} \times (5 + 3 \cdot 4771) \\ &= 1 \cdot 6954; \end{aligned}$$

$$\therefore \sqrt[5]{\cdot 03} = \cdot 4959.$$

Note.—When dividing a logarithm with a negative characteristic by a number, increase the negative characteristic until it is a multiple of the divisor, adding the increment to the logarithm.

Example IV. Find the value of $\frac{.056 \times 81.32}{953.1}$.

Let the expression be denoted by E.

$$\begin{aligned}\text{Then } \log E &= \log .056 + \log 81.32 - 953.1 \\ &= \bar{2}.7482 + 1.9102 - 2.9791 \\ &= \bar{3}.6793; \\ \therefore E &= .004779.\end{aligned}$$

EXAMPLES. XXXII 32

Tables are not to be used in Questions 1-11.

1. Write down the logarithms of 1, 100, .001, 1000, .0001.
2. Given that $3 = 10^{.4771}$, write down $\log 3$, $\log 300$, $\log .003$, $\log 300000$, $\log 3 \times 10^{-7}$.
3. Given that $7 = 10^{.8451}$, write down the numbers whose logarithms are .8451, 3.8451, 3.8451, 1.8451, $\bar{7}.8451$.
4. Graph 10^x , taking x from 0 to 1. Do this by finding first $\sqrt{10}$, i.e. $10^{\frac{1}{2}}$, then $10^{\frac{1}{3}}$, $10^{\frac{1}{4}}$. Thence find $10^{\frac{2}{3}}$, for example, by writing down $10 = 10^{\frac{1}{3}} \times 10^{\frac{2}{3}}$ and so on. Use this graph in Questions 5-10.
5. From the graph find the value of 10^{-3} , and hence find the number whose logarithm is .3.
6. Find the values of $10^{.48}$, $10^{.85}$, 10^{-9} .
7. Deduce $\log 2$, $\log 3$, and $\log 4.5$ from the graph.
8. Find $\log 6.5$ and write down $\log 6500$ and $\log .0065$.
9. Find the number whose logarithm is .77, and hence the numbers whose logarithms are 2.77 and $\bar{2}.77$.
10. Find $\log 2$ and $\log 3$. Write down the logarithms of 4, 5, 6, 8, 9, 10, 12, 15, 27, 64, 120.
11. Giving x the values 4 to 120 as in the above question, graph $\log x$. Find from the graph $\log 7$, $\log 13$, and $\log 23$.

Given $\log 51 = 1.7076$, $\log 440 = 2.6435$, and $\log 129 = 2.1106$, find the values, correct to as many figures as possible, of—

- | | |
|-------------------------------------|--|
| 12. $\log (51 \times 440)$. | 13. $\log (.51 \times 4.4)$. |
| 14. $\log (12.9 \times .44)$. | 15. $\log \sqrt[3]{.51}$. |
| 16. $\log (.00129 \times 440000)$. | 17. $\log \frac{51 \times 44}{12.9}$. |

18. $\log \frac{.051 \times .044}{.0129}.$

19. $\log (.44)^5.$

20. $\log \frac{.44^{\frac{1}{3}} \times .051^{\frac{1}{4}}}{129}.$

21. $\log {}^5\sqrt{4 \cdot 4^2 \times 5 \cdot 1^3 \times 1 \cdot 29^4}.$

22. Given $\log 49 = 1.6902$, find the value of $\log {}^6\sqrt{.0049}.$

23. Given $\log 49 = 1.6902$, find $\log 7$, $\log (7^{\frac{1}{3}})$, and $\log (7^{\frac{1}{4}})$.

24. Find from the tables the value of x satisfying (i) $\log x = .6990$, (ii) $10^x = .7243$, (iii) $\log 1.9 = x$.

25. How many digits are there in 2^{100} ?

26. Find how many noughts there are after the decimal point in 3^{-50} .

27. Read from the tables the values of 71.32 , 8.005 , $.00561$, 77.77 , 310000 .

28. Write down the logarithms of $.0093$, $.1762$, 81.33 , 9560000 , 5×10^{-3} .

29. Using the table of antilogarithms, find the numbers whose logarithms are 1.7170 , $.5754$, 2.9919 , 6.2333 , 4.2088 .

30. Find the values of $.32 \times 10^5$, 32×10^{-5} , $.81 \times 10^{-2}$, 7153×10^6 , $.008 \times 10^{-3}$, and write down their logarithms.

Find the value of—

31. $25.69 \times 16.05.$

32. $4.217 \times .915.$

33. $.819 \times .918.$

34. $.0051 \times 6532.$

35. $82.63 \times 63.82.$

36. $(.07)^3.$

37. $1152000 \times .000115.$

38. $35 \div 1.912.$

39. $.715 \div .013.$

40. $7612 \div 6127.$

41. $.0053 \div .119.$

42. $(.005)^6.$

43. $\sqrt{1725}.$

44. ${}^3\sqrt{72.51}.$

45. $(.0072)^{\frac{1}{4}}.$

46. $(1.005)^{15}.$

47. $\frac{8 \cdot 16 \times 5 \cdot 32}{39 \cdot 11}.$

48. $1.56 \div .056 \times 1152.$

49. ${}^5\sqrt{.00071}.$

50. $(.001067)^{11}.$

51. ${}^3\sqrt{.053} \times {}^4\sqrt{.035}.$

52. $(84.32 \times 6.15)^{\frac{1}{4}}.$

53. $\frac{61.32}{\sqrt{.005 \times 71.2}}.$

54. $\frac{81 \cdot 81 \times \sqrt[3]{3 \cdot 9}}{\sqrt[4]{87 \cdot 38}}$

55. $(\cdot 0015)^{\frac{1}{2}}$

56. $\sqrt{\frac{11 \cdot 5 \times 12 \cdot 6}{8 \cdot 028}}$

57. $\sqrt[3]{\frac{97 \cdot 6 \times 67 \cdot 9}{\cdot 019}}$

58. $\frac{\sqrt{05716} \times \sqrt[3]{\cdot 0751}}{79 \cdot 93}$

59. $\sqrt[7]{\frac{\frac{1}{2}(\cdot 0056)}{(\cdot 017)^3}}$

60. $(2 \cdot 718)^{1 \cdot 5}$

61. $(1 \cdot 097)^{2 \cdot 561}$

62. $\frac{3 \cdot 142}{(8 \cdot 03)^{-2 \cdot 5}}$

63. $(81\frac{1}{8})^3 \times (72\frac{1}{2})^{\frac{1}{2}}$

64. $12 \cdot 6^{\pi}$

65. $\frac{4}{3}\pi(1 \cdot 087)^3$

66. $\frac{10 \times \cdot 18 \times 6 \cdot 05}{2\pi \times 105}$

Note.— $\log \pi = \cdot 4971$.

67. Find the area of a circle of radius 1·079 cm.

68. A circular cylinder stands on a base diameter 14·75 in. and is 5 ft. $7\frac{1}{4}$ in. high. Find its volume.

69. Find the cubical contents of a tank, inside dimensions 75·36 cm. by 52·09 cm. by 37·37 cm.

70. The area of surface of an anchor ring is given by $4\pi^2 rR$, where r is the radius of the circular section and R is the mean radius of the ring. Find the surface area of a ring for which $R = 19 \cdot 35$ in. and $r = 1 \cdot 37$ in.

71. The perimeter of an ellipse of semi-axes a and b is approximately $2\pi \sqrt{\frac{1}{2}(a^2 + b^2)}$. Find the perimeter of an ellipse of semi-axes 2·75 in. and 1·56 in.

72. The velocity of light in miles per hour is given by the formula $V = \frac{4r}{1980}$. If $r = 92 \cdot 8 \times 10^6$, find V .

73. Find the value of the expression $\frac{\sqrt{3 \times 10^7}}{24 \times 589}$ giving the radius of a glass lens in cm.

74. In the monthly report of a tea company the crop for April 1912 was 227649 lb. as compared with 203841 lb. in the preceding month. Find the increase per cent. to as many figures as your logarithms will allow.

75. $y = 15.69 \times e^{-194x}$. Find y when $x = 9.5$, given that $e = 2.718$.

76. The pressure of water flowing spirally in a horizontal plane is given by $p = c - \frac{wa^2}{2gx^2}$. Find p , given that $c = 10^4$, $w = 62.3$, $g = 32.2$ and $\frac{a}{x} = 70$.

77. Find the value of $(1 - P^{\frac{1}{\gamma} - 1})$, where $\gamma = 1.37$, $P = 26$.

78. Prove that $4 \log 3 > 1 + 3 \log 2$, given only that $\log 3 = .4771$.

79. Find the velocity of longitudinal waves in glass from the formula

$$\sqrt{\frac{(4.2 \times 10^{11}) + (\frac{4}{3} \times 1.5 \times 10^{11})}{2.5}}$$

80. The time of oscillation (t) of a small drop of liquid under surface tension T is given by

$$t = 2\pi \sqrt{\frac{l a^3 T^{-1}}{8}}$$

Find T when $t = \frac{1}{32}$, $l = .99$, and $a = .25$.

81. The sum of a number of terms in a geometrical progression is $\frac{1 - r^n}{1 - r}$. Find the sum when $r = .25$ and $n = 56$.

82. A is the amount realised when £ P is put out at $r\%$ per annum for n years. Find the amount when £750. 7s. 3d. is invested for 5 years at $3\frac{1}{8}\%$, from the formula $A = P \left\{ 1 + \frac{r}{100} \right\}^n$.

83. The quantity of the water which flows out per second in a long straight tube was discovered by Poiseuille to be $\frac{\pi p a^4}{8kl}$. Calculate the quantity per sec. when $a = .05$, $l = 72.06$, $p = 42.5$, and $k = .01$.

84. The electrostatic capacity of a cable is given by

$$C = \frac{K}{2 \log \frac{R}{R_0} \times 2.306}$$

Find C when $K = 4.08$, $R = .45$, $R_0 = .17$.

85. $\phi = \left(2.303 \log_e \frac{t+273}{273} \right) + \frac{606.5 - .695t}{t+273}$. What is the value of ϕ when $t = 125$? ($\log_{10} n = .4343 \log_e n$).

86. A formula used in investigations with induction coils is $\omega = \frac{4.8f^2\beta^2l^2}{10^{12}}$. Find ω when $f = 10^3$, $\beta = 25$, and $l = .03162$.

87. If s be the semi-perimeter of a triangle, its area is $\sqrt{s(s-a)(s-b)(s-c)}$, a , b , and c being the lengths of the sides of the triangle. Calculate the area of a triangle of sides 17.36 in., 12.08 in., 14.44 in.

88. Under certain conditions the pressure of a fluid of density D on the containing vessel is

$$\frac{1}{2}g^2D^2\{b^2c^4 + a^2c^4 + \frac{1}{4}\pi^2a^2b^2c^2\}.$$

Find the pressure, given that $D = .912$, $g = 32.2$, $a = 12.1$, $b = 13$, $c = 10.2$.

89. Using the formula in Question 87, find the area of a triangle of sides 20.03 cm., 19.7 cm., 16.51 cm.

90. Find h from the equation used in telephone working:—

$$h = \sqrt{\frac{kgr}{2} \left\{ \left(1 + \frac{q^2l^2}{r^2} + \frac{l^2}{r^2} \right)^{\frac{1}{2}} - \left(\frac{ql}{r} - \frac{1}{q} \right) \right\}^{\frac{1}{2}}}$$

given that $k = .0096 \times 10^{-6}$, $g = .0302$, $r = 2.97$, $q = 5 \times 10^3$, $l = .0033$.

91. Find the sum of $5^{2.43}$, $3^{.246}$, $.042^{.476}$.

92. The load that will produce bending in a strut is given by

$$L = \frac{EI\pi^2}{4l^2}. \text{ Find } L, \text{ given } E = \frac{10.75}{10^{11}}, I = \frac{59}{12}l^2.$$

93. The mean radius (R) of an anchor ring of iron is 17.5 cm.; the radius of its circular section (r) is 2.8 cm. The ring is melted down, and a sphere is cast out of the material so as to have the same volume as the ring. Find its radius. (Vol. of anchor ring $= 2\pi^2r^2R$.)

§ 33. SIMPLE INTEREST

Example I. Find the simple interest on £350 for 3 years at 5 %.

Interest on £100 for 1 year is £5.

$$\begin{aligned} \therefore \quad & \dots\dots\dots \text{£}350 \dots\dots\dots \text{£}(5 \times 3\cdot5). \\ \therefore \quad & \dots\dots\dots 3 \text{ years} \dots\dots\dots \text{£}(3 \times 5 \times 3\cdot5). \\ & \text{Interest} = \text{£}(52\cdot5) \\ & = \text{£}52. 10s. \end{aligned}$$

It will be seen from the above example that

$$I = \frac{P \times R \times T}{100},$$

where

I = simple interest,

P = principal,

R = rate per cent.,

and

T = time in years.

Example II. Find the simple interest on £729. 12s. 3d. for 28 days at 4 %.

$$\begin{aligned} I &= \frac{P \times R \times T}{100} \\ \therefore \text{Interest} &= \text{£} \frac{729\cdot6125 \times 4 \times \frac{28}{365}}{100} \\ &= \text{£} \frac{7\cdot296125 \times 4 \times 28}{365} \\ &= \text{£}2\cdot239\dots \text{ to three places} \\ &= \text{£}2. 4s. 9d. \end{aligned}$$

Note.—The **Amount** is the sum of the Principal and the Interest.

Example III. What sum of money will amount to £345 in 5 years at 3 %?

£115 is the amount when the principal is £100.

$$\therefore \quad \text{£}345 \dots\dots\dots \text{£} \frac{100 \times 345}{115}.$$

$\therefore$ required principal = £300.

Example IV. In what time will the interest on £560 at $2\frac{1}{2}$ % be £42?

Now
$$I = \frac{P \times R \times T}{100}.$$

$$\therefore 42 = \frac{560 \times 2\frac{1}{2} \times T}{100}.$$

$$\therefore T = \frac{42 \times 100}{560 \times 2\frac{1}{2}}$$

$$= 3.$$

$\therefore$ time is 3 years.

Example V. What is the rate per cent. when £720 amounts to £846 in 5 years?

$$\text{Interest} = £846 - £720$$

$$= £126.$$

$$I = \frac{P \times R \times T}{100}.$$

$$\therefore 126 = \frac{720 \times R \times 5}{100}.$$

$$\therefore R = \frac{126 \times 100}{720 \times 5}$$

$$= 3\frac{1}{2}.$$

$\therefore$ rate per cent is $3\frac{1}{2}$.

As mentioned on page 123, we may use the method of practice to advantage in finding simple interest.

Example VI. Find the interest on £221. 11s. 4d. for 3 years at $3\frac{3}{4}\%$.

$$£221. 11s. 4d. = £221.5667.$$

$$£221.5667$$

At 3 %	6.6470
$\frac{1}{2}$ %	1.1078
$\frac{1}{4}$ %	.5539

$$8.3087... \text{ interest for 1 year.}$$

$$24.9261... \text{ 3 years.}$$

$$\text{Interest} = £24.9261$$

$$= £24. 18s. 6d.$$

Discount.

If a bill is due at the end of a given time and is paid before that time a certain amount is deducted from the value of the bill. For example, a bill of £500 is due at the end of the year,

the value of the money being reckoned at 3 per cent., and is paid at once. A certain amount, viz. £(5 × 3), is deducted from the bill, and the sum paid is £485. The sum deducted is called **Banker's** or **Commercial Discount**; the sum paid now is called the **Present Value**.

If, instead of reckoning the discount on the **Amount** of the bill (in the above case £500), it is reckoned on the **Present Value**, the discount is called **True Discount**.

In daily life True Discount is rarely, if ever, used, Banker's Discount taking its place.

Banker's Discount, though *mathematically unsound*, is used in practical life as it is easy to reckon a percentage discount on the actual amount of the bill to be discharged, and less easy to find the True Discount. The error involved is not very large and is compensated for by the practical convenience.

Example VII. How much will be given for a bill of £5500 discounted 3 months before it is due? Rate per cent. = 4.

$$\begin{aligned}\text{Discount} &= \text{£} \frac{5500 \times 4 \times \frac{3}{12}}{100} \\ &= \text{£}55.\end{aligned}$$

$$\begin{aligned}\therefore \text{Amount given} &= \text{£}5500 - 55 \\ &= \text{£}5445.\end{aligned}$$

Example VIII. Find the discount on a bill of £356. 12s. 9d. due on August 30th and discounted on July 1st at 4 %.

Three extra days, called **three days of grace**, are always allowed on discounting bills. The bill is therefore due on September 2nd.

The bill is paid 63 days before it is due.

$$\begin{aligned}\text{Discount} &= \text{£} \frac{356 \cdot 6375 \times 4 \times 63}{100 \times 365} \\ &= \text{£} \frac{3 \cdot 566375 \times 4 \times 63}{365} \\ &= \text{£}2 \cdot 462... \text{ to 3 places} \\ &= \text{£}2. 9s. 3d.\end{aligned}$$

(Note.—Only one of the limiting days is included.)

EXAMPLES. XXXIII 33

Find the simple interest on—

1. £500 for 2 years at 4 %.

2. £600 for 1 year at $3\frac{1}{2}\%$.
3. £450 for 2 years at 4% .
4. £800 for 3 years at $2\frac{1}{4}\%$.
5. £750 for 4 years at $3\frac{1}{4}\%$.
6. 2500 dollars for 6 years at $2\frac{1}{4}\%$.
7. £370 for 4 years at $3\frac{1}{4}\%$.
8. £1540 for 5 years at $6\frac{1}{4}\%$.
9. £538 for 4 years at 4% .
10. £730 for 4 months at 3% .
11. £8500 for 6 months at $4\frac{1}{2}\%$.
12. £753. 10s. for 3 years at 3% .
13. £818. 15s. for 4 years at $2\frac{1}{4}\%$.
14. £400. 12s. 6d. for 5 years at 4% .

Find to the nearest penny the simple interest on—

15. £156 for $2\frac{1}{4}$ years at $3\frac{1}{2}\%$.
16. £250 for 100 days at 5% .
17. £173. 6s. for 210 days at 4% .
18. £1445. 13s. for 3 years at 4% .
19. £725. 10s. for 2 years 5 months at 3% .
20. £1883. 12s. 6d. for 5 years at 5% .
21. £3394. 10s. for 45 days at $2\frac{1}{2}\%$.
22. £3643. 10s. for $3\frac{1}{2}$ years at $4\frac{3}{4}\%$.
23. £573. 10s. for 1 year 250 days at $3\frac{1}{8}\%$.
24. £1324. 15s. for 28 days at 4% .
25. £734. 8s. 6d. for 7 months at $3\frac{1}{2}\%$.
26. £5796 for 1 year 9 months at $4\frac{1}{4}\%$.
27. £1368. 15s. for 106 days at $4\frac{3}{4}\%$.
28. £240. 12s. 6d. for $8\frac{3}{4}$ years at $2\frac{1}{2}\%$.
29. Show that interest at 5% per annum is equivalent to 1d. per £ per month.

30. A man borrows £26. 5s. on January 1st and promises to pay on February 3rd. If interest be charged at $12\frac{1}{2}\%$ per annum, what does he pay on February 3rd?

31. The National Debt is £659187451; find how much is paid in interest on this amount at 3 %.

32. The funds of a certain Friendly Society amount to £121803. If this were put out to interest at the rate of £2. 12s. 6d. %, to what would it amount at the end of 5 years?

33. Use the method of practice to find the interest on £512. 1s. 3d. for 5 years at $3\frac{5}{6}$ %.

34. Find the interest on £1500 from November 1st to December 23rd at $3\frac{1}{4}$ %.

35. Interest is charged on a loan at the rate of $3\frac{1}{8}$ % per annum. A sum of £2500 is borrowed on April 15th. How much must be paid back on July 12th?

36. What sum of money will amount to £660 in 5 years at 4 %?

37. Find the principal which will amount to £328. 13s. 5d. in 3 months at 4 %.

38. In what time will £225 amount to £315 at 5 %?

39. Find the rate per cent. if £227. 10s. amounts to £259. 7s. in 4 years.

40. In how many years will £227. 10s. amount to £259. 7s. at 4 %?

41. How many years must elapse before the interest on £1500 is £365. 12s. 6d. at $3\frac{3}{4}$ %?

42. A sum of money doubles itself in 25 years at simple interest. Find the rate per cent.

43. After how many days will the interest on £540 at 4 % exceed £10?

44. In $4\frac{3}{4}$ years a certain sum of money amounts to £4249. 0s. 9d. at $3\frac{1}{2}$ %. Find the sum of money.

45. Find the rate per cent. necessary in order that the interest on £1560 should be £245. 14s. in $3\frac{1}{2}$ years.

46. In how many years will £200 double itself at $2\frac{1}{2}$ %?

47. £481. 5s. amounts to £516. 8s. 10d. in $2\frac{1}{4}$ years. Find the rate per cent.

48. A sum of £50 amounts to £56 at 3 %. To what would it amount at $2\frac{3}{4}$ %?

49. If £275 amounts to £291. 10s. in 3 years, what will be the amount in 5 years?

50. What sum of money will amount to £284. 4s. in six months and £284. 8s. in a year?

51. Find the rate per cent. per annum represented by 1 cent per dollar per month.

52. Calculate the amount when £3725. 15s. is put out at simple interest for $3\frac{1}{2}$ years at $3\frac{1}{3}\%$.

53. After how many days will £985. 10s. amount to £997. 13s. at 3% ?

54. What sum of money will amount to £726. 15s. in three months at $3\frac{3}{4}\%$?

55. Find the interest on £1256 from May 15th to June 29th. (Rate per cent. = 5.)

56. Use the method of practice to find the amount of £916. 10s. 9d. after $4\frac{1}{2}$ years at $3\frac{9}{16}\%$.

57. In how many years will a sum of money treble itself at 5% ?

58. A sum of money amounts to £197. 15s. in $3\frac{1}{4}$ years at 4% . Find to how much the principal amounts at the end of 2 years.

59. Find the interest on £717. 10s. 5d. for 171 days at $2\frac{1}{8}\%$.

60. A man borrowed £20 and agreed to pay 1s. a month for the use of the money. At what rate per cent. per annum did he pay interest?

61. 5143 dollars 65 cents is lent for 302 days at $4\frac{1}{2}\%$. What is the interest?

Note.—In the following questions discount is taken to be banker's discount.

Find the discount on—

62. £1500 due 6 months hence at 3% .

63. £75. 10s. ... 3 years 4% .

64. £1172 ... 9 months 5% .

65. £385 ... 6 months $2\frac{1}{8}\%$.

66. £7509 ... 2 years $3\frac{3}{4}\%$.

67. How much will a banker give for a bill of £2500 discounted 28 days before it is due at 3% ?

68. Find the present value of £1700 due 3 months hence at 5 %.

69. What sum should be deducted from a bill of £3500 due 6 months hence at $3\frac{1}{2}$ % if paid at once?

70. A bill of £357. 10s. at 3 months is discounted at once for £353. 18s. 6d. Find the discount rate.

71. A merchant allows 6 months credit for goods for which he charges £317. 15s. The discount rate is 4 %; what should he charge the purchaser if he paid cash down?

72. A bill of £348 is drawn on Jan. 5th payable after 3 months. For how much would it be discounted on Jan. 31st at 4 %?

73. Find the discount on a bill of £800 drawn June 26th for 7 months, and discounted July 5th at 5 %.

74. The discount on a bill of £9645. 1s. 6d. due three months hence is £119. 1s. 6d. : what is the discount rate?

75. A man owes £903. 11s. 3d. : how much should he pay if he discharged his debt at once, the money not being due for 11 months? The interest is charged at the rate of $3\frac{1}{8}$ %.

76. Find the discount on a bill of £739. 12s. drawn on May 9th for 9 months and discounted on July 30th at 4 %.

§ 34. COMPOUND INTEREST

Example I. Find the compound interest on £753. 4s. 6d. for 4 years at 3 %.

$$\begin{array}{r}
 \text{£} \\
 753.225 \\
 22.5968 \dots \dots \dots (a) \\
 \hline
 775.8218 \\
 23.2747 \dots \dots \dots (\beta) \\
 \hline
 799.0965 \\
 23.9729 \\
 \hline
 823.0694 \\
 24.6921 \\
 \hline
 847.7615 \dots \dots \dots (\gamma) \\
 753.225 \\
 \hline
 94.5365
 \end{array}$$

$$\begin{aligned}
 \text{The interest} &= \text{£}94.5365 \\
 &= \text{£}94. 10s. 9d.
 \end{aligned}$$

Line (α) is obtained by multiplying the principal by the rate per cent. and moving the decimal point two places to the left. The new principal, obtained by adding the principal for the first year and the corresponding interest, is multiplied by the rate per cent., and thus line (β) is obtained. This process is carried out until we obtain line (γ)—the amount at the end of the specified time. Subtracting the original sum of money from this, the compound interest is obtained.

Example II. Find the compound interest on £753. 4s. 6d. for 2 years at 3 %, interest being payable half-yearly.

£	
753·225	
7·5323.....	(α)
3·7662.....	(β)
764·6235	
7·6462	
3·8231	
776·0928	
7·7609	
3·8805	
787·7342	
7·8773	
3·9387	
799·5502	
753·225	
46·3252	

Interest = £46·3252
= £46. 6s. 6d.

Note that here, since the interest is paid half-yearly, 3 % per annum = $1\frac{1}{2}$ % per half-year. Line (α) is 1 % of the principal, and line (β) $\frac{1}{2}$ %.

The method used above is an example of Practice.

The formula used in finding the compound interest on a sum of money corresponding to the formula on page 152 for simple interest is

$$A = P\left(1 + \frac{R}{100}\right)^n,$$

where £A is the amount of a principal £P for n years at R per cent.

Compound Interest tables are compiled giving the amount

of £1 for numbers of years at varying rates of interest. In these cases if a be the amount of £1 at the end of 1 year, $A = Pa^n$.

Example III. Find the amount of £700 for 2 years at 3 %.

$$\begin{aligned}
 & A = Pa^n. \\
 \text{Here} \quad & a = £1.03, \\
 \therefore \quad & A = £\{700 \times (1.03)^2\} \\
 & = £\{700 \times 1.0609\} \\
 & = £742.63 \\
 & = £742. 12s. 7d. \text{ (to the nearest penny).}
 \end{aligned}$$

When the number of years is large it is important to notice that four-figure logarithms give only a very approximate result. More correct results can be obtained by using seven-figure logarithms, but with large sums errors occur even then.

Example IV. Find the amount of £5000 for 20 years at 3 % compound interest.

$$\begin{aligned}
 & A = Pa^n. \\
 \therefore \log A &= \log P + n \log a, \\
 \text{i.e. } \log A &= \log 5000 + 20 \log 1.03 \\
 &= 3.6991 + 20 \times .0128 \\
 &= 3.6991 + .2560 \\
 &= 3.9551. \\
 \therefore A &= 9018. \\
 \therefore \text{Amount} &= £9018 \text{ (approximately).}
 \end{aligned}$$

It will be seen that the above result is only very approximate, and that a better result would have been obtained by using more accurate tables.

Example V. What is the present value of a sum of £1000 to be paid in 10 years at 2 % compound interest?

$$\begin{aligned}
 & A = Pa^n. \\
 \therefore 1000 &= P \times (1.02)^{10}. \\
 \therefore P &= 1000 \times (1.02)^{-10}. \\
 \therefore \log P &= \log 1000 - 10 \log 1.02 \\
 &= 3 - 10 \times .0086 \\
 &= 3 - .086 \\
 &= 2.914. \\
 \therefore P &= 820.4. \\
 \therefore \text{Present value} &= £820. 8s.
 \end{aligned}$$

Example VI. At what rate per cent. will £1500 amount to £1687 in 3 years?

$$\begin{aligned}
 A &= Pa^n. \\
 \therefore \log A &= \log P + n \log a, \\
 \text{i.e. } \log 1687 &= \log 1500 + 3 \log a. \\
 \therefore \log a &= \frac{1}{3} \{ \log 1687 - \log 1500 \} \\
 &= \frac{1}{3} \{ 3.2271 - 3.1761 \} \\
 &= \frac{1}{3} \times .0510 \\
 &= .017. \\
 \therefore a &= 1.04.
 \end{aligned}$$

$\therefore$ Rate per cent. = 4 (approximately).

A good illustrative example involving the principles of Compound Interest without the use of logarithms is the following:—

Example VII. An amount of £1000 has to be collected; only £880 can be raised. How long must it lie at compound interest at 4 % till £1000 is obtained, and how much will be left over to meet general expenses?

The sum cannot be taken out in the middle of a year, and hence will be greater than £1000.

As in Example I. :—

£	
880	
35.2	
915.2	...Amount at end of 1st year.
36.608	
951.808	 2nd year.
38.072	
989.880	 3rd year.
39.595	
1029.475	 4th year.

$\therefore$ it must lie for 4 years, and £29.475, *i.e.* £29. 9s. 6d. will be left over for general expenses.

EXAMPLES. XXXIV

34

Find the amount at compound interest of—

1. £100 in 3 years at 3 %.

2. £500 in 2 years at 5 %.
3. £750 in 4 years at $2\frac{1}{2}$ %.
4. £280 in 3 years at $2\frac{1}{4}$ %.
5. £759 in 3 years at $3\frac{1}{4}$ %.
6. £5625 in 3 years at 6 %.
7. £100. 15s. in 4 years at 3 %.
8. £1572. 16s. in 3 years at $2\frac{1}{4}$ %.
9. £3600 in $2\frac{1}{2}$ years at 3 %.
10. £3782 in $2\frac{1}{2}$ years at $2\frac{1}{2}$ %.
11. £3643. 10s. in $3\frac{1}{2}$ years at $4\frac{3}{4}$ %.
12. £1661. 8s. in 3 years 4 months at 3 %.
13. £560 in 2 years at 4 %, interest payable half-yearly.
14. £729. 10s. in 1 year 6 months at 3 %, interest payable half-yearly.
15. £350. 12s. 6d. in 5 years at 4 %.
16. £1250 in 18 months at 3 %, interest payable quarterly.
17. £3000 in 2 years at $2\frac{1}{2}$ %, interest being added half-yearly for the first year and then quarterly for the second year.
18. £726. 14s. 9d. in $3\frac{1}{2}$ years at $3\frac{1}{2}$ %.

Use the formula $A = Pa^n$ to obtain the following results as correctly as your logarithms will allow :—

Find the amount of—

19. £500 in 21 years at 3 %.
20. £756 in 12 years at 4 %.
21. £356. 12s. in 14 years at $3\frac{1}{2}$ %.
22. £6500. 10s. in 33 years at 2 %.
23. £3009 in 20 years at $3\frac{1}{2}$ %.
24. £1759. 16s. in 40 years at $3\frac{1}{3}$ %.
25. £1668. 12s. in 31 years at $2\frac{1}{4}$ %.

Find the principal which will amount to—

26. £874. 3s. in 3 years at $5\frac{1}{4}$ %.
27. £580. 15s. 5d. in 3 years at 6 %.

28. £604. 4s. in 3 years at 4 %.
29. £143. 16s. 8d. in 3 years at $2\frac{1}{2}$ %.
30. £100 in 2 years at $2\frac{1}{8}$ %.

What is the present value at compound interest of—

31. £1000 due 15 years hence at 2 %?
32. £6720 due 5 years hence at 4 %?
33. £100 due 21 years hence at 3 %?
34. £5000 due 14 years hence at $2\frac{1}{4}$ %?

At what rate per cent. will—

35. £1250 amount to £1447 in 3 years?
36. £5432. 11s. amount to £6199. 9s. in 3 years?
37. £4500 amount to £4890. 12s. 6d. in 2 years?
38. £100 amount to £126. 13s. 6d. in 8 years?

39. By means of an example show that compound interest at 4 % payable yearly is not the same as 2 % payable half-yearly.

40. In how many years will £466. 9s. amount to £495 at 2 % compound interest?

41. A sum is put out to interest at 5 % per annum, the interest at the end of each year being added to the capital: by how much per cent. will it have increased in 3 years?

42. £1640 is borrowed to be paid back in 2 years by two equal annual payments, allowing 5 % compound interest. Find the annual payment.

43. The amount of a certain sum of money at the end of 2 years is £100: at the end of 3 years it is £102. Find the rate per cent.

44. What sum of money put out at compound interest will amount to £212. 3s. 7d. at the end of 2 years, and £225. 4s. at the end of 4 years?

45. A father invests £10 for his son on the day the latter is born. How much interest can the son claim on his twenty-first birthday if the £10 is invested at $3\frac{1}{2}$ % compound interest?

46. What is the difference in the compound interest on £100 lent at 4 % for 20 years if the interest is added on (i) yearly, (ii) half-yearly?

47. Find by how much the compound interest on £57800 for 4 years at $3\frac{1}{8}\%$ exceeds the simple interest on the same sum for the same time and at the same rate.

48. In how many years will a sum of money double itself at 4% compound interest?

49. If a penny had been put out at compound interest at 2% a hundred years ago, what would it have amounted to by now?

50. A sum of money amounted to £1650 in 1 year and £1815 in 2 years. Find the principal if (i) simple interest, (ii) compound interest, is reckoned.

51. A boy was to get from his father the difference between the compound and simple interest on £350 for 3 years at 4% if he calculated it correctly. He did so; how much did he get?

§ 35. STOCKS AND SHARES

There are no new principles involved in questions on stocks and shares. A few examples will suffice to explain the method.

Example I. How much Royal Mail Steam Ordinary stock at $112\frac{1}{2}$ can be bought for £5400?

$$\begin{aligned} & \text{£}112\frac{1}{2} \text{ cash will purchase £}100 \text{ stock} \\ \therefore \text{£}5400 & \dots\dots\dots \text{£}\left(\frac{100 \times 5400}{112\frac{1}{2}}\right) \text{ stock.} \\ \therefore \text{Amount of stock} &= \text{£}\frac{100 \times 5400 \times 2}{225} \\ &= \text{£}4800. \end{aligned}$$

Example II. £500 Western Australia stock is bought at 98: what does it cost?

$$\begin{aligned} & \text{£}100 \text{ stock costs £}98 \text{ cash.} \\ \therefore \text{£}500 & \dots\dots\dots \text{£}\left(\frac{98 \times 500}{100}\right) \text{ cash.} \\ \therefore \text{Cost} &= \text{£}\frac{98 \times 500}{100} \\ &= \text{£}490. \end{aligned}$$

Example III. £1 shares are at 18s. If a man invests £90 in these shares when they pay a dividend of 1s., how much dividend does he get?

He can buy $\left(\frac{90 \times 20}{18}\right)$ shares.

Each share pays 1s.

$\therefore$ His dividend is $\left(\frac{90 \times 20}{18}\right)$ s.,

i.e. £ $\frac{90}{18}$, or £5.

Example IV. Find the income derived from investing £750 in South Manchurian $4\frac{1}{2}\%$ stock at 90.

£90 yields an income of £ $4\frac{1}{2}$.

$\therefore$ £750 £ $\frac{4\frac{1}{2} \times 750}{90}$.

$\therefore$ Income = £ $\frac{9 \times 750}{2 \times 90}$
= £37. 10s.

Example V. By investing £3150 in the 3 per cents. a man obtains an income of £90: what is the price of stock?

£90 income is obtained by investing £3150

$\therefore$ £3 £ $\frac{3150 \times 3}{90}$.

$\therefore$ Price of stock = £ $\frac{3150 \times 3}{90}$
= £105.

Example VI. When 3 per cents. are at 88 I invest £5000. I sell out at 91 and invest the proceeds in $4\frac{1}{2}\%$ per cents. at 120. Find the change in income.

£88 cash yields an income of £3.

$\therefore$ £5000 £ $\frac{3 \times 5000}{88}$.

$\therefore$ First income = £ $\frac{3 \times 5000}{88}$
= £170. 9s. 1d.

The amount of money obtained by selling the stock at 91
is £ $\frac{5000 \times 91}{88}$.

$$\begin{array}{l} \text{£120 cash yields an income of } \text{£}4\frac{1}{2} \\ \therefore \text{£} \frac{5000 \times 91}{88} \dots\dots\dots \text{£} \frac{4\frac{1}{2} \times 5000 \times 91}{88 \times 120} \end{array}$$

$$\begin{aligned} \therefore \text{Second income} &= \text{£} \frac{9 \times 5000 \times 91}{2 \times 88 \times 120} \\ &= \text{£}266. 12s. \end{aligned}$$

$$\therefore \text{Change in income} = \text{£}96. 2s. 11d.$$

Note.—Stockbrokers charge a commission to the buyer or seller of stocks or shares. This commission—called **brokerage**—varies from $\frac{1}{8}$ per cent. to $\frac{1}{2}$ per cent. of the amount of stock.

Example VII. I invest £4500 in Burma Railways stock at $99\frac{1}{4}$. How much stock do I obtain, brokerage being $\frac{1}{8}\%$?

Price of stock is $\text{£}(99\frac{1}{4} + \frac{1}{8})$, i.e. $\text{£}99\frac{3}{8}$.

$\therefore$ Since $\text{£}99\frac{3}{8}$ will buy £100 stock,

$$\text{£}4500 \dots\dots\dots \text{£} \frac{100 \times 4500}{99\frac{3}{8}} \text{ stock.}$$

$$\begin{aligned} \therefore \text{Amount of stock} &= \text{£} \frac{100 \times 4500 \times 8}{795} \\ &= \text{£}4528. 6s. \end{aligned}$$

To illustrate the difference between the **marked** price and the **market** price we may notice the following:—

No. 39.

1 SHARE.

£100 Stock.

THE NORTHERN AND WESTERN RUBBER COMPANY.

This is to certify that John Smith is the owner of **1** share of One Hundred Pounds fully paid Capital Stock of the Northern and Western Rubber Company.

LONDON, *January 11, 1912.*

J. JAY,
Secretary.

A. BEE,
Chairman.

The above is a copy of a *Share Certificate*. Although it is **marked** at £100, it may be selling in the **market** for £107.

Again: Tickets for the pit of a theatre cost 2s. 6d. each. There is a scarcity of seats owing to the fact that a famous actor is appearing. A man holding one of these tickets is quite likely to be offered 5 or 6 shillings for it; but he still has a ticket marked 2s. 6d.

Stock is merely what a man is entered on the books of the company as having, in exactly the same way as the theatre company would state that the man who paid 6s. for his seat in the above case was only the holder of a pit ticket at 2s. 6d.

The following is a copy of the upper part of a Dividend Warrant (the lower part is merely a receipt):—

*Three Hundred and Seventieth
Dividend.*

No. 1902.

SUN INSURANCE OFFICE
(Founded 1710),
63 THREADNEEDLE STREET, E.,
February 26, 1914.

I hand you annexed warrant for Dividend on.....10.....
Shares as follows, viz. :—

	£	s.	d.
Interim Dividend of Seven Shillings			
per Share	3	10	0
Less Income-tax at 1s. 2d. in the £	0	4	1
	<u>£3</u>	<u>5</u>	<u>11</u>

I hereby certify that the Income-tax has been or will be paid by the Company to the proper Officer for the receipt of taxes.

THOMAS EVANS,
Manager and Secretary.

To J. JONES, Esq.

EXAMPLES. XXXV 35

1. How much stock can be obtained by investing £300 in Alberta 5% stock at 90?

2. I invest £735 in Egypt 4 % stock at 98. What amount of stock do I obtain?
3. Metropolitan Tramway £1 shares are quoted at $\text{£}3\frac{7}{8}$. How many shares could be obtained for £64?
4. How much money must be invested to purchase 100 £20 shares of the Rock Island Railway Company at £17. 5s.?
5. A man intends to buy £1500 Peru Land Company stock at 104. What must he lay out?
6. Canadian Iron 6 % stock is at 94. How much stock can be obtained by investing £1500 in this company? Give your answer to the nearest shilling.
7. Find how much it will cost to buy 386 Nitrate Produce £10 shares at £7. 15s.
8. How much will be realised from the sale of £5600 Brazil Railway $4\frac{1}{2}$ % stock at $88\frac{1}{2}$?
9. A man wishes to realise £1500 on the sale of L. & S.W. stock at 115. What amount of stock must he sell?
10. Paraguay £5 shares are at £5. 5s. How many shares could be obtained by investing £100, and how much money would be left over?
11. Find the cost of £7560 Natal $3\frac{1}{2}$ % stock at 89.
12. A man invests £7560 in Natal $3\frac{1}{2}$ % stock at 89. How much stock does he get?
13. I have 370 shares in the Cunard Company, which has its capital in £1 shares. Find the amount of money I should receive if I sold out all my shares at 28s. 6d.
14. How much stock will £4181 buy if invested in Bank of Australasia stock at 111? If when the price was 113 this amount of stock was sold out, how much money would be realised?
15. A company has as capital £25000 in £1 shares. Find the market value of the capital when the price of each share is 24s. 9d.
16. Find the annual income derived from £1500 Newfoundland 5 % stock at 99.

17. I invest £3750 in Aranco 5 % debentures at par. What income do I get from the investment?

18. £1 Pekin Syndicate shares yield a dividend of 2s. per share. What dividend is derived from investing £30 when each share costs 15s.?

19. What is the annual income derived from £21000 British Electric 6 % stock at 84?

20. Cape $3\frac{1}{4}$ % stock is at 89. I hold £4895 stock in this company. Find the income I obtain from this investment.

21. A man invests £5874 in Liverpool $3\frac{1}{2}$ % stock at par. What income does he derive from this?

22. If in the above question the man had waited till the price had fallen to 1 below par, what would have been his income?

23. Russian 4 % stock is at $88\frac{1}{2}$. Find the income to the nearest penny derived from investing £5000 in this stock.

24. If I invest £1800 in South Australian stock at 90, how much stock do I get? If I sell out at 88, how much do I lose?

25. £1 shares in a certain industrial company cost 23s. 6d.; they pay a dividend of 2s. 3d. How many shares could be bought for £50? What dividend would be derived from these shares?

26. A man invests in Irish $2\frac{3}{4}$ % stock and finds that, after paying 6d. in the pound income-tax, his net income is $\frac{1}{6}$ of the sum invested. Find the price of the stock.

27. What is the annual income derived from investing £5000 in Portugal 3 % stock at 64?

28. I invest £576 in £8 Bank shares at $18\frac{5}{8}$, the dividend being at the rate of 15 %. What income do I derive from my investment?

29. How much per cent. does an investor obtain for his money if he invests £1450 in Japan $4\frac{1}{2}$ % stock at $90\frac{5}{8}$?

30. By investing £1000 in Uruguay $3\frac{1}{2}$ % stock a man obtains a yearly income of £50. Find the price of the stock.

31. How much Ottoman 4 % stock at 87 must I buy in order to obtain an annual income of £52?

32. What is the price of Sheffield $3\frac{1}{2}$ % stock when a sum of £1175 invested will produce an annual income of £43. 15s.?

33. Queensland stock is at 87. A man invests £1450 and thereby obtains an income of £58. 6s. 8d. What dividend per cent. does the stock pay?

34. Venezuela 4 % stock is at 60. How much per cent. per annum will be obtained by investing in this?

35. Find to the nearest penny the income derived from investing £5000 in South Manchurian $4\frac{1}{2}$ % stock at 89.

36. How much stock can I buy for £3500 when $2\frac{1}{2}$ % Consols are at 73? What is my income?

37. What is the price of a £1 share in a certain gold-mining company paying 2s. dividend per share, when an investment of £70 brings in as dividend £6. 13s. 4d.?

38. I buy Rand 6 % stock at 93. Find correct to the second decimal place how much the investment brings me in per cent.

39. A man invests £5025 in Johannesburg 4 % stock and obtains £5000 stock. What is the price of the stock? What is the man's yearly income?

40. I invest £5712. 10s. in Great Western stock at 114 $\frac{1}{4}$: I sell out when the price of stock has risen to 22 $\frac{3}{4}$ above par. Find how much money I gain.

41. If 62 £1 shares are bought when the price of a share is 22s. 6d. and are sold again when the price has risen to 23s. 9d., find the gain on the investment.

42. The buying price of West Australian $3\frac{1}{2}$ % stock is 87 and the selling price 89. How much would it cost to buy £1560 of this stock? How much would be realised by selling it again?

43. A man invests £8517 in Canadian Northern 4 % stock at 85. What is his income? If he sells out and invests the proceeds in Russian 5 % at par, what is the change in his income?

44. Find which is the better investment, China 5 % stock at 99 or Hungary 4 % stock at 83.

45. A man wishes to invest some money. Which will be the more profitable to buy : Glasgow 3 % stock at 91, or Sheffield 3½ % stock at 94 ?

46. Find the difference in the income obtained by investing £615 in Winnipeg 5 % at 82 and £600 in Canadian Iron 6 % at 90.

47. A man sells out £1500 stock at 96 yielding 3 %, and invests the proceeds in 3¼ % stock at par. What is the change in his income ?

48. After investing £2800 in Mexican 5 % at 42 and receiving one yearly dividend, a person sold out at 44¾. Find how much he gained by the transaction.

49. Is it more advantageous to invest in 2½ % Consols at 75 or 3½ % London County stock at 91¼ ?

50. When Chile 5 % stock is at 98, I sell out £3000 stock and invest the money in £2 shares in the Bengal Iron Company at 35s. 6d. The dividend paid by the latter company is 4s. 6d. per share. What is the difference in the dividend I obtain ?

51. A broker charges 2s. 6d. per cent. How much will it cost me to buy £7500 Mercantile 5 % stock at 119¼ ?

52. I buy £3000 Brazil Traction stock at 90¼ and sell out at 96⅞. If the brokerage in each case is ⅙ per cent., what do I gain by the transaction ?

53. A man invests his money partly in Royal Mail stock at 112½ and partly in Southern Punjab at 95. The first investment yields 5 % and the second 4 %. In each case the income derived is £57. How much money did the man invest ?

54. What annual income is obtained by investing £1700 in Argentine 3½ % stock at 85 ? If the stock is sold out at 83¼ and the proceeds invested in Brazil 5 % stock at 111, what is the change in income ?

55. I invest equal sums in a 4 % stock at 90 and a 3 % stock ; I get 5 % for my money. Find the price of the 3 % stock.

56. I want to invest £3270 partly in 2¾ % Consols at 92 and partly in India 4 % stock at 104 so that the income from the two investments may be the same. How much must I invest in each ?

57. A person invests £6030 in India 3 % stock at $100\frac{1}{2}$ and pays income-tax at 1s. 2d. in the pound; on the stock rising to 102 he sells out and invests the proceeds in railway stock at 180 paying 5 % free of income-tax. Find the change in his net income.

58. A sum of £1000 was invested in the purchase of a $2\frac{1}{2}$ % stock at $88\frac{3}{4}$. Find how much stock was obtained, and the annual income that will be received. What would have been the income if the £1000 had been invested in a $3\frac{1}{2}$ % stock at 104?

59. A man invests a sum of money in Russian 3 % stock at 98. If he had invested in the same stock when the price was 99 he would have received £2. 10s. less. How much did he invest?

60. What must be the market value of 7 % stock in order that, after deducting income-tax of 1s. 2d. in the pound, it may yield 7 % interest?

§ 36. MISCELLANEOUS EXAMPLES ON TIME AND WORK

Example I. A train, 120 yards long, travelling at 40 miles per hr. overtakes another, 100 yards long, travelling at 30 miles per hr. How long will the first train take to pass the second?

The speed of the first train **relative** to the second will be $(40 - 30)$ miles per hr.

The problem therefore reduces to the following: "How long will it take to cover $(120 + 100)$ yards at a speed of $(40 - 30)$ miles per hr.?"

By the unitary method :—

$$\begin{aligned}\text{Time taken} &= \frac{220}{1760 \times 10} \text{ hr.} \\ &= \frac{60 \times 60 \times 220}{1760 \times 10} \text{ sec.} \\ &= 45 \text{ sec.}\end{aligned}$$

Note.—If the trains had been travelling in opposite directions, the speed of either **relative** to the other would have been $(40 + 30)$ miles per hr.

Example II. At what time between 4 and 5 o'clock are the hands of a watch together?

Now the large hand moves through 60 minute spaces while the small hand moves through 5.

$\therefore$ the large hand moves through 12 while the small hand moves through 1.

$\therefore$ for every 12 spaces through which the large hand moves it gains 11 on the small hand.

i.e. for every $\frac{12}{11}$ spaces through which the large hand moves it gains 1 on the small hand.

Now, in the question, the large hand starts 20 spaces behind the small hand: to be coincident with the small hand it must therefore gain 20 spaces on the small hand. It does this in $\frac{12}{11} \times 20$ spaces,

i.e. in $\frac{12}{11} \times 20$ minutes.

$\therefore$ Hands are together at $(\frac{12}{11} \times 20)$ minutes past 4,

i.e. at $21\frac{9}{11}$ minutes past 4.

Example III. A tap A can fill a bath in 40 minutes; a tap B can fill it in 50 minutes. How long will it take to fill the bath if both taps are turned on at once?

A can fill the bath in 40 min.

$\therefore$ it can fill $\frac{1}{40}$ of the bath in 1 min.

B can fill the bath in 50 min.

$\therefore$ it can fill $\frac{1}{50}$ of the bath in 1 min.

$\therefore$ A and B together can fill $(\frac{1}{40} + \frac{1}{50})$ in 1 min.,

i.e. $\frac{9}{200}$ in 1 min.

$\therefore$ they can fill it in $\frac{200}{9}$ min.,

i.e. in $22\frac{2}{9}$ min.

EXAMPLES. XXXVI 36

1. A man can row 4 miles an hour in still water. How long will it take him to row 2 miles with the stream, the stream flowing at the rate of a mile an hour?

2. How long will it take a train 195 yards long to pass a wall 25 yards long if the speed of the train is 30 miles an hour?

3. At what time between 8 and 9 o'clock are the hands of a watch opposite?

4. Three men, A, B, and C, are engaged on a piece of work. If A worked alone, he could complete the work in 6 hours; if B worked alone, he could do it in 4 hours. Working together,

the three men take 1 hour 20 minutes. How long would it take C to do the work alone?

5. A man standing on the platform of a station notices that a train whose speed is 36 miles an hour passes the platform in 20 seconds. If the length of the platform is 200 yards, what is the length of the train?

6. In a race of 100 yards A can give B 5 yards start. When C and A race the same distance, C can give A 5 yards start. How many yards start should C give B in 100 yards?

7. A swimming bath can be filled by one tap alone in 10 hours and by another in 15 hours. Supposing the waste-pipe, which can empty the bath in 12 hours, to be left open just as the taps are turned on together, how long will it be before the bath is full?

8. A train 105 yards long is travelling at 45 miles an hour; another, 115 yards long, is travelling at 30 miles an hour. How long will they take to pass one another completely if they are travelling (i) in the same direction, (ii) in opposite directions?

9. At what times between 1 and 2 o'clock are the hands of a watch at right angles?

10. In a hurdle race of 120 yards, A owes 5 yards, B owes 1 yard, and C receives 5 yards. They finish together. The next year C is placed on the scratch mark; how many yards should A and B owe?

11. A crew row from A to B, a distance of 3 miles, and back in $37\frac{1}{2}$ minutes. If the rate at which the tide is flowing is 2 miles an hour, find the speed of the boat in still water.

12. If a train maintains an average speed of 42 miles an hour it arrives at its destination punctually: if, however, the average speed is 40 miles an hour, it arrives 15 minutes late. Find the length of the journey.

13. A man starts on a piece of work which he could finish by himself in 10 days. After he has been working for 2 days a second man is employed. The two men complete the work in 3 more days. Find how long the second man would have taken had he worked by himself.

14. Two men are walking in opposite directions along a footpath parallel to a railway. A train passes one man in 3 seconds, and the other in 10 seconds. If the men are walking

at the same speed, find how long the train would take to describe a distance equal to its own length.

15. Two watches are set right at 12 noon. One gains 3 minutes during 24 hours, and the other 1 minute in the same time. When will the first watch be a quarter of an hour ahead of the second?

16. Working together, A and B can complete a piece of work in 6 days. A can do as much in 4 days as B can do in 5 days. How long would each take to complete the work singly?

17. Two men are cycling in opposite directions; their rates of travelling are $9\frac{1}{2}$ and $10\frac{1}{2}$ miles an hour respectively. They pass each other at 12 o'clock noon; how far will they be apart at 12.15 p.m.?

18. 8 men, 6 women, and 18 boys, working together, make a certain number of articles in an hour. The same number of articles is made in an hour by 14 men, 3 women, and 9 boys. Employing men alone, how many would have to work for an hour to make the same number?

19. A man sets out to walk from London to Harrow, a distance of 9 miles. After covering two-thirds of the distance he turns back, and when he has walked for half an hour he meets another man who started from London half an hour after he did. If the speed of the second man is two-thirds that of the first man, find their rates of walking.

20. A watch gains 2 seconds every 5 minutes. It is put right at 4 p.m. What is the true time on the following day when the watch indicates 2.30 p.m.?

21. Two watches, one of which gains 3 seconds and the other 8 seconds every hour, are correct at 12 noon. What is the true time when one watch is 4 minutes ahead of the other?

22. A motor car sets out from Brighton on a 200-mile journey, keeping up a uniform speed of 20 miles an hour. 15 minutes later another car leaves Brighton on the same journey as the first. For 25 miles the speed of this car is 15 miles an hour; after this it increases its speed to 25 miles an hour. How far will the cars be from Brighton when the second car overtakes the first?

23. The distance between Euston and Rugby is 84 miles. An express train starts from Rugby at 4.25 a.m. and reaches

Euston at 6.10 a.m.; the 5.0 a.m. from Euston reaches Rugby at 6.52 a.m. At what time will the trains meet, and how far will the meeting-place be from Euston? (The trains are assumed to travel at uniform speeds between the two stations.)

24. At what times between 4 and 5 o'clock are the hands of a watch (i) opposite, (ii) at right angles to each other?

25. 20 men, each working 7 hours a day, do half of a piece of work in 12 days. How many extra men, each working 8 hours a day, must be employed in order that the work may be completed within 4 more days?

26. A can give B 5 yards start in 100 yards; B can give C 4 yards start in the same distance. If A's time for 100 yards is 11 seconds, find C's time for the whole distance.

27. A can do a piece of work in 10 days; B can do the same piece of work in 8 days. A works for a certain number of days and then leaves off; B takes up the work and finishes it. If the total time of working was 9 days, find how many days each worked.

28. The tap A of a barrel of liquid could fill a tank in an hour; the tap B of a second barrel could fill the tank in $1\frac{1}{2}$ hours; the tap C of a third barrel could fill it in 2 hours. A and B are turned on together and after 10 minutes C is turned on as well. What will be the proportions of the mixture in the tank when it is full?

29. A watch which gains 1 second in 15 minutes is 10 minutes fast at 3 p.m. (true time); at the same time another watch which loses 1 second in 10 minutes is 15 minutes slow. What will be the correct time when the two watches indicate the same time?

30. Two trains, 90 and 86 yards long respectively, are travelling along parallel lines in the same direction. The faster passes the slower in 1 minute 12 seconds. If they had been travelling in opposite directions with the same speeds as above, they would have passed in $4\frac{1}{2}$ seconds. Find the speeds of the trains.

31. A circular track is $\frac{3}{4}$ mile round. A and B start on a walking match round the track, A having half a lap start. A's rate of walking is $4\frac{1}{2}$ miles an hour, and B's is $6\frac{3}{4}$. When will they be together, and how long after the start will they next be together?

32. A contractor is allowed 14 days in which to complete a piece of work. He engages 10 men, who do $\frac{3}{8}$ of the work in 8 days, working 7 hours a day, and he pays them $9\frac{1}{2}d.$ an hour. How many men working 8 hours a day, and whose rate of pay is $10d.$ an hour, must he employ in addition to those already at work, so that the contract may be fulfilled within the specified time, and how much will he have to pay altogether in wages?

33. A man is cycling along a road parallel to a railway when a train of length 88 yards overtakes him and passes him in $4\frac{1}{2}$ seconds; 16 minutes later the train passes a telegraph pole by the line in $3\frac{3}{4}$ seconds. Assuming that both the train and the cyclist are travelling at uniform speeds, how far ahead will the train be when the cyclist reaches the telegraph pole?

34. A and B, two crews, row a race. At the start, A row at the rate of 33 strokes a minute, and at each stroke the boat travels 37 feet; B row at the rate of 32 strokes a minute, their boat travelling through 36 feet at each stroke. At the end of a minute A drop to 32 strokes and B increase their speed to 33, each stroke sending the boat through the same distance as before. How long after the start will the boats be level?

TEST PAPERS

[First Four Rules, Prime Factors, Compound Rules]

I 1

1. Find the value of $(872193 + 2152) - (1572 + 813015)$.
Write the answer in words.

2. Shares in a certain company are worth £97 each. What is the value of 3125 such shares?

3. The following items are taken from the private accounts of a tradesman:—

March 3. Cash in hand, £32.

„ 4. Paid to A, £16. Received from B, £3.

„ 6. Received from C, £22.

„ 7. Paid to D, £40. Paid to E, £7.

„ 8. Received from F, £18.

Find by drawing a diagram on squared paper his financial position after the above transactions.

4. Find the prime factors of 1485 and 2160. Which prime factors are common to both numbers? Find the product of these common factors.

5. Divide £417. 14s. $1\frac{1}{2}d.$ among 106 people so that each may have the same amount.

6. How much shall I save if I buy 1460 articles at 5s. 3d. per dozen instead of paying $5\frac{1}{2}d.$ each for them?

II 2

1. Find the total majority (L or U) in Wolverhampton, the figures for the divisions being—

E. Wolverhampton.	S. Wolverhampton.	W. Wolverhampton.
L 5072	U 4784	U 5925
U 3881	L 4440	L 5631

2. What is the smallest number above 15000 which will contain 689 an exact number of times?

3. In 1912 the following copper coins were turned out of the Mint: 23079168 pennies, 12570880 halfpence, 5196800 farthings. Find their total value in £ s. d.

4. Every weekday I have one halfpenny paper and one penny one; I have a penny Sunday paper; two threepenny periodicals a week. How much do I owe my newsagent at the end of a month? (Take a month as 28 days.)

5. If a grocer buys currants at $4\frac{1}{4}d.$ per lb. and sells them at 12s. per quarter, what does he gain on each ton?

6. A bankrupt pays 4s. 7d. in the £. How much will a creditor to whom he owes £183. 10s. receive?

III 3

1. Find the total value of imports and exports for 1910 from the following table:—

Imports into China.		Exports from China.	
	£		£
Opium	7460263	Beans	4912001
Cotton Goods . . .	17594511	Cotton (Raw) . . .	3817158
Woollen Goods . . .	833359	Oils (Vegetable) . .	1883826
Metals	2545660	Sesamum Seed . . .	1925705
Cigarettes	929287	Silk	13382651
Coal and Coke . . .	1103467	Skins and Furs . . .	1260575
Oil (Kerosene) . . .	3010406	Cow & Buffalo Hides	1429642
Rice	4216825	Straw Braid	1034714
		Tea	4837607
Total		Total	

2. Simplify $(110000 - 6200) \div (4200 + 130 - 5)$.

3. A man earns 5s. 1d. per day except on Saturdays, when he is paid 2s. 6d. If he does no work on Sundays, and holidays amount to 21 full working days, what was his income in 1912?

4. Two boxes weigh together 1 qr. 2 lb. 14 oz., and one is 1 lb. 4 oz. heavier than the other. Find the weight of the smaller box.

5. Divide £118539. 1s. 10½*d.* by 693.

6. Find the sum of $1^3 + 3^3 + 5^3 + 7^3 + 9^3$.

IV 4

1. Fill in the vacant spaces:—

HOLDINGS UNDER IRISH LAND PURCHASE ACT (1910)

Division.	Number of Agreements.	Purchase Money (£).
Ulster	84869	19219269
Leinster	53440	(b)
Connaught	26163	5227106
Munster	(a)	24502959
Total	222733	74684265

2. The cost of loaf-sugar per cwt. was 19s. 6*d.* in 1902, 19s. 0*d.* in 1907, and 24s. 11*d.* in 1912. Find by how much the cost per ton in 1912 exceeded that in (i) 1902, (ii) 1907.

3. How many articles costing 9s. 2*d.* can be bought for 10 guineas? How much money will be left?

4. From London to Plymouth is 225 miles. A train travelling 45 miles an hour arrives at Plymouth at 3.10 p.m. At what time did it start from London?

5. Find the total cost of 171 articles at 19s. 3½*d.* each, and a gross of articles at 4¼*d.* per dozen.

6. $2s = \frac{44^2 - 36^2}{32}$; find *s*.

V 5

1. Find the number of tons and the value of the lead produced in the United Kingdom in 1910:—

Metal.	Tons.	Value (£).
Iron	4975735	17008812
Tin	4797	738025
Lead	(a)	(b)
Zinc	4168	99824
Copper	449	27570
Total	5006671	18157425

2. A Turkish lira is equal to 18 shillings. Find the value in pounds and shillings of 382156 lire.

3. Find the prime factors of 2744, 3375, 4096.

4. A tradesman has 15 articles worth 7s. $2\frac{1}{2}$ d. each, 12 worth 6s. $8\frac{1}{4}$ d., and 4 worth 8s. 1d. each. He sells them all at 7s. 6d. each; find the profit.

5. A field is in the shape of a rectangle of sides 183 ft. and 606 ft. Find the cost of fencing the field at 1s. $4\frac{1}{2}$ d. per yd.

6. £10000 is to be divided among 798 people so that each has the same share; the amount that each receives is to be as large as possible, and is not to include parts of a penny. How much will be left over?

VI 5

1. The cost of the National Debt for the years 1910–1912 is detailed below:—

	1910–1911.	1911–1912.
	£	£
Perpetual Annuities	15036175	14861556
Government Debts	(a)	(b)
Terminable Annuities	3481490	3517569
Unfunded Debt	1353178	1158841
Cost of Management	175050	173181
New Sinking Fund	4112961	447707

If the cost of the Debt remain at £24500000, find the Government Debts for 1910–1911 and 1911–1912.

2. A piece of wire 4 inches long weighs 50 grains. Find the weight (in lb. and grains) of a mile of this wire.

3. A box contains weights of 5 pounds, 3 pounds, 1 pound, 8 ounces, and 4 ounces. There is the same number of each kind of weight, and the total weight of the box and weights is 21 lb. If the box weighs 1 lb. 8 oz., how many weights are there in the box?

4. Which prime factors when multiplied together will make 14400? Find the value of $3^2 \times 4^2 \times 5^2 \times 6^2$.

5. Find the difference in weight between 96 sovereigns and 96 pence, given that a sovereign weighs 123 grains and a penny weighs half an ounce (avoir.). (2 oz. = 875 gr.)

6. $D = \frac{M}{V}$. If $M = 183725$ and $V = 7349$, find D .

VII 7

1. The following is the balance sheet of a certain Political Union. Find the balance at the end of the year.

Total Income	.	.	.	£28502
Expenditure :—				
Rent, Lighting, etc.	.	.	.	£2662
Salaries	.	.	.	6321
Printing and Stationery	.	.	.	1612
Travelling	.	.	.	2045
Hire of Halls	.	.	.	2816
Postages and Telegrams	.	.	.	1512
Bye-Election Expenses	.	.	.	751
Legal Expenses	.	.	.	894
Other Expenses	.	.	.	4516
				£

2. In a certain sum the dividend is a million and the quotient 89. Find the divisor and remainder.

3. A vertical pole has six marks on it A, B, C, D, E, F. Show by a diagram on squared paper the fact that B is 3 ft. above A, C 6 ft. below B, D 10 ft. above C, E 14 ft. below D, and F 2 ft. below E. How far is F above B?

4. A halfpenny is an inch across. How many, placed so that each touches that next it, would be required to stretch a mile? What would be the value in £ *s. d.* of these coins?

5. A man leaves £1191. 13s. 4d. to be divided equally among his seven sons and six daughters. How much does each receive?

6. Find the value of 2^{30} .

VIII 8

1. Find the number of emigrants from the British Isles whose destination was not stated:—

Year.	Destination of Emigrants.			
	To British N. America.	To U.S.A.	To Australasia.	To S Africa.
1909	85887	109700	37620	22017
1910	156990	132192	45701	27297
1911	184860	121814	80770	30767

The totals for the three years were 288761, 397848, and 454527.

2. A is 723 ft. below sea-level, B is 120 ft. above A, C is 200 ft. above sea-level, D is 612 ft. above B. Find by drawing a diagram the distance between C and D.

3. Find the excess of £10000 over 183 times £51. 2s. 4½d.

4. 1 cu. ft. of copper weighs 550 lb. How many copper discs each of volume 1 cu. in. will weigh 1 ton 4 cwt. 62 lb.?

5. How many days are there from March 26, 1897, to June 2, 1898? (Include both days.)

6. $U = GT$: given that $U = 16677$ and $T = 17$, find G .

IX 9

1. Find the value of $7^3 + 9^3 + 11^3 - (3 \times 7 \times 9 \times 11)$.

2. It was high water at London Bridge on July 15, 1913, at 11.29 a.m.; at 10.55 p.m. on August 28, 1913, it was also high water. Find the number of seconds between these two times.

3. A wheel is 8 ft. 4 in. in circumference. How many times does it turn in travelling $2\frac{1}{2}$ miles?

4. A tradesman buys 15 articles at 2s. 6d. each, 21 at 3s. 4d. each, and 29 at 3s. 0d. each. At what price must he sell each article (charging the same amount for them all) so as to make a total profit of £3. 5s. 6d.?

5. Find the prime factors of 44100 and 210. Hence find the quotient of 44100 by 210.

6. £16. 3s. 0d. is to be divided equally among 19 people. Two fail to come, and the sum of money is distributed among the remainder in equal shares. How much does each gain owing to the non-arrival of the two?

X 10

1. Simplify $\{183 \times (3030 \div 101)\} - (582 - 1109)$.

2. The pendulum of a clock swings 5 times in 2 seconds. How many times does it swing in a year of 365 days?

3. A grocer pays 15s. $8\frac{1}{2}$ d. for a cwt. of sugar. He wastes 8 oz. on every 7 lb. and sells the remainder at $2\frac{1}{4}$ d. per lb. How much does he gain?

4. On an average there are 9 words on a line in a book of 287 pages. If there are 100737 words in the book, how many lines are there to a page?

5. A flask weighs 6 oz. when empty, and 3 lb. 9 oz. when filled with a liquid half as heavy again as water. What does it weigh when filled with water?

6. If the Sun be represented on paper as a point an inch distant from another point (the Earth), then the nearest star will be represented by a point 4 miles away. Reckoned in millions of miles, the Sun is 93 distant from the Earth. Find (in millions of miles) the distance of the nearest star.

[Decimals, Metric System, Vulgar Fractions, Simple Areas and Volumes, Averages.]

XI 11

1. Add the following, giving the answer in (i) cm., (ii) Hm., (iii) inches. ($\cdot 39$ inch = 1 cm.)

117 cm., 39 mm., 10 m., 91 Km.

2. Find which is the more expensive and by how much—

300 articles at £1. 1s. 6d.

or 500 articles at 12s. 3d.

3. A box without a lid, length 6 cm., breadth 3 cm., height 2 cm., is to be covered with paper on the inside. How much paper must be used?

4. Add the greatest and least of the following fractions:—

$$\frac{7}{18}, \frac{4}{9}, \frac{15}{32}, \frac{31}{63}.$$

5. The average of nine numbers is 73.58; the average of eight of them is 74.02. Find the remaining number.

6. The length of the Lötschberg tunnel is 9 miles 210 yards. Express this as miles and decimal of a mile correct to five places.

XII 12

1. How many times is 10 cwt. 14 lb. contained in 506 tons 5 cwt.?

2. Find the value of—

(i) $(2.646)^2 - 7.$

(ii) $7 - (2.645)^2.$

3. The distance from Berlin to Malente is 595 Km. An airman accomplished the flight between these two places in 6 hr. 9 min. What was his speed in Km. per hr.? Give your answer correct to two places of decimals.

4. How many cm. cubes can be cut from a rectangular block of wood, length .85 m., breadth .42 m., height 5 cm.?

5. Find the H.C.F. and L.C.M. of—

170, 2890, 500, 85.

What is the least number I should have to add to 500 so that the H.C.F. of the above numbers should contain 85 as a factor?

6. Simplify $\frac{\frac{1}{1\frac{1}{2}} + \frac{1}{1\frac{1}{3}} + \frac{1}{1\frac{1}{4}}}{\frac{3}{5} - \frac{16}{15} + \frac{17}{60}}.$

XIII 13

1. Express in £ s. d., £3.905 + £.091875.

2. Express the following sums of money as decimals of £1 :—

3s. 6d. ; 4s. 9d. ; 17s. 4½d. ; 2s. 1½d.

Write down the results (i) exactly, (ii) correct to two decimal places.

3. Find the weight in Kg. of a cubical block of metal each of whose edges is 1.02 m., having given that 1 cu. cm. of the metal weighs 8.95 gm.

4. The numbers of copies of an evening paper sold during ten days in March 1912 are :—

Date.	Copies.
1 . . .	568604
2 . . .	459220
4 . . .	547962
5 . . .	538644
6 . . .	552715
7 . . .	556391
8 . . .	537903
9 . . .	446513
11 . . .	526739
12 . . .	531889

By how many does the number of copies sold on March 1 exceed the average?

5. Find the excess of—

(i) $\frac{1}{2} + \frac{5}{9}$ of $1\frac{1}{8}$ over $\frac{1}{24}$ of $3 - \frac{1}{17}$.

(ii) $(\frac{1}{2} + \frac{5}{9})$ of $1\frac{1}{8}$ over $\frac{1}{24}$ of $(3 - \frac{1}{17})$.

6. In the formula $K = \frac{10Hr}{2\pi n}$, $H = .18$, $r = 7$, $\pi = 3.14$, $n = 2$.

Find the value of K.

XIV 14

1. The silver coins turned out from the Mint in 1912 were :— shillings, 20065901 ; sixpences, 9155310 ; florins, 5951284 ; threepences, 5843075 ; half-crowns, 2914573. Express the total amount in pounds and decimals of £1.

2. A rectangular piece of zinc 8 cm. by 6.3 cm. is cut from another rectangular piece 9.3 cm. by 7.6 cm. Find the area of the remaining part.

3. Find the value of—

(i) 82.735×3.1416 ;

(ii) $17.85 \div 2.54$, correct to three places of decimals.

4. A rupee = 1s. 4.03d. How many rupees can be exchanged for £48. 8s. 5 $\frac{3}{4}$ d.?

5. By how much does the sum of $\frac{1}{11}\frac{1}{2}$ and $\frac{7}{37}$ exceed their product?

6. In 1863 there were 234 depositors in the Western Australian Savings Bank; their deposits amounted to £2485. In 1893 there were 4745 depositors with total funds amounting to £74025. Find by how much the average deposit in 1893 exceeded that in 1863. Assuming the rate of increase to be the same from 1893 to 1923 as it was for the thirty years previous to 1893, what would be the average deposit in 1923? Answers to be given to the nearest penny.

XV 15

1. Find with as little working as possible the total cost of 39 articles at £1. 15s. 4 $\frac{1}{4}$ d. each, and 40 articles at 4s. 7 $\frac{3}{4}$ d. each.

2. What is the weight in Kg. of a beam of wood 5.4 m. by .35 m. by .28 m., if 1 cu. cm. of wood weighs .75 gm.?

3. Express the following vulgar fractions as decimals:—

(i) $\frac{3}{625}$.

(ii) $\frac{9}{17}$ correct to 5 places.

(iii) $\frac{51}{3300}$ correct to 5 places.

4. A man invests $\frac{4}{15}$ of his money in Canadian Stock, $\frac{1}{11}$ of the remainder in Consols, and $\frac{1}{4}$ of what is left in Rubber Shares. He has £9000 left. How much did he invest in Consols?

5. Six boys measure the area of a triangle. Their results are 17.75 sq. cm., 18.05 sq. cm., 17.8 sq. cm., 10.05 sq. cm., 17.95 sq. cm., and 18 sq. cm. Neglecting the obviously incorrect answer, what is the average of the results?

6. Simplify $\frac{3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{4} \text{ of } \frac{6}{7}}{3\frac{1}{2} - 4\frac{1}{3} \text{ of } \frac{7}{26} + 1\frac{4}{5}}$.

XVI 16

1. Express 18s. 11 $\frac{1}{4}$ d. as the decimal of £2. 10s.

2. How many sq. cm. of paper would be required to cover a rectangular board 2·75 m. long and ·85 m. broad, leaving a border 5 cm. wide all the way round unpapered?

3. Simplify $\frac{81\cdot03 + (7\cdot01 \times 11\cdot3)}{6\cdot25}$.

4. Find the average speed in knots and length in feet of six of the largest English liners:—

Vessel.	Speed in knots.	Length in feet.
Mauretania . . .	25	790
Lusitania . . .	25	785
Olympic . . .	22·75	883
Campania . . .	22	622
Oceanic . . .	21	685
Aquitania . . .	23·5	902

5. A man owns $\frac{17}{182}$ of an estate and sells ·35 of his share. How much of the estate has he now?

6. Find the value of $\frac{3}{4 - \frac{5}{6 - \frac{1}{7}}}$ of £6. 9s.

XVII 17

1. 100 dm. = 11 yards. Express a litre in cu. in.

2. A solid cylinder, height 19 cm., radius of base 1·8 cm., fits tightly into a box so as just to touch the sides: the height of the box is the same as the height of the cylinder. How much water must be poured in to fill up the space between the cylinder and the box?

3. Express as decimals of a ton correct to three places—

(i) 110 lb. ; (ii) 3 cwt. 10 lb. ; (iii) 3 stone.

4. The average weight of the crew of an eight-oared boat is 11 st. 1 lb. When the weight of the coxswain is included, the average is decreased by 4 lb. Find the weight of the coxswain.

5. The velocity of light in miles per sec. is given by the formula $V = \frac{4r}{1980}$, where V = vel. of light and $r = 92\cdot8 \times 10^6$. Find V correct to the nearest whole number.

6. Simplify $\frac{3}{3-1 \div (3-\frac{1}{3})} \times \frac{6\frac{3}{8}}{6-1 \div (6-\frac{1}{6})}$.

XVIII 18

1. 86400 mean solar seconds = 1 mean solar day. Also 86164 sidereal seconds = 1 mean solar day. Express a mean solar second as the fraction of a sidereal second in its simplest form.

2. Find the volume of a glass cube, side 3.7 cm. If 1 cu. cm. of glass weighs 2.25 gm., find the weight of the cube in Kg.

3. By how much does $\frac{2}{5}$ of 7s. 6d. exceed $\frac{5}{12}$ of a guinea? Express the answer as the decimal of £1.

4. Express the following decimals as fractions in their lowest terms:—

(i) .3125; (ii) .576; (iii) .0001024.

5. Find the cost of—

(i) 100 articles at 3s. $4\frac{1}{2}d.$ each;

(ii) 1500 articles at $1\frac{1}{2}d.$ each.

(Use decimals.)

6. The number of visitors to the Science Museum, South Kensington, from July 1, 1909 to April 13, 1913, was 174507. Excluding days on which the Museum was shut (amounting to 232), find the average daily attendance.

XIX 19

1. Find the value of $\frac{2}{11}$ of a furlong + $\frac{3}{7}$ of $33\frac{1}{4}$ yards - $\frac{2}{3}$ of 16 ft. $1\frac{1}{2}$ in.

2. A salesman has to make up rice into packages containing either 4 lb. 6 oz., 4 lb. 11 oz., or 7 lb. 14 oz. Find the least quantity of rice that can be used so that none is wasted. How many packages containing 4 lb. 11 oz. will there be?

3. Find the value of $(.0125)^3$ —

(i) by reducing the decimal to a vulgar fraction;

(ii) by actual multiplication.

4. A tank is 10.3 metres long, 4.2 metres broad, and

2 metres deep. How many litres will be left after filling it with water to a depth of 30 cm.?

5. The volume of a certain quantity of gas at different temperatures is given below. Find the average volume.

	Temperature.	Volume in cu. cm.
1	0° C.	300
2	15° C.	316·5
3	30° C.	332·9
4	50° C.	354·9
5	70° C.	376·9
6	100° C.	409·9

6. Simplify $\frac{\frac{4}{7} \text{ of } 1\frac{5}{9}}{\frac{2}{3} \text{ of } 8\frac{1}{2}} \times \frac{10\frac{3}{4} - 6\frac{1}{2}}{1\frac{1}{3} - \frac{8}{9}}$.

XX 20

1. A square courtyard, side 10 m., is to be paved with tiles 10 cm. sq. How many will be wanted? How many more tiles would be required if, instead of being square, they were 8 cm. by 5 cm.?

2. Find the value of

$$\frac{174 \cdot 13238}{5 \cdot 615} - \frac{8 \cdot 31183}{23 \cdot 05}$$

3. In finding the answer to a question a boy worked it by calling the value of a farthing £·001. The question was to find the cost of 1900 articles at 3d. and 300 articles at 4½d. By how much was his answer incorrect?

4. By how much does the volume of a cylinder, height 20·04 cm., radius of base 10 cm., exceed the volume of a cone of the same height and the same base area?

5. Find the L.C.M. of 999999 and 1955, expressed in prime factors. Hence divide the product of these two numbers by $37 \times 13 \times 17 \times 7 \times 23$.

6. Simplify $\frac{(\frac{4}{7} \times \frac{5}{8}) \text{ of } 3}{\frac{4}{7} + \frac{5}{8}} - \frac{1}{1 + \frac{1}{4 - \frac{3}{14}}}$.

[Unitary Method, Areas and Volumes, Density, Graphs,
Square Root, Proportion.]

XXI 21

1. Supply the missing figures in the following table :—

Year.	Revenue (£).	Expenditure (£)
1907-8	156537700	(a)
1908-9	151578300	152292400
1909-10	131697900	157945000
1910-11	(b)	172000000
1911-12	185090000	178545000
Average :—	165750700	162518800

2. The distance between two towns is 50 miles : on a map this is represented by 6·25 inches. If a circle of radius 2 inches be drawn on the map, what area does it represent?

3. Find the value of—

$\frac{1}{8}$ of a sovereign + ·0625 of £3 + ·15 of £5 + $\frac{1}{4}$ of 4s. 1d.

4. In 1912 the output of rubber by a certain company was 565335 lb. against 380735 lb. for 1911 ; the average price realised per lb. was 4s. 6d. in 1912 and 5s. 1½d. in 1911. Find whether the receipts for 1912 exceeded those for the previous year, and if so by how much.

5. The specific gravity of coal is 1·2. Find the volume taken up by a ton of coal if 1 cu. ft. of water weighs 1000 oz.

6. A cogwheel which has 30 cogs fits into another which has 56 cogs : if the former turns 16 times in 15 sec., how often does the latter turn in 14 sec.?

XXII 22

1. The election of members to a certain association does not take place in August and September. The following table shows the numbers enrolled during the other months :—

Month.	1911-12.	1912-13.
April . . .	65	95
May . . .	37	42
June . . .	49	45
July . . .	10	60
October . . .	73	92
November . . .	242	261
December . . .	214	147
January . . .	121	29
February . . .	44	46
March . . .	100	65

Find by how much the numbers for November exceeded the average in the years 1911-12, 1912-13.

2. If 5 men or 7 women can do a piece of work in 37 days, in what time will 7 men and 5 women do the work?

3. Draw a triangle of sides 7, 8, 9 cm. Find its area. If a piece of zinc, thickness 1 mm., be cut to this shape, find how much it would weigh. (Density of zinc = 7.1 gm. per cu. cm.)

4. Find the value of $\frac{3}{8} + \frac{9}{16} + \frac{11}{32}$ (i) by vulgar fractions, (ii) by decimals. Show that both results are the same.

5. A man can run 100 yd. in 11 sec. Assuming that he can run only half as fast as this when running 10 miles, find how long he would take over this distance.

6. $C = \frac{V}{R \cdot \frac{b+n}{n}}$: find C when $V = 1.4$, $b = 3$, $R = 5.6$, and $n = 6$.

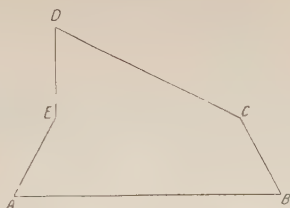
Give your answer as a decimal correct to three places.

XXIII 23

1. Find the cost of carpeting a room 19 ft. 4 in. long and 14 ft. 3 in. wide, at 4s. $1\frac{1}{2}d.$ per yard, the carpet being 33 in. wide.

2. Divide £1753 among two men so that for every 11 pence that one gets the other may have 1s. 9d.

3. Find the area of the following figure :—



$AB = 3$ in. ; $BC = DE = EA = 1$ in. ; angles at A, B, are 60° .

4. The deflection of a steel beam varies, within certain limits, directly as the load. When the load is 16 lb. the deflection is 1.58 cm., and when the load is 8 lb. the deflection is .78 cm. Plot a curve connecting load and deflection, and find the deflection when the load is 35 lb.

5. Energy is supplied electrically at 5.7. per unit. A unit is 1000 watts per hour. In a certain experiment 450 watts were used for 20 minutes: find the cost of the energy supplied.

6. Find the density of brass, given that a cube of the material, side 1.91 cm., weighs 57.12 gm.

XXIV 24

1. The density of a liquid may be measured by a hydrometer. In a certain case the law of the hydrometer was unknown, but the following readings were taken for liquids of different densities :—

Density.	1.82	1.75	1.57	1.39	1.27	1.12	1.105	1
Reading.	63.5	61	50.2	40.5	30	13	12	0

Draw a graph and estimate the density of a liquid which gave a reading 15.1.

2. A greengrocer bought 1500 oranges at 20 for a shilling. He sold them at such a price that he gained £2. 10s. How many oranges would a purchaser get for a shilling?

3. 25.22 lire = 9.46 roubles. A Russian exchanges 100 roubles for Italian money and receives 260 lire: find how many lire (to the nearest whole number) he loses.

4. The volume of a rectangular prism is $14\cdot415$ cu. cm. The length of one side is $3\cdot1$ cm. If the length of one of the other sides is three times this length, how long is the remaining side?

5. At $6s. 4d.$ in the pound a man could pay all his creditors, his assets being £4750. Omitting the account of one of his creditors, he could pay $7s. 11d.$ in the pound. How much does he owe to this creditor?

6. The friction in a certain machine is given by the formula $F = P - \frac{W}{k}$; find F , given that $P = 14\frac{3}{16}$, $W = 21\cdot5$, and $k = 2\frac{1}{10}$. Express your answer as a fraction.

XXV 25

1. The average age of a class of 25 boys is 12 yr. 2 mo. One boy leaves and two join the class; the average age rises to 12 yr. 3 mo. If the ages of the new boys are 12 yr. 4 mo. and 12 yr., find the age of the boy who left.

2. (i) How many times is $\frac{3}{7} + \frac{1}{3}$ contained in the difference between $4\frac{1}{20}$ and $3\frac{3}{44}$?

(ii) Divide $3\cdot246 \times \cdot00187$ by $\cdot066$ without unnecessary working.

3. The premium to be paid on a certain class of property for insurance is $2s. 6d.$ on every £100. How much would be paid at this rate on property worth £75864?

4. Find the amount of metal required for a cylindrical tube, outside radius $6\cdot2$ cm., length 1 metre, and thickness 5 mm.

5. $1\cdot293$ gm. of normal air occupies a litre. Draw a graph connecting litres and gm. Hence find the space taken up by a Kg. of air.

6. A tobacconist mixes tobacco at $7\frac{1}{2}d.$ per oz., another brand at $5d.$ per oz., and a third at $4\frac{1}{2}d.$ per oz. in the proportion 1:4:5. He sells the mixture at $6d.$ per oz. How much does he gain per lb.?

XXVI 26

1. Plot a graph showing how the price of a carpet varies with its area; and find the probable price of a carpet 8 ft. 6 in. by 4 ft.

Area in sq. ft.	Price.
18	£1. 7s. 6d.
23 $\frac{3}{4}$	£1. 19s. 6d.
31 $\frac{1}{2}$	£2. 9s. 6d.
38 $\frac{3}{4}$	£2. 15s. 0d.
40 $\frac{1}{2}$	£3. 1s. 6d.
44	£3. 7s. 6d.
49 $\frac{1}{2}$	£3. 17s. 6d.
60	£4. 12s. 6d.

2. Coffee is sold at 1s. 4d. per lb. Is it more expensive to buy it in England or in France, where the price is 4 fr. 50 c. per Kg.? 453 gm. = 1 lb., and 25 fr. = £1.

3. Extract the square roots of—

(i) $\frac{121}{361}$. (ii) 4.41. (iii) 255025.

4. Find the value of $(\frac{95}{96})^2 - (\frac{31}{48})^2$. Show that the answer is equal to the product of $\frac{95+62}{96}$ and $\frac{95-62}{96}$.

5. A piece of timber is cut up. Of two pieces, the first is 12 ft. 6 in. long, $10\frac{1}{2}$ in. wide, and 10 in. thick; it weighs 320 lb. The second is 10 ft. long, 9 in. wide, and weighs 153.6 lb. What is its thickness?

6. A length of copper wire, radius .5 mm., weighs 69.08 gm. The specific gravity of copper is 8.8: how long is the wire?

XXVII 27

1. Find the value of $(\frac{12^3 - 10^3}{8^3 - 6^3})(\frac{8^2 - 6^2}{12^2 - 10^2})$.

2. London, with a population of 4521301, sends 61 members to Parliament. Assuming that the number of members is proportional to the population, find how many the following towns should send:—

Town.	Population.
Birmingham . . .	525903
Liverpool . . .	747566
Manchester . . .	716180
Glasgow . . .	784621

3. A cistern can be filled by three taps which take separately 3, $4\frac{1}{2}$, 6 hours to fill it. A waste-pipe can empty the cistern in 9 hours. At 3 p.m. a man turns on the three taps and goes away without noticing that the water can flow out through the waste-pipe. When he returns he finds that the cistern is just full. What time did he come back?

4. The twist of a metal wire varies inversely as the fourth power of the diameter of the wire. In a wire of diameter $\frac{1}{4}$ in. the twist is 48; find the twist for a wire of diameter $\frac{1}{3}$ in.

5. Which is the greater density, gm. per cu. cm. or half-oz. per cu. in.? ($453\cdot6$ gm. = 1 lb.; $2\cdot54$ cm. = 1 in.)

6. The sum of a number of terms in a geometrical progression is $\frac{a(r^n - 1)}{r - 1}$, where a is the first term, n the number of terms, and r the common ratio. Find the sum of eight terms of a progression of which the first term is 64 and the common ratio 5.

XXVIII 28

1. The efficiency of a screw-jack varies with the load according to the following table:—

Load in lb. . .	1·80	2·78	3·53	4·41	5·22	6·84	8·48
Efficiency % . .	7	9·3	11·2	11·8	12·4	13·3	13·8

By drawing a graph, find the efficiency for a load of 6 lb.

2. Find the value of (i) $\frac{7^4 + 7^2 5^2 + 5^4}{7^2 + 7 \times 5 + 5^2}$, (ii) $\sqrt{21893041}$.

3. Given the times of trains as under, at what time would you expect the second train to arrive at Carlisle?

Euston . . .	10.0 p.m.	2.0 a.m.
Crewe . . .	1.57 a.m.	5.15 a.m.
Carlisle . . .	5.23 a.m.	

4. The specific gravity of gold is 19·3. Find the thickness of a piece of gold-leaf weighing 7·72 gm., its area being 4 sq. m.

5. A grocer pays £1. 1s. for a consignment of sugar. Find the weight of this, knowing that 88 lb. of sugar are worth as much as 9 lb. of tea, 15 lb. of tea are worth as much as 22 lb. of coffee, and coffee is 1s. 4d. per lb.

6. A tent in the form of a pyramid is to be constructed on a hexagonal base, side 6 ft. If the volume of air inside is 360 cu. ft., find the height of the top of the tent above the ground.

XXIX 29

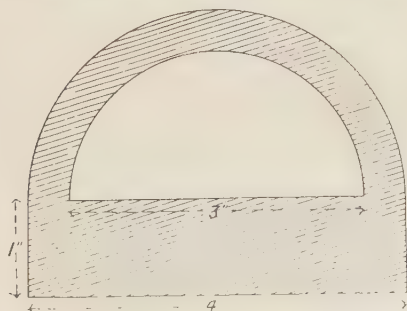
1. In an experiment on the straining of a wire, the lower end was placed against a fixed scale and readings were taken.

Straining force in lb.	·3	3·3	8·4	12·5	17·6	21·6	27·6	31·6
Scale reading in in. .	·15	·16	·18	·20	·24	·99	4·40	9·30

Find from a graph the scale reading when the straining force was 25·6 lb.

2. The average speed of a train from London to Bristol was 48 miles per hr. It left London at 3.0 p.m. and reached Bristol, 118 miles distant, $5\frac{1}{2}$ minutes late. At what time did the train arrive at Bristol?

3. Find the area of the shaded part in the figure :—



4. Find the value of $\frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$.

5. A contractor will cart 13033 cu. yd. of earth $\frac{2}{7}$ mile for £70; how far should 181 cu. yd. be carted for the same sum?

6. Find the square root of (i) 277729, (ii) $22\frac{1}{9}$, (iii) 6.4, correct to three places.

XXX 30

1. From the following table find the approximate weight of a water-engine of horse-power $\frac{1}{20}$ by drawing a graph connecting the given values:—

Weight in Kg. .	12	22	29	55	84	176
H.P.	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{1}{30}$	$\frac{1}{17}$	$\frac{1}{16}$	$\frac{1}{5}$

2. In the report of a tea company it was stated that the total crop for 1912 was 1689285 lb., at an average price of 6.64d. per lb. Find the amount realised for the crop. In the previous year the average price realised was 7.59d. per lb. What amount of tea ought to have been produced so that the receipts for the two years should have been the same?

3. Find the volume of wood required for a box with a lid 11 in. long, $4\frac{1}{2}$ in. wide, and $6\frac{1}{4}$ in. deep, the thickness of the box being $\frac{1}{2}$ in.

4. The railway statistics for 1912 state that the decrease of passengers was as follows:—First class, 130733; second class, 10516232; third class, 21184925. Show that these are in the ratio 1 : 80.4 : 162 approximately.

5. Find correct to two places of decimals the value of (i) $\sqrt{17} - \sqrt{15}$, (ii) $\sqrt[4]{17}$.

6. If x men take y days to dig a trench z yards long, how many men will it take to dig a trench c yards long in b days?

[General Arithmetic.]

XXXI 31

1. In a mine 536 tons of ore were produced containing 530 tons crude copper, 190 oz. gold, and 5110 oz. silver. What percentage of the ore was (i) gold, (ii) silver?

2. A garden is in the shape of a trapezium, parallel sides 90 yd. and 53 yd.; the perpendicular distance between the sides is 17 yd. Find its value at 6*d.* per sq. yd.

3. The fuel used in manufacturing the 29726937 tons of pig-iron produced in the United States during 1911 included 35721127 tons of coke. Find correct to two decimal places the number of tons of coke used per ton of pig-iron.

4. Use the method of practice to find the simple interest on £450. 12*s.* 6*d.* for 3 years at $4\frac{3}{4}\%$.

5. The velocity of an aeroplane of given horse-power is inversely proportional to the cube root of the supporting surface of the planes. Given that in certain units the supporting surface is $\frac{1}{2160} S$ when the velocity is 60 miles per hr., find what velocity is represented by $\frac{1}{10000} S$.

6. A man on looking at his watch notices the time to be 4.20. Less than five minutes afterwards he again looks at his watch and finds that the hands are together. What is the time?

XXXII 32

1. A tradesman sells an article for 35*s.*, and thereby gains 5%. At what price should he sell it so as to gain 10%?

2. The formula $\delta = \frac{444.53 + .082h}{15.32 + .0065h}$ is used in working with a platinum thermometer. Find δ correct to three places of decimals when $h = 2$.

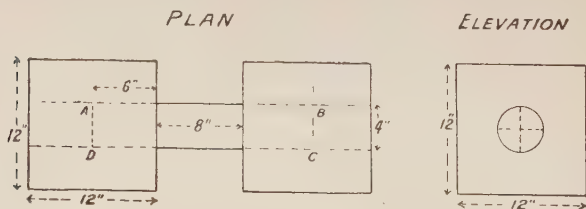
3. What discount will a banker give on a bill of £600 drawn on September 8th for 3 months and discounted on November 1st at $3\frac{1}{2}\%$?

4. (i) How many digits are there in 5^{50} , given $\log 2 = .3010300$?

(ii) Find the value of $M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$ giving the moment of inertia of a circular cylinder, where $M = 17.05$, $l = 8.31$, $r = .66$.

5. Compare the dividend obtained by investing a sum of money in 3 per cents. at $82\frac{1}{2}$ and $3\frac{1}{2}$ per cents. at $93\frac{1}{2}$. Express your answer as a decimal.

6. A bar of iron is of the following shape: find its volume (each cube has a cylindrical hole through it, and the joining cylinder ABCD is solid):—



XXXIII 33

1. In 1888 there were 37957 policemen in England and Wales; in 1911 there were 51203. Find the percentage increase to the nearest whole number.

2. What weight of hydrogen, density .09 gm. per litre, will fill a hollow sphere, external and internal diameters 35 and 28 cm. respectively?

3. The shortest time that a liner can cross the Pacific from Yokohama to Victoria, B.C., a distance of 4200 miles, is 9 days 5 hours. Find the average speed in knots, given that a knot is 6080 ft. per hour. What is the increase in average speed if the previous best time was 10 days 10 hours?

4. Find the compound interest on £1 for 3 years at $4\frac{1}{2}\%$ expressed as a decimal of a pound, and use your result to find the compound interest on £350. 16s.

5. When stock is at $92\frac{3}{8}$ paying 3% an investor obtains a yearly income of £630. He sells out at 94, and invests the proceeds so as to produce $3\frac{1}{2}\%$ on his investment. What change takes place in his income?

6. Find the value of P, the drop in pressure when air flows through a cast-iron pipe, from

$$P = .125 \times 10^{-5} \times \frac{34^{n-1} \times 10^n \times 400}{6.6^n \times 12^{3-n}},$$

when $n = 1.8$.

XXXIV 34

1. A costermonger bought 60 oranges at 5 for 2d. He sold 30 at 3 for 1d. and 30 at 2 for 1d. What was his gain or loss per cent.? Explain why the amount of money he received for his oranges is more than the amount he paid for them.

2. The total output of ore from a mine was 538·86 tons, containing 74·5 % metallic tin. Deducting a certain amount for smelting losses, the weight of metallic tin finally returned was 390·8 tons. What percentage of the ore is represented by smelting losses?

3. During 1912 the Middlesex County Council Tramways carried 92 million people, the traffic receipts amounting to £461942. Find in pence and decimals of a penny (correct to two decimal places) how much each traveller paid on the average.

4. A cylinder of lead of height 12 cm. and radius of base 3 cm. is melted down, and then rolled into a sphere without any waste of material. Find, correct to as many figures as your tables will allow, the radius of the sphere.

5. Find what sum will amount to £8334. 18s. in 3 years at 5 % compound interest.

6. Two-thirds of a man's capital is invested in stock yielding $4\frac{1}{2}$ %, and the other third in stock yielding $3\frac{1}{2}$ %. The income-tax at 11*d.* in the pound on the whole of the annual interest amounts to £38. 10s. Find the capital invested.

XXXV 35

1. The number of paupers admitted to workhouses in London during 1911 was 101311; the number during 1912 was 105084. The first number represented 22·4 per 1000 of the population, and the second 23·2 per 1000. Find whether the population of London was taken to be the same in both years.

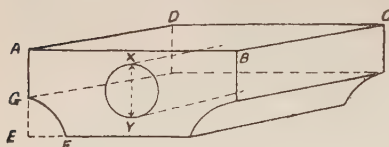
2. A field in the shape of a rectangle whose sides are in the ratio 1 : 2 is to be fenced with wire. The area of the field is 181202 sq. yd. Assuming that four rows of wire will be required, and that the length of wire is 4 % greater than the perimeter of the field, find to the nearest yard how much wire will be wanted.

3. $\pi = 3\cdot141592\dots$ Find π^2 correct to four significant figures.

4. Find the discount on a bill of £485. 17s. 6*d.* drawn April 15th for 6 months, and discounted on July 8th at $3\frac{1}{2}$ %.

5. The side of a cube is 7·6 in. correct to the nearest tenth of an inch. Find as accurately as the data will allow the volume of the cube.

6. Find the volume of the stone slab:—



AE = 6 in.

AB = BC = 13 in.

EF = 2.5 in.

XY = 4 in.

GF is a quadrant of a circle, and there is a cylindrical hole through the slab.

XXXVI 36

1. A bookseller allows a regular customer 10 % discount off the marked price of a certain book. This book is marked at 15s., and on selling it to this customer the tradesman makes a profit of $12\frac{1}{2}$ %. Find the cost of the book to the tradesman. If the book had cost him 6d. less, and the marked price had been 14s. 9d., find what his percentage of profit would have been, discount being allowed as before.

2. From London to Berlin is 724.5 miles, and the journey takes 21 hr.; from London to Cairo is 2499 miles, and it takes 7 days to accomplish the distance. Find how much time would be saved if it were possible to travel from London to Cairo at the same average rate as one can travel to Berlin.

3. A chemist has to mix equal weights (1 gm.) of liquids of S.G. 1.05 and 1.45 respectively. By mistake he mixes equal volumes (1 cu. cm.) of the liquids. What is the percentage error in the weight of the mixture?

4. Use the method of practice to find the area of a square pane of glass, side 2 ft. 5 in. Find the weight of the glass if it is $\frac{1}{8}$ in. thick, and a cu. ft. of glass weighs 2800 oz.

5. Find the difference between the simple interest and the compound interest on £1678 for 3 years at 3 %.

6. P watts are required to keep a disc of paper of diameter d inches revolving at the rate of 1000 revolutions a minute. The formula connecting P and d is $P = k d^s$. Evaluate k , knowing that $s = 5.5$ and $P = 154.9$ when d is 47.1 in.

XXXVII 37

1. A shipment of 95 tons of ore showed 32.15 dwt. of gold and 171.68 dwt. of silver. Express the weights of gold and silver as percentages of the total weight of ore.

2. Find the cost of 7 tons 15 cwt. 1 qr. 27 lb. of copper at £58. 6s. 8d. per ton. If the price increased $\frac{3}{4}d.$ per lb., what would be the cost of 8 tons?

3. Use contracted methods to find the volume of a cylinder of height 17.375 cm., radius of base 3.066 cm., correct to the nearest cu. cm. ($\pi = 3.141592\dots$)

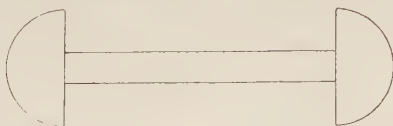
4. What change of income would be occasioned to an investor by selling out £10000 Tasmanian $3\frac{1}{2}\%$ stock at $100\frac{1}{2}$ and investing the proceeds in New Zealand 4% stock at $103\frac{1}{2}$?

5. Find Z from the formula

$$Z = \frac{P}{r} \left\{ 1 + \frac{\log r}{\log e} \right\} + E,$$

where $P = 170$, $r = \frac{1}{2.649}$, $e = 2.718$, $E = 101.8$.

6. A ladder has 20 rungs; each rung is cylindrical in shape, 9 in. long, and of diameter 1 in. The sides of the ladder are semicylindrical, and of diameter 4 in. The ladder is 11 ft. long. How much wood is required to make the ladder? The following is an end-on view:—



XXXVIII 38

1. During the year ending March 31st, 1913, 402432 people emigrated to Canada. Of these, 150542 were British, 139009 were from the United States, and the remainder from other countries. What percentage of the number of emigrants (correct to the first decimal place) came from other countries?

2. In 1831 the population of the British Isles was 16539318. The value of the exports from this country was £71429004, and the value of the imports was £49713889. Use contracted methods to find the value of (i) exports, (ii) imports, per head of population, to the nearest shilling.

3. A train 175 yards long travelling at 45 miles per hr. overtakes another train 150 yards long travelling at 30 miles per hr. How long will the first train take to pass the second?

4. On a certain investment the tax amounts to 5 % per annum for the first month, and an additional 1 % per annum per month until the maximum of 10 % has been reached. What would be the tax on £500 during a year at this rate?

5. A man sells out £5000 Argentine 5 % stock at 99 $\frac{3}{4}$, and invests the proceeds in Electric Lighting 4 $\frac{1}{2}$ % stock at 91. Find the change in his income.

6. A certain formula in trigonometry is

$$f(x) = 1 - \frac{x^2}{1.2} + \frac{x^4}{1.2.3.4} - \dots$$

The actual value of $f(x)$ when $x = \frac{\pi}{4}$ is $\frac{1}{\sqrt{2}}$. To how many significant figures does the result obtained by taking the three terms above for $f(x)$ agree with the actual result?

XXXIX 39

1. A rectangle of paper 2 ft. by 1 ft. can be rolled into a hollow cylinder in two ways. Compare the volumes of solid cylinders of these dimensions.

2. The value of a ruby is proportional to the 1.5th power of its weight. The weights of two stones are in the ratio 6.5 : 13.4; if the first one is of unit value, find the value of the second. Use contracted methods, and give your answer correct to the second decimal place.

3. A and B are two stations 3 miles apart. Just as a train starts from A it emits a whistle; a man on the platform at B hears it and notices that the time is exactly 1 o'clock. At what time will the train arrive at B? The train is travelling at 30 miles per hour, and the velocity of sound may be taken as 1089 ft. per sec. (Answer to nearest second.)

4. If a sum of money be allowed to accumulate at 3 % per annum compound interest, how many years must elapse before it has increased by more than 12 $\frac{1}{2}$ %?

5. A man can row 5 $\frac{1}{2}$ miles an hour in still water. He rows from X to Y and back again in a stream flowing at the rate of 2 miles an hour in 2 hr. 12 min. How far is X from Y?

6. (i) Given $\log 43 = 1.6335$, find how many noughts there are immediately after the decimal point in 43×10^{-15} .

(ii) Find x from $\sqrt[3]{x} \cdot 0537 = .6165$.

XL 40

1. Professor Wood has stated that a 2-lb. loaf loses an ounce a day in weight as a result of the evaporation of water in it. After how many days will the value of such a loaf, sold when freshly baked at $3\frac{1}{4}d.$, decrease to $2\frac{3}{4}d.$?

2. The profits of a certain company were 476775 dollars for the year ending June 1911, 622456 dollars for the year ending June 1912, and 588194 dollars for the nine months ending March 1913. Find the percentage increase of profit in 1912, and assuming the same percentage increase of the 1912 profit for 1913 find how much extra profit must be made from March to June 1913 to obtain the required amount.

3. A cyclist was travelling with uniform velocity along a straight level road on a foggy day when objects were just visible a furlong off. He saw a milestone in front of him for half a minute and then passed it. Afterwards he overtook a carriage the back of which he saw for a minute and a half. What was the speed of the carriage?

4. A company starts a reserve fund to meet an anticipated expenditure of £10000 in 10 years' time. The fund is to be maintained by 10 annual contributions from the profits of the company, these contributions being invested at 3 % compound interest, and each contribution being 20 % larger than the previous one. Find the value of the first contribution.

5. When are the hands of a watch together between 7 and 8 o'clock?

6. Find K from the formula

$$\frac{15 \times 10^6}{.8 \times .1266} = \frac{15 \log 2.718}{K \log \frac{30}{26.43}}$$

MISCELLANEOUS EXAMPLES

1. Simplify $\frac{2\frac{1}{5} - 1\frac{2}{3}}{3\frac{1}{3} + 1\frac{2}{5}} \div \frac{\frac{2}{7} - \frac{1}{8}}{\frac{2}{7} + \frac{1}{8}}$ [L.U.]

2. A dealer bought 45 bullocks for £607 and sold one for £15. 10s. Find, to the nearest shilling, the average selling price of the others, if he made altogether a profit of £48. 12s.

[L.U.]

3. 13680 persons were brought to trial before the English superior criminal courts during 1910. The average for the 5 years 1906-1910 was 13381. Find the average for the 4 years 1906-1909.

4. The price of bread per cwt. increased from 8s. 4·6d. per cwt. in 1902 to 9s. 8·3d. per cwt. in 1912. What is the increase per cent.?

5. In 1903 December 18th is Friday. On what day will January 1st, 1920, fall?

[C.U.H.]

6. A carpet 8 ft. 6 in. by 10 ft. 9 in. is priced at 10 guineas. How much should a carpet 9 ft. by 10 ft. 7 in. cost?

7. The animals at the Zoological Gardens consume annually, among other things, 219 loads of straw and 426 bushels of wheat. Find by how much the amount of straw exceeds the amount of wheat, given that a load = 18 stones and a quarter weighs 512 lb. Show also that the ratio of the amount of straw consumed to the amount of wheat consumed is a little greater than 63 : 32.

8. A piece of wire 4·7 in. long weighs 55 grains. Find the weight of a foot of the wire to the nearest whole number.

9. Find the dimensions of a tank which is 1·28 metres deep, and which holds 3000 litres, the length of the tank being three times the width.

[L.U.]

10. A formula for a photographic developer is as follows :—

Hydroquinone	.	.	.	3.5 gm.
Metol	.	.	.	.5 "
Sodium Sulphite	.	.	.	25.0 "
Sodium Carbonate.	.	.	.	45.0 "
Water up to.	.	.	.	1000.0 "

Express the weights of the solids in grains when the added water makes up the total to 20 oz. (7000 grains = 1 lb.)

[L.U.]

11. A tradesman marks his goods at 20 per cent. above the cost price ; if he then allows a discount of 1s. 3d. in the £ off the marked price, what percentage of profit does he make ?

[C.U.S.]

12. Find the value of

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \frac{1}{5.6.7}.$$

13. A circular plate of iron of uniform thickness is 35 cm. in diameter and weighs 17.2 Kg. Find what will be the weight of the plate correct to the nearest tenth of a Kg. after 25 circular rivet holes, each 1 cm. in diameter, have been drilled out of it.

[C.W.B.]

14. A tank measuring 8 in. by 11 in. by 17 in. is to be filled with a liquid which is sold in France at 13.7 fr. per litre. Find the cost of the liquid to the nearest shilling. (£1 = 25 fr. ; 1 in. = 2.54 cm.)

[L.U.]

15. How much tea which costs 1s. 5½d. a pound must be mixed with 1 cwt. which cost £9. 16s. 8d. in order that 15 % may be gained by selling the mixture at 1s. 11d. per lb. ?

[C.U.S.]

16. A cylindrical vessel 4 cm. in diameter and 30 cm. long is two-thirds filled with water. How many marbles each 2 cm. in diameter must be dropped into the vessel in order to bring the surface of the water to the top of the vessel ?

[L.C.C.]

17. The number of seamen, excluding fishermen, employed in British ships in 1911 was 236418 of whom 165159 were British, 43127 Lascars, and the rest foreigners. What percentage of seamen were neither British nor Lascars ?

18. A formula in mechanics is $n^2 = \frac{g}{Wh}$. Find n^2 when $g = 32$, $W = 64 \cdot 4$, and $h = \cdot 01$. Find also the value of n correct to two decimal places.

19. The annual output of the public elementary schools in England is 600000 scholars. Of these, 1 in 22 proceeds to a secondary school and 1 in 46 receives free education there. What percentage of those proceeding to a secondary school receives free education at that school?

20. Find, correct to the nearest penny, (i) the compound interest on £250 for 3 years at $2\frac{1}{4}$ per cent. per annum, (ii) the compound interest on £125 for 3 years at $4\frac{1}{2}$ per cent. per annum.

[C.W.B.]

21. Find the area of a spherical ball of $3\frac{3}{4}$ in. diameter. If the ball weighs $5\frac{3}{4}$ oz., find, correct to two places of decimals, the weight of one cubic inch of the ball.

[L.U.]

22. A and B enter into partnership; A puts £3500 into the business, but at the end of 3 months withdraws £450, and again 7 months later withdraws £315 of this capital, while B withdraws nothing. If at the end of a year they are entitled to equal shares of the profits, how much capital did B provide?

[C.U.J.]

23. Find, by contracted methods, the values of—

(a) the product $7 \cdot 2351 \times 121 \cdot 0234$,

(b) the quotient $6 \cdot 0251 \div 2 \cdot 0143$,

each to three decimal places only.

[L.U.]

24. In Paris gas costs 20 c. per cu. m.; in London, 3s. per 1000 cu. ft. Taking 1 cu. m. = $35 \cdot 287$ cu. ft. and 10 c. = $1d.$, find to the nearest penny the difference in English money of the cost of 1000 cu. m. of gas in Paris and in London.

[L.C.C.]

25. A man has £1000 invested partly at $3\frac{1}{2}\%$ and partly at 5%. His total income is £44. 17s. 3d. Find the amounts of his investments.

[L.U.]

26. How many times can a hemispherical bowl, radius 12 cm., be filled from a tank 10 ft. by 3 ft. by 4 ft.? (1 inch = 2.54 cm.)

27. The time of oscillation of a pendulum varies as the square root of its length, or $t = k\sqrt{l}$.

The following observations were made on a swinging pendulum whose length could be altered :—

Length of pendulum—	6	5	4	3	2	1
Time of one complete } osc. (sec.).	2·7	2·46	2·2	1·9	1·56	1·1.

Plot a curve, and deduce from these observations the value of k in the above equation. [L.C.C.]

28. Prove by actual multiplication that the fifth root of 423 lies between each of the pairs of numbers (3, 4), (3·3, 3·4), (3·35, 3·36), and find by trial the next pair (that is, the pair containing four significant figures). [L.U.]

29. The area of a rectangular plot is 2960 sq. ft., and the area of a path 3 ft. wide surrounding the plot is 720 sq. ft. Find the length and breadth of the plot. [C.U.S.]

30. The electrical resistance of a wire of diameter d cm. and length l cm. is given by

$$R = \frac{4el}{\pi d^2},$$

where e is a constant for the material of which the wire is made. Find R for a wire of diameter ·5 mm. and length 1 m., for which $e = 40·8 \times 10^{-6}$.

31. Since 1885 there have been 2101 men promoted from the ranks in the British Army, 640 being made second lieutenants, 98 riding-masters, and the rest quartermasters. Express the number of quartermasters as a percentage of the whole.

32. A company has a pension fund to which employees contribute at the end of each year 2 per cent. of their salaries, and an amount equal to that contributed by each employee is added by the company. On retirement, each employee receives his own and the company's contributions, with simple interest on both reckoned at 4 per cent. per annum. If an employee starts with a salary of £400, which is increased to £500 after ten years, find how much he would receive on retiring after 21 years' service. [L.U.]

33. Two men were walking, each at 3 miles per hour, in opposite directions at the side of a railway, and a train passed one of the men in 5 secs., and the other in 6 secs. Find the length of the train. [C.U.S.]

34. A fathom is 6·08 feet; a nautical mile is 1000 fathoms; a metre is 39·37 inches. Find a convenient factor for reducing nautical miles approximately to kilometres. [L.C.C.]

35. From April 1st, 1908, to March 31st, 1909, the numbers of letters, etc., delivered by the Post Office were estimated as follows :—

England and Wales	.	.	.	2482 $\frac{6}{10}$ millions
Scotland	.	.	.	256 $\frac{9}{10}$ „
Ireland	.	.	.	167 $\frac{9}{10}$ „

and the average number of letters, etc., delivered per head of the population as—

England and Wales	.	.	.	70
Scotland	.	.	.	53
Ireland	.	.	.	38

From these data estimate the number of letters delivered per head of the whole population of Great Britain and Ireland.

[L.U.]

36. A nut is in the shape of an octagon, side 11 mm., with a circular hole, radius 5 mm. If it is 2 mm. thick, what is its volume?

37. Two men A and B can walk at 3 and 4 miles per hour and cycle at 12 and 15 miles per hour respectively. Having only one bicycle between them, they have to travel 120 miles, starting and arriving simultaneously. They agree that A shall ride first then dismount, leave the bicycle, and finish the journey on foot; when B reaches the bicycle, he will rest for three hours, and then complete the journey, on the machine. Find the time taken on the journey, and also how soon A must dismount, giving each result to the nearest minute.

[L.U.]

38. Given that the volumes of spheres of different diameters are proportional to the cubes of their diameters, find how many leaden pellets, each 2 mm. in diameter, can be made from a solid ball of lead 13 cm. in diameter.

[C.W.B.]

39. A man sold £2356. 14s. 8d. Consols at 90 $\frac{7}{8}$. How much money did he get from the sale? If £2000 were invested in the purchase of 3% Railway debentures at 106 $\frac{1}{2}$, what would be the amount of half-yearly income from the investment?

[C.U.S.]

40. The horse-power H , the displacement D in tons and the speed v in knots of a steamer are connected by the formula $H = \frac{D^{\frac{2}{3}} v^3}{219}$. This can be written as $v^3 = 219HD^{-\frac{2}{3}}$. Find the speed for a vessel of 1314 horse-power giving a displacement of 1500 tons. (Use logarithms.)

41. In 1912 the total quantity of tea produced on a certain plantation was 1663401 lb., being 118351 lb. in excess of the estimate and 444920 lb. more than the 1911 crop. Find the increase per cent. ; and, if the selling price averaged 6·42*d.* per lb., find how much was realised over and above the estimate.

42. A man standing on a long straight road saw a motor car turn a corner into the road and come towards him. He heard the horn of the car $5\frac{1}{2}$ sec. after he first saw the car, and the car passed him 3 min. $39\frac{1}{2}$ sec. after he heard the horn. Assuming that the horn was sounded exactly as the car turned the corner, and that sound travels at the rate of 1140 ft. per sec., find the number of miles per hour the car was travelling.

[L.C.C.]

43. A regiment contains 1024 men. Show, with the aid of diagrams, that the regiment can be drawn up into any one of the following formations : (1) a solid square, (2) a hollow square 1 deep, (3) a hollow square 2 deep, (4) a hollow square 4 deep, (5) a hollow square 8 deep.

What is the number of men in a side of the square in each formation?

[C.W.B.]

44. A dealer bought some sugar, half of it at 25*s.* 3*d.* per cwt., one-third of it at 26*s.* $4\frac{1}{2}$ *d.* per cwt., and the remainder at 27*s.* per cwt. How many cwt. did he buy and sell if he made a profit of £14. 4*s.* by selling all the sugar at 27*s.* 3*d.* per cwt.?

[L.U.]

45. At what o'clock will a train, which leaves London for Crewe at 11.0 a.m., and goes at the rate of 45 miles an hour, meet a train which leaves Crewe for London at 11.40 a.m., and goes at the rate of 35 miles an hour, the distance between London and Crewe being 158 miles?

[C.U.I.]

46. Find the value correct to four places of decimals of

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} + \frac{1}{9^3}$$

47. When the engines of a certain battleship develop 18000 h.p. she steams at the rate of 18 nautical miles per hour and her rate of coal consumption is 1·98 lb. per h.p. per hour. When they develop 15160 h.p. her rate of steaming is 17 nautical miles per hour and her rate of coal consumption is 1·85 lb. per h.p. per hour. She is ordered to steam to a place 1530 nautical

miles away. She has 1150 tons of coal on board. At which speed must she steam in order to be able to reach her destination without taking any more coal on board? How much coal will be left? [L.C.C.]

48. A manufacturer makes an article at a cost of £ A and sells it to a wholesale dealer; the dealer sells it to a shopkeeper at a profit of p per cent., and the shopkeeper sells it to a customer for £ b , making a profit of q per cent. Find the manufacturer's profit per cent., and also find in what ratio the price paid by the customer is increased if the manufacturer doubles his profit, the dealer and shopkeeper taking the same percentage of profit as before. [L.U.]

49. One vessel contains 12 gallons of wine and another 6 gallons of water. One gallon is taken from each and transferred to the other. Find the proportion of water to wine in the first vessel, after the operation of transferring liquid from one to the other has been performed three times in all. [C.U.S.]

50. If p is the pressure in pounds per sq. in. at a temperature t° C., then

$$\log p = 6.1 - \frac{1518}{t + 273} - \frac{122500}{(t + 273)^2}.$$

Find the pressure at a temperature of 120° C.

51. A manufacturer increases his output by 50 %. He finds that the cost per article is decreased by 10 %. He sells an amount equal to his original output at the same price as before. Show that his profits will be unaltered if he can sell the remaining articles at 70 % of what they would have cost to manufacture before he increased his output.

52. A square patchwork quilt is made up of a number of pieces, measuring 9 in. each way, arranged parallel to the diagonal of the quilt. In order to make the edges straight, the necessary number of triangular pieces equal to half the square pieces are fitted in round the sides, and a triangular piece equal to a quarter of the square piece is placed at each corner. The number of whole square pieces along a diagonal of the quilt is 9. Find the area of the quilt in any way you please.

53. A railway on leaving a station S rises uniformly for half a mile at a slope of 1 in 160; it then runs on the level for three-quarters of a mile and afterwards falls for the next mile

at a slope of 1 in 110. Find at how many yards distance from S it is at the same level with S, the distance being measured along the railway.

54. The receipts of a mountain railway in a certain year were £35. 5s. on the average for every day the railway was open. In the next year the railway was open for 18 days more than in the preceding year, and the accounts showed an increase over the results for the preceding year of £1. 5s. in the daily average receipts, and an increase of £857 in the total receipts for the year. What were the total receipts for the first year?

[C.W.B.]

55. In tests with an engine using oil, the brake horse-power (B) and the weight of oil used per hour (W) were found to be connected according to the following table:—

B	275	250	187·5	125	62·5
W	126	112	84·4	62·5	40

Plot a graph connecting B and W, and find the approximate brake horse-power corresponding to a consumption of 55 units of oil per hour.

56. A recipe directs the mixture of a cu. cm. of a substance A with b cu. cm. of a substance B, but by mistake a grams and b grams respectively are taken. Find the proportions of A and B in the mixture estimated by volume, it being given that p cu. cm. of A weigh x grams, and q cu. cm. of B weigh y grams.

[L.V.]

57. The owner of 50 American coupons for 30 dollars each wishes to convert them into English money. He can either sell them in London at the rate of 4s. 1d. for a dollar; or he can send them to be cashed in America and have the money remitted to London at the rate of 4·86 dollars to the pound. What is the difference between the amount received in the two cases?

[C.U.H.]

58. A cigar-shaped balloon consists of a cylinder 70 feet long and 14 feet in diameter capped by a hemisphere at each end. Find the cost of varnishing it at 1s. 6d. per sq. foot.

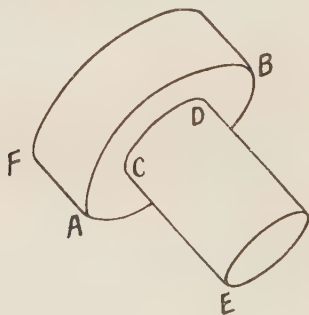
[L.C.C.]

59. Find the value of the following expression giving the difference in temperature of the melting-point of paraffin wax due to a diminution of pressure of 1 atmosphere :—

$$d = \frac{325.7 \times .125 \times 1.01 \times 10^6}{35.35 \times 4.27 \times 10^7} \text{ degrees centigrade.}$$

60. A owes B £1500, but it is not due for payment till the end of three years from now. How much ought B to be willing to accept now in order to clear off the debt, (a) taking money to be worth 4 per cent. per annum simple interest, (b) taking it to be worth 4 per cent. per annum compound interest? [L.U.]

61. Find the weight of the brass bolt-head in the diagram. $AB = 1.2''$, $CD = .6''$, $CE = .8''$, $FA = .4''$. Specific gravity of brass, 8.5. Weight of one cubic foot of water = 62.5 lb.



62. A contractor undertook to carry out a certain work in 100 days, reckoned from the date of the beginning of the work. He employed 160 workmen, and, after 30 days, one-third of the work was done. Owing to unforeseen difficulties, work was suspended for 30 days, and it was found that 2 men would be required to do as much work in a day as was at first done in a day by 1 man. How many additional workmen must be employed in order to finish the work within the stipulated time? [C.W.B.]

63. A man's gross income in 1902 was 20 % greater than in 1901, and in both years £150 of his income was exempt from income-tax ; the tax on the rest of his income being 14*d*. in the £ for 1901 and 15*d*. in the £ for 1902. Also, the amount of

income-tax paid for 1902 exceeded the amount for 1901 by £6. 4s. 2d. What was the man's gross income in 1901?

[C.U.S.]

64. If a pears cost 1d. more than b apples, and c pears cost twice as much as d apples, find the cost in shillings of a dozen apples. Give a numerical answer when

$$a = 6, b = 8, c = 7, d = 5.$$

[L.C.C.]

65. A clock gains 1 minute per day and a watch loses 1 min. 40 sec. in 18 hours. They are both set right at noon on Monday. Find the true time when the clock is 5 minutes ahead of the watch.

[L.U.]

66. A tent in the form of a pyramid is to be constructed on a hexagonal base, side 6 ft. If the volume of air inside is 300 cu. ft., find the amount of canvas required for the tent.

67. The time spent in a particular passage by ships of the same class travelling at high speeds varies inversely as the sixth root of the displacement of the ship in tons. If a steamer of 10000 tons displacement crosses the Atlantic in 6 days, what must be the displacement of a vessel which makes the passage in 5 days?

68. A horizontal pipe for main water supply is 6 feet in diameter. If it is filled with water to a depth of 3 feet, find the number of gallons of water in a mile of the pipe. A gallon of water weighs 10 lb., and a cubic foot of water 1000 oz.

69. A man invests his money equally in two stocks, a 3 per cent. stock at 88 and a 4 per cent. stock at 96. Had he with the same money purchased equal amounts of the two stocks, his annual income would have been 25s. more. How much did he invest?

[C.W.B.]

70. A stream flows from A to B, four miles distant, at the rate of $2\frac{1}{4}$ miles per hour. Two men, who can each row at the rate of 6 miles per hour in still water, start simultaneously from A and B to row towards each other. When they meet, each at once turns, rows back to his starting-point, and then again turns and rows to meet the other. Find the distance between the two meeting-points and the interval which elapses between the two meetings.

[L.U.]

71. Twenty men can make a number of articles in 1 hour which a machine, tended by 1 man, can make in 10 minutes.

Reckoning interest and depreciation at 12 per cent. per annum and a man's wage at 33s. a week, find the capital value of such a machine to an employer. [L.U.]

72. A mixture consists of equal weights of two substances, one of which costs four times as much per pound as the other. The mixture is altered by using, instead of the more expensive substance, another costing only $\frac{4}{5}$ as much per pound, but of which twice as much has to be used. What fraction is the cost of a pound of the new mixture of the cost of a pound of the original mixture? [L.C.C.]

73. Under certain conditions a particle in a circular tube of radius a cm. oscillates, and the time of one oscillation is $2\pi \frac{\sqrt{\pi}}{\sqrt{\pi+4}} \sqrt{\frac{a}{981}}$ secs. Find the time of oscillation in a tube of radius 16 cm.

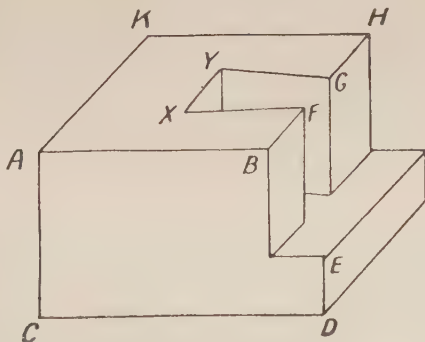
74. Two steamers travel between two ports by the same route, one in 10 days, the other in 5. The cost of coals per voyage varies as the square of the speed, and the general expenses as the duration of the voyage. If on the faster voyage the expenditure for coals exceeds that on the slower by £1000, and the general expenses on the slower voyage exceed those on the faster by £250, find the cost of running the vessels per annum, the faster vessel making 40 voyages and the slower 20. [C.U.S.]

75. The following are taken from the railway statistics for 1911 and 1912:—

Year.	Mileage travelled by Trains.	Gross Receipts.	Working Expendi- ture.
		£	£
1911	428633062	127199570	78617824
1912	412546000	128568000	81174000

Use contracted methods to find (i) the gross receipts per mile, (ii) the working expenditure per mile, during each of the two years. Calling the difference between the gross receipts and working expenditure profit, find the difference of profit per mile in the two years expressed as a percentage of the profit in 1911.

76. In the adjoining figure of a stump dovetail you are given the following measurements:— $AB = 3''$, $CD = 3\frac{3}{4}''$, $DE = \frac{3}{4}''$, $BF = GH = \frac{3}{4}''$, $XY = \frac{3}{4}''$, $AC = AK = 2''$. The mid-



point of XY is the intersection of the diagonals of ABHK. Find the amount of wood (in cu. in.) in the dovetail.

77. The Welsh merchants used to wind their silk on a reel of diameter 1 foot and sell the material reckoning one revolution as equal to a yard. Supposing that a purchaser had bought 15 yd. 2 ft. 9 in., how much silk too much would the merchant have given him?

78. Evaluate the following expression, which gives the difference in level between two stations:—

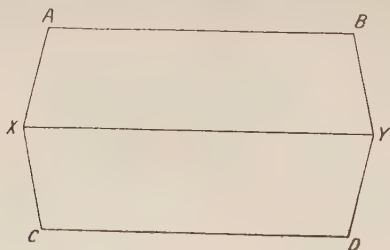
$$\frac{13.59 \times 13.32 \times 760 \times 282.25}{.001293 \times 666.6 \times 273} \text{ cm.}$$

79. A distillation was made in four operations, and the percentages of alcohol in the portions which came over were 72, 69, 62, 52, while the weights of these portions were in the ratios 4 : 3 : 2 : 1 ; what is the percentage of alcohol in the whole liquid that came over? [L.U.]

80. Three bicyclists, A, B, and C, ride round a circular track half a mile in length, B receiving 240 yd. start and C 720 yd. If A travels at the rate of 18 miles per hr. and B at 16 miles per hr., find at what rate C must travel in order that B may overtake him at the instant when A overtakes B. Find also the time that will elapse before they are again all abreast.

[C.U.S.]

81. The diagram represents the elevation of a coal-scuttle; every section parallel to XY is a circle. $AB = CD = 10$ in.;



$XY = 12$ in.; $AX = BY = 3\frac{1}{3}$ in. If coal is 1.25 times as heavy as water, find the weight of coal the scuttle will contain when just full.

82. When the pressure on 1 lb. of dry steam is p lb. per sq. in. the volume of the steam is $329(p^{-.94})$ cu. ft. Find the volume of 1 lb. of steam which is subjected to a pressure of 180 lb. per sq. in.

83. Two models, the one a right cone and the other a regular pyramid on a square base, are apparently solid, but are in fact hollow. Their external measurements are: height, 10 inches; diameter of cone and edge of base of pyramid, 15 inches each. The corresponding internal measurements are 8 inches and 12 inches. The wood of which they are made weighs 12 lb. to the cu. ft. Which is the heavier, and by how much? ($\pi = \frac{22}{7}$.)

[L.C.C.]

84. Two railroads cross each other at right angles at a point P. Two trains A and B are moving on them with velocities of 30 and $22\frac{1}{2}$ miles per hour respectively; at a given instant the front of A is 32 yards short of P, and the front of B is 100 yards past P; find how far the two fronts are apart at the end of $\frac{3}{5}$ min. from the given instant.

85. Find the difference between the compound interest on £1572. 10s. for 3 years at $3\frac{1}{2}\%$ per annum, according as the interest is added on (i) yearly, (ii) half-yearly.

86. The average of a numbers is x ; the average of b of them is $\frac{x}{2}$; and the average of c of the remainder is $\frac{3x}{4}$. Find the average of those still left, if $a = 2(b + c)$ and $b = \frac{c}{2}$. Give your result in terms of x alone.

87. A man realised £2730 by selling a 3% stock at $113\frac{3}{4}$, and invested one-third of this sum in a 4% stock at 130, and the remainder in a $2\frac{3}{4}$ % stock at $110\frac{5}{8}$; find how much of each kind of stock he bought, and the difference in his income.

88. On a certain day the price of English $2\frac{1}{2}$ % stock was $81\frac{3}{4}$ while that of French 3% stock was $97\frac{1}{2}$. Find the amounts of English stock (in pounds) and French stock (in francs) which could be bought for £100, and find which investment is the more profitable. The French stock is bought with French money at the rate of 25 fr. 17·25 c. to the £1, and the interest on it is converted into English money at 25 fr. to the £1. [L.U.]

89. The diameter of a bowl is 20 inches. If the bowl be filled to a depth d inches, then the number of pints of water in the bowl is given by the expression $100p = 90d^2 - 3d^3$. Draw a graph connecting p and d , and hence find the depth of the water when the bowl is holding 5 pints.

90. Without using logarithms, find the surface area of a sphere whose volume is 1000 cu. cm.

91. At what time between 7 and 8 o'clock are the hands of a watch (i) together, (ii) at right angles?

92. In London there were in one week 1577 deaths, equivalent to a death-rate of 19 per 1000 per annum, while in Greater London (which includes an additional suburban ring) there were 2047 deaths, equivalent to a death-rate of 17·8. Find the population of London, and calculate the death-rate for that week of the outer ring alone. Take one year equal to 52 weeks. [C.U.H.]

93. A wrought-iron plate is made in the form of a quadrilateral ABCD; the lengths of the sides in feet are as follows: $AB = 10$, $BC = 4\cdot5$, $CD = 10\cdot75$, $DA = 5\cdot5$, and the diagonal $BD = 11\cdot5$. Find the area of the plate; if the thickness is $\frac{1}{2}$ ", find its weight. (1 cu. ft. of wrought-iron weighs 480 lb.) [L.C.C.]

94. Three men, A, B, C, are required to fill a tank holding 450 gallons by carrying buckets of water. A pours in $3\frac{1}{2}$ gallons every 20 sec., B pours in 4 gallons every 25 sec., and C pours in $4\frac{1}{2}$ gallons every 30 sec. If each pours in his first instalment at noon, find at what time the tank is full, how many journeys each man has made, and which poured in last. [L.U.]

95. Find the value of $\frac{1}{x}(e^{.99a} - 1.23)$, where $x = .55$, $a = 23.43$, and $\log e = .4343$.

96. A rectangular cistern is 10.5 ft. long, 5 ft. 4 in. wide, and 3.5 ft. deep. It contains 5 tons of water. How high does the water rise? Find also how many bricks (9 in. by 4.5 in. by 3 in.) must be dropped into the water so that it shall just overflow. (A brick is assumed to absorb an amount of water equal to a fifth of its volume; 1 cu. ft. of water weighs 62.5 lb.) [L.C.C.]

97. An express train leaves Euston at the same time that a slow train leaves Watford (18 miles from Euston), and a goods train leaves Bletchley (46 miles from Euston), going in the same direction. The express overtakes the slow train in one hour twelve minutes, and the goods train in two hours. Assuming the trains to travel uniformly, find in what time the slow train will overtake the goods train. [C.U.S.]

98. Find by how much the diagonal of a cube of side 1.59 in. exceeds the diagonal of one of the faces.

99. A debt of £200000 is to be paid off as follows:—At the end of every year £13000 is to be paid, part of which is in payment of 5% interest on the part of the debt not paid off, and the remainder in reduction of the debt. Find how soon the debt will be liquidated. [L.U.]

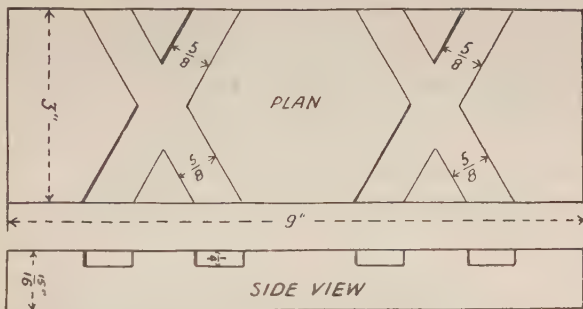
100. A square number which is less than 10000 is divisible by 2, by 9, and by 11. Find the number. Find the length in yards of the side of a square field whose area is $4\frac{4}{9}$ acres. [C.U.H.]

101. At the end of t seconds the current through a circuit is given by $C = \frac{E}{R} \left(1 - \frac{1}{e^{Rt/L}}\right)$. If $R = 2$, $L = 1$, and $t = 4$, find the percentage error in the value of the current if C is taken to be $\frac{E}{R}$. ($\log e = .4343$.)

102. After how many years will the compound interest on £7153 exceed £1500, at $3\frac{1}{4}\%$?

103. The lifting power P of a horse-shoe magnet of weight W was found by Häcker to be proportional to W . If the weight be expressed in kilograms, then $P = 10.3W^{\frac{2}{3}}$. Plot a curve connecting P and W , and hence find the weight of the magnet whose maximum lifting power is 1000 gm.

104. A rectangular piece of wood has grooves cut out as shown in the diagram. Find the volume of the wood after the grooves have been cut out. (The central figure is a hexagon.)



105. 3.24 lb. of petroleum, S.G. .87, are poured into a conical vessel and fill it full. If the depth of the petroleum is 8 in. and a cu. in. of water weigh .036 lb., what is the diameter of the base of the cone to the nearest inch? [L.C.C.]

106. On a circle railway $16\frac{1}{2}$ miles long the stations are $\frac{3}{4}$ miles apart. Each train stops 20 sec. at a station, and runs at an average speed of 20 miles per hr. between the stations. If the distance between the leading carriages of two successive trains is never to be less than $\frac{1}{4}$ mile, find the greatest number of trains that can pass through a station in an hour, and the number of trains required to maintain this continuous maximum service round the circle. [L.U.]

107. The holder of 80 four per cent. debenture shares of £20 each in a company which pays interest on Jan. 1st and July 1st invests the interest regularly in a bank which allows $2\frac{1}{2}$ per cent. interest, and adds the interest to the sum invested at the end of each year. Find the total amount at the bank at the end of six years. [L.U.]

108. How long will a train 400 ft. long travelling at 50 miles per hr. take to pass a train 370 ft. long travelling at 30 miles per hr.?

109. If $\frac{x}{y} = e^{\mu\theta}$, and if $\mu = 0.25$, $\theta = 3$, and $e = 2.718$, find $\frac{x}{y}$. If it is known that the difference between x and y is 1000, find x and y .

110. An approximate relation that exists between A , the number of degrees in the smallest angle of a right-angled triangle, and the sides is $\frac{A}{172} = \frac{a}{b+2c}$, where c is the hypotenuse.

Given that in such a triangle $A = 30$, $a = 2\sqrt{3}$ in., $b = 6$ in., find the value of c from the formula, and the percentage error made in taking this value for c .

111. A cubical box 6 in. side contains a sphere of radius 2 in. Another sphere is put in so that it just fits in when the lid is shut; find the radius of the second sphere.

112. 1016 Kg. = 1 ton; 567 litre = 1 pint. Find a convenient factor for converting ounces per gallon to grams per litre.

113. A man has £5500 invested in 3 per cent. stock at 110, and £3800 in 5 per cent. stock at 95. He sells out and invests part of the proceeds in 3 per cent. stock at 105 and part in $5\frac{1}{2}$ per cent. stock at 99. How must the investments be apportioned if there is no change in his income?

[L.V.]

114. A vessel is formed by cutting a conical hole out of a cylinder, radius 6 in., the vertex of the cone being at the centre of the base of the cylinder. If the height of the vessel be 12 in., and if it will hold 314 cu. in. when just full, find the thickness of the top of the vessel.

115. Find the value of—

(i) 39.056×11.915 , correct to two places of decimals,

(ii) $39.056 \div 11.915$, correct to two places of decimals,

using contracted methods.

116. $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$, where x is less than 1. If $x = .24$, find how many terms must be taken in the series on the right-hand side of the above identity in order to obtain a value for $\frac{1}{1-x}$ correct to the second decimal place.

117. Draw a circle of radius 5.1 cm. and take any diameter AB. From O, the centre of the circle, draw OX perpendicular to AB, 2.1 cm. long. Join AX, and produce it to meet the circumference in Z. Find the area of the triangle ABZ. Calling AO r and OX x , find a general expression for the area of this triangle.

118. A grocer has three different kinds of tea which cost him 1s. 3*d.*, 1s. 6*d.*, and 1s. 8*d.* a pound. With these he makes two mixtures of equal value; each mixture is made of two sorts of tea combined in the same proportion by weight as in the other. How is this done? [L.U.]

119. The radii of the two plane faces of a frustum of a cone are 7 in. and 8·3 in. respectively; the distance between these two faces is 3·5 in. Find the volume of the frustum, given that

$$V = \frac{1}{3} \{A_1 + A_2 + \sqrt{A_1 A_2}\} H,$$

where A_1 and A_2 are the areas of the faces, and H is the height of the frustum.

TABLES

I. THE ENGLISH SYSTEM

Money.

4 farthings	= 1 penny (<i>d.</i>).
12 pence	= 1 shilling (<i>s.</i>).
20 shillings	= 1 pound (£).

A florin	is worth 2 <i>s.</i>
A half-crown	 2 <i>s.</i> 6 <i>d.</i>
A crown	 5 <i>s.</i>
A half-sovereign	 10 <i>s.</i>
A guinea	 21 <i>s.</i>

Weight.

(a) *Avoirdupois.*

This is used for most purposes except for the weighing of precious stones and the precious metals (gold, silver, and platinum).

16 drams (dr.)	= 1 ounce (oz.).
16 ounces	= 1 pound (lb.).
{ 14 pounds	= 1 stone (st.).
{ 28 pounds	= 1 quarter (qr.).
4 quarters	= 1 hundredweight (cwt.).
20 hundredweight	= 1 ton.

Apothecaries use the following:—

20 grains	= 1 scruple.
3 scruples	= 1 drachm.

(b) Troy.

This is used for the weighing of precious stones and the precious metals.

20 grains (gr.)	= 1 pennyweight (dwt.).
20 pennyweights	= 1 ounce (oz.).
12 ounces	= 1 pound (lb.).

1 lb. (avoir.)	= 7000 gr.
1 lb. (troy)	= 5760 gr.

Length.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
22 yards	= 1 chain (ch.).
220 yards } 10 chains } 40 poles }	= 1 furlong (fur.).
8 furlongs	= 1 mile (mi.).

100 links	= 1 chain.
6 feet	= 1 fathom.
100 fathoms	= 1 cable.

6080 feet	= 1 nautical mile.
-----------	--------------------

A knot is a speed of 1 nautical mile per hour.

Area.

144 (12^2) sq. inches	= 1 sq. foot.
9 (3^2) sq. feet	= 1 sq. yard.
484 (22^2) sq. yards	= 1 sq. chain.
4840 sq. yards } 10 sq. chains }	= 1 acre (ac.).
640 acres	= 1 sq. mile.

1210 sq. yards } 40 sq. poles }	= 1 rood.
4 roods	= 1 acre.

Volume.

1728 (12^3) cu. inches = 1 cu. foot.

27 (3^3) cu. feet = 1 cu. yard.

Capacity.*Liquids.*

4 gills = 1 pint (pt.).

2 pints = 1 quart (qt.).

4 quarts = 1 gallon (gall.).

Dry Measure.

2 pints = 1 quart.

4 quarts = 1 gallon.

2 gallons = 1 peck (pk.).

4 pecks = 1 bushel (bush.).

8 bushels = 1 quarter (qr.).

Time.

60 seconds (sec.) = 1 minute (min.).

60 minutes = 1 hour (hr.).

24 hours = 1 day (d.).

7 days = 1 week (wk.).

365 days = 1 ordinary year (yr.).

366 days = 1 leap year.

Angle.

60 seconds (") = 1 minute (').

60 minutes = 1 degree (°).

90 degrees = 1 right angle.

II. THE METRIC SYSTEM**Money.**

100 centimes (c.) = 1 franc (f.).

100 pfennigs (pf.) = 1 mark (M.).

100 cents (c.) = 1 dollar (\$).

Length.

10 millimetres (mm.)	= 1 centimetre (cm.).
10 centimetres	= 1 decimetre (dm.).
10 decimetres	= 1 metre (m.).
10 metres	= 1 dekametre (Dm.).
10 dekametres	= 1 hectometre (Hm.).
10 hectometres	= 1 kilometre (Km.).

The fundamental unit of **area** is the are (1 are = 1 sq. Dm.).
..... **weight** gram (gm.).
..... **volume** litre (l.).
..... (1 litre = 1 cu. dm.)

1 metric ton (or tonne) = 1000 Kg.
1 quintal = 100 Kg.

These Tables of Logarithms and Antilogarithms are taken, with permission, from Mr. F. Castle's Tables, published by Messrs. Macmillan & Co.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0013	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1611	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	0	1	1	2	2	3	3	4	4
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
-00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
-01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
-02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
-03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
-04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	1	2	2	2
-05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	1	2	2	2
-06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	1	2	2	2
-07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	1	2	2	2
-08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	1	2	2	2
-09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	1	2	2	3
-10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	1	2	2	3
-11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	1	1	2	2	3
-12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	1	1	2	2	3
-13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	1	1	2	2	3
-14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	1	1	2	2	3
-15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	1	1	2	2	3
-16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	1	1	2	2	3
-17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	1	1	2	2	3
-18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	1	1	2	2	3
-19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	1	1	2	2	3
-20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	1	1	2	2	3
-21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
-22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
-23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
-24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
-25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
-26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
-27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
-28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
-29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
-30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
-31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
-32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
-33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
-34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
-35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
-36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
-37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
-38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
-39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
-40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
-41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
-42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
-43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
-44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
-45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
-46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
-47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
-48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
-49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
-50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
-51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
-52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
-53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
-54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
-55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
-56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
-57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
-58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
-59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
-60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
-61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
-62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
-63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
-64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
-65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
-66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
-67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
-68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
-69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
-70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
-71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
-72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
-73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
-74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
-75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
-76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
-77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
-78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
-79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
-80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
-81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
-82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
-83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
-84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
-85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
-86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
-87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
-88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
-89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
-90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
-91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
-92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
-93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
-94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
-95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
-96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
-97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
-98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
-99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

ANSWERS TO EXAMPLES

EXAMPLES. I 1

- | | | |
|---|-----------------|---------------|
| 1. 214. | 2. 1452. | 3. 1827. |
| 4. 11120. | 5. 888812. | 6. 24094. |
| 7. 11500. | 8. 81109. | 9. 106397. |
| 10. 1030910. | 11. 10226. | 12. 106322. |
| 13. 50386. | 14. 9182736. | 15. 104684. |
| 16. 132153. | 17. 298316. | 18. 1704125. |
| 19. 181514. | 20. 1644412. | 21. 228. |
| 22. 114. | 23. 103 m. | 24. 127. |
| 25. 510. | 26. 458. | 27. 387. |
| 28. 1016. | 29. 247. | 30. 39 pence. |
| 31. (a) 55500; (b) 1623. | | |
| 32. (a) 90343856; (b) 153867214. | 33. £217180184. | |
| 34. (a) 4836708; (b) 1160328; (c) 4258. | | |
| 35. (a) 718562; (b) 749491; (c) 736636; (d) 751600; | | |
| (e) 752356; (f) 768358; (g) 765275; (h) 3134858; (i) | | |
| 2107420; (l) 5242278. | | |
| 36. (a) 388539; 374777; 367132; 362313; 368267; | | |
| 392253. (b) 635892; 627750; 624191; 630529; 626367; | | |
| 662323. (c) 1024431; 1002527; 991323; 992842; 994634; | | |
| 1054576. | | |

EXAMPLES. II 2

- | | | | |
|---|------------|-------------|-------------|
| 1. 54. | 2. 65. | 3. 465. | 4. 727. |
| 5. 589. | 6. 127. | 7. 5013. | 8. 64569. |
| 9. 8915. | 10. 90909. | 11. 613. | 12. 9088. |
| 13. 10202. | 14. 54. | 15. 22. | 16. 9453. |
| 17. 1132. | 18. 15938. | 19. 323211. | 21. 467814. |
| 22. 16. | 23. 43. | 24. 1899. | 25. 89. |
| 26. 6281. | 27. 13. | 28. 9326. | |
| 29. (a) 2523; (b) 439; (c) 18810; (d) 45. | | | |

36. 79200. 37. 42000. 38. 61425. 39. 86400.
 40. 421200. 41. 918; 91800; 91800.
 42. 6150690; 60080; 6759000. 43. £2812.
 44. 39998848 cu. ft. 45. 54405 millions.
 46. 4095. 47. 88023. 48. 43250.
 49. 368 miles. 50. 450.
 51. 96100; 96721; 97344; 97969; 98596; 99225.
 52. (i) 4925; (ii) 10375; (iii) 94644; (iv) 998001.

EXAMPLES. V S

1. 27, 9, 36. 2. 18, 14, 12. 3. 14, 11, 144.
 4. 13, 22, 115. 5. 624, 7496, 953.
 6. $10269\frac{1}{8}$, $14393\frac{1}{5}$, $2837\frac{2}{11}$. 7. $10212\frac{7}{9}$, $4566\frac{5}{7}$, $14376\frac{5}{8}$.
 8. $2079\frac{3}{11}$, $40618\frac{6}{20}$, $1021\frac{62}{70}$. 9. $234\frac{331}{500}$, $745\frac{41}{110}$, $32\frac{752}{1200}$.
 10. $11699\frac{49}{50}$, $263\frac{2}{120}$, $206\frac{487}{600}$. 11. 9. 12. 15.
 13. 2; 100 lb. 14. 82. 15. 140.
 16. $201\frac{15}{36}$, $43\frac{4}{19}$, $58\frac{14}{17}$. 17. $440\frac{36}{23}$, $3676\frac{19}{25}$, $908\frac{39}{91}$.
 18. $80\frac{20}{131}$, $701\frac{1}{121}$, $126\frac{67}{121}$. 19. $39\frac{373}{816}$, $4792\frac{56}{191}$, $82\frac{113}{811}$.
 20. 1600, $900\frac{333}{370}$, $1731\frac{586}{1131}$. 21. 33. 22. 99.
 23. 298. 24. 40 pence. 25. 33 runs.
 26. 504. 27. 1121. 28. 11.
 29. 16. 30. 121. 31. 500.
 32. 240 min. 33. 325. 34. 46.
 35. 6000 min. 36. 22 yd. 37. 3 p.m.
 38. 16416. 40. 1400 and 140.
 41. (i) 5199; (ii) 1280; (iii) 2857.

EXAMPLES. VI 6

1. 2^3 , 3^2 , 5; 2, 3^2 , 5, 11. 2. 2, 43; 2, 3, 5, 13.
 3. 2, 3, 7, 11; 2^6 , 3^3 . 4. 2^5 , 3, 5, 7; 2^2 , 3^2 , 11.
 5. 2, 5, 13^2 ; 2^2 , 5^3 , 17. 6. 3^2 , 5^3 , 7; 5^2 , 11, 13.
 7. 2^3 , 5, 11; 11, 17. 8. 2, 7, 13, 37; 3, 19, 367.
 9. 7^2 , 11, 37; 3^2 , 13, 19^2 .
 10. 2, 3^2 , 13, 17, 19; 5^3 , 7^2 , 11.
 11. 2^2 , 3^3 ; 3^4 , 5^2 , 7^2 .
 12. 2, 5^2 , 11, 293; 2^2 , 5, 29, 61.
 13. 2, 7, 1523; 3^5 , 11^2 , 13.
 14. 3^3 , 7, 11, 13, 37; 2^2 , 5, 13, 31^2 .
 15. 2. 16. 13. 17. 16. 18. 71.

EXAMPLES. VII 7

1. 28; 42; 62; 100; 386; 539.
2. 3s. 5d.; 14s. 7d.; 2s. 5d.; 3s. 2d.; 5s. 2d.; 7s. 11d.; 9s. 9d.; 32s.
3. 15 ft. 2 in.; 3 ft. 1 in.; 7 ft. 7 in.; 3 ft. 8 in.; 10 ft. 11 in.; 5 ft. 6 in.; 6 ft. 4 in.; 7 ft. 5 in.
4. 66 in.; 27 in.; 115 in.; 122 in.; 48 in.; 141 in.
5. (i) 215; 296; 409. (ii) 1290; 1776; 2454.
6. (i) 95; 98; 136; 204; 239. (ii) 380; 392; 544; 816; 956.
7. (i) 198; 75; 32; 331; 660. (ii) 2376; 900; 384; 3972; 7920.
8. 16; 5376; 35840; 5648; 48395.
9. 1770½d.; £31. 14s. 0½d. 10. £21. 12s. 6d.; 808d.
11. 15 cwt. 32 lb.; 85 qr. 8 lb. 7 oz.
12. 31363200 sq. in.; 57596 sq. yd.
13. 1088 pints; 245 bush. 2 pk. 1 gall. 1 pt.
14. 21 cu. yd. 11 cu. ft. 1216 cu. in.; 27878400 sq. ft.
15. 31536000 sec.; 277 hr. 46 min. 40 sec.
16. 182000 grains; 4224 grains.
17. 25200d.; £1. 0s. 10d. 18. 5662 far.; £1027. 7s. 6d.
19. 94556 lb.; 4 st. 6 lb. 8 oz.
20. 5517 gr.; 253471 lb. 21. 857 lb. 8 oz.
22. 26 tons 18 cwt. 4 lb. 23. 1240 grains.
24. 113 tons 5 lb. 25. 166608576 cu. in.
26. 1 cwt. 22 lb. 9 oz. 27. 352d.; £1. 9s. 4d.
28. 56 times. 29. 3 hr. 12 min.
30. 316800 ft. per hr.; 88 ft. per sec.

EXAMPLES. VIII 8

1. £22. 14s. 9½d. 2. £26. 2s. 1¾d.
3. £41. 16s. 6¾d. 4. 13 yd. 1 ft. 1 in.
5. 12 miles 4 fur. 6. 15 tons 9 cwt. 1 qr.
7. 8 bush. 3 pk. 1 gall. 2 qt.
8. 20 a. 2 sq. ch. 216 sq. yd.
9. £386. 11s. 11¼d. 10. 46 lb. 11 oz. 4 dwt. 5 gr.
11. £13. 10s. 11½d. 12. £5. 2s. 2¾d.
13. £55. 18s. 10¾d. 14. 3 yd. 0 ft. 4 in.
15. 4 tons 9 cwt. 1 qr. 13 lb. 2 oz.
16. 1 fur. 1 ch. 12 yd. 1 ft. 5 in.
17. 2 rd. 1190 sq. yd. 18. 3 bush. 3 pk. 1 gall. 3 qt.

19. 11 hr. 43 min. 44 sec.
20. 4 lb. 3 oz. 16 dwt. 18 gr.
21. £157. 4s. $1\frac{1}{2}d.$
22. £35. 16s. $3\frac{1}{2}d.$
23. $7\frac{1}{2}$ hr.
24. 2 yd. 1 in.
25. 1 ton 11 cwt. 3 qr.
26. 4 miles 47 yd.
27. £56. 15s. 6d.
28. 3.24 p.m.
29. £32. 0s. $0\frac{1}{2}d.$
30. 99 yd. 1 ft. 3 in.
31. 1 ft. 2 in.
32. 7283 sq. yd.
33. 438.
34. 110 tons 11 cwt. 17 lb.
35. £2. 14s. 3d. too small.
36. 8 st. 5 lb.; 8 st. 12 lb.
37. (a) -£40. 14s. 9d.; (b) -£1721. 17s. 5d.; (c) +£1795. 14s. 2d.; (d) +£7709. 17s. 1d.; (e) +£6726. 7s. 6d.; (f) +£1159. 4s. 6d.; (g) +£1799. 8s. 4d.
38. 1 lb. 7 oz.
39. (a) 4 hr. 2 min.; (b) 4 hr.; (c) 3 hr. 35 min.; (d) 3 hr. 42 min.; (e) 4 hr. 23 min.; (f) 4 hr. 51 min.; (g) 4 hr.; (h) 4 hr. 19 min.; (k) 4 hr. 48 min.; (l) 5 hr. 5 min.
40. 1 ch. 34 links too large.
41. (a) £1247868. 15s. 9d.; (b) £9586. 1s. 10d.; (c) £99072. 8s. $11\frac{1}{2}d.$; (d) £24. 2s. 0d.; (e) £2091. 3s. 11d.; (f) £205. 14s. $3\frac{1}{2}d.$; (g) £16829. 6s. $2\frac{3}{4}d.$; (h) £2206. 2s. $11\frac{3}{4}d.$; (k) £94576. 13s. 3d.; (l) £1664. 8s. $4\frac{1}{2}d.$; (m) £13880. 3s. 3d.; (n) £379. 13s. 8d.; (p) £1224669. 18s. 7d.
42. (a) £11907312. 2s. 5d.; (b) £1124593. 8s. 0d.; (c) £82. 11s. 1d.; (d) £910682. 18s. 6d.; (e) £16. 17s. 9d.; (f) £10. 18s. 2d.; (g) 1s. 8d.; (h) £20156. 1s. 2d.; (k) £55. 6s. 9d.; (l) £7140399. 17s. 9d.; (m) 11s. 11d.; (n) £1543. 4s. 9d.; (p) £2. 1s. 4d.; (q) 18s. 11d.; (r) £825048. 12s. 1d.; (s) £6298. 13s. 10d.; (t) £7. 18s. 3d.; (v) £1878411. 19s. 8d.

EXAMPLES. IX 9

1. £58. 2s. 6d.; £79. 18s. 8d.; £57. 0s. 3d.
2. £703. 12s. $7\frac{1}{2}d.$; £1001. 9s. 2d.; £364. 16s. 6d.
3. 1 ch. 18 yd.; 2 ch. 19 yd. 2 ft. 3 in.; 2 ch. 19 yd. 4 in.
4. 4 tons 6 cwt. 1 qr.; 213 tons 10 cwt.; 125 tons 10 cwt. 3 qrs.
5. £10914. 10s. 6d.; £9713. 10s. $7\frac{1}{2}d.$; £279. 14s. 2d.
6. £665. 6s. 9d.; £1264. 9s. $3\frac{3}{4}d.$; £1326. 13s. $9\frac{1}{4}d.$
7. 223 a. 1 r. 1170 sq. yd.; 932 miles 3 ch.
8. £15926. 11s. $6\frac{1}{2}d.$; 1 ton 11 cwt. 4 st. 6 lb. 4 oz.

9. 449 hr. 50 min. 11 sec. ; £17891. 8s. $6\frac{3}{4}d.$
 10. 2173 bush. 2 pk. ; 78 lb. 2 oz. 4 dwt. 1 gr.
 11. £29754. 11s. $1d.$; 15 tons 14 cwt. 20 lb. 2 oz.
 12. £1534. 11s. $0\frac{3}{4}d.$; 8 miles 1 fur. 3 ch. 13 yd. 1 ft. 4 in. ;
 7 tons 1 cwt. 2 qr. 26 lb. 13 oz.
 13. 1s. $10\frac{1}{4}d.$; 2s. $6d.$; 3s.
 14. £14. 9s. $2d.$; 1s. $8d.$; £25. 11s. $6\frac{1}{2}d.$
 15. 3 yd. 1 ft. 2 in. ; 4 yd. 7 in. ; 1 yd. 2 ft. 10 in.
 16. £21. 1s. $4d.$; £1. 1s. $1\frac{1}{4}d.$; £91. 2s. $1\frac{1}{2}d.$
 17. 2 lb. 4 oz. ; 1 ton 2 cwt. 1 qr. ; 1 ton 2 cwt. 1 qr. 5 lb.
 5 oz.
 18. £3. 11s. $2\frac{1}{4}d.$; £12. 4s. $0\frac{1}{2}d.$; $2\frac{1}{4}d.$
 19. 3 yd. 2 ft. 4 in. ; 6 cwt. 86 lb.
 20. 4 lb. ; 7 bush. 2 pk. 1 gall.
 21. £9. 18s. $4d.$; £3. 3s. $4d.$
 22. 3 hr. 57 min. 21 sec. 23. £18137. 11s. $2d.$
 24. 3 cwt. 57 lb. 25. 52. 26. 24.
 27. 9. 28. 339. 29. 25. 30. 112.
 31. 24. 32. 732. 33. 38. 34. 625.
 35. £16. 4s. $4\frac{1}{2}d.$ 36. £30. 18s.
 37. 17s. $2d.$ 38. 300 men.
 39. 5 times ; 95 yd. 40. 18 tons 1 cwt. 7 lb.
 41. $6\frac{1}{2}d.$ 42. 4 days 22 hr. 21 min.
 43. £4. 6s. $7\frac{1}{2}d.$ 44. 89.
 45. 1 yd. 9 in. 46. 480 times.
 47. £32. 19s. $10\frac{1}{2}d.$ 48. 2s. $9\frac{3}{4}d.$
 49. 15 times. 50. 9 lines.
 51. £1. 17s. 52. 16335.
 53. 184 coins. 54. 4 tons 15 lb. 8 oz.
 55. 10 st. 5 lb. 12 oz. 56. £14. 6s. $8d.$
 57. 75950 francs. 58. 2 ; 8 in.
 59. 14. 60. 65700 times. 61. 107340 tons.
 62. 16800 rats. 63. 15. 64. 4s. $1\frac{1}{2}d.$
 65. £554367. 14s. $11\frac{1}{2}d.$
 66. 2790453600 sq. yd. ; 2467 sq. yd.
 67. 50 lb. 9 oz. 16 dwt. 3 gr. 68. 16 cwt. 104 lb.
 69. 8299 miles 1720 yd.
 70. 7698801 oz. 18 dwt. 3 gr.

EXAMPLES. X

1. 835 ; 9152 ; 318 ; 82000.
 2. 9·15 ; ·08001 ; ·005 ; ·000005.
 3. 69 ; ·0069 ; ·069 ; ·69.

4. $\cdot 721$; $\cdot 00721$; 72100; $\cdot 0721$.
5. 156 $\cdot$ 41087. 6. 82 $\cdot$ 08. 7. 965 $\cdot$ 533.
8. 1097 $\cdot$ 26. 9. 79 $\cdot$ 889. 10. 110 $\cdot$ 62185.
11. 136 $\cdot$ 056. 12. 78 $\cdot$ 795. 13. 126 $\cdot$ 2.
14. 92 $\cdot$ 6729. 15. $\cdot$ 0198. 16. 1 $\cdot$ 4705.
17. 973 $\cdot$ 21315. 18. 2727 $\cdot$ 273. 19. 57644 $\cdot$ 1389.
20. 77197 $\cdot$ 12247. 21. $\cdot$ 008 m.; $\cdot$ 00008 Dm.
23. £11392. 24. 2 $\cdot$ 24 in. 25. 1 $\cdot$ 72825 l.
26. 1 $\cdot$ 77 in. 27. (i) 18 $\cdot$ 4 ft.; (ii) -14 $\cdot$ 1 ft.
28. 3687 $\cdot$ 131 thousands of pounds. 29. 1 $\cdot$ 11 gm.
30. 45 $\cdot$ 4° F. 31. 581 $\cdot$ 7; 5 $\cdot$ 817; $\cdot$ 05817.
32. 71385 $\cdot$ 6; 7 $\cdot$ 13856; $\cdot$ 00713856.
33. $\cdot$ 0945; 94500; $\cdot$ 0945.
34. 10046 $\cdot$ 4; 10 $\cdot$ 0464; 1 $\cdot$ 00464.
35. 73 $\cdot$ 719; 3 $\cdot$ 85; $\cdot$ 01164. 36. 6 $\cdot$ 721; 11983 $\cdot$ 2; $\cdot$ 696.
37. $\cdot$ 326; 73170; $\cdot$ 00825. 38. 9 $\cdot$ 01; 20045; 3 $\cdot$ 6018.
39. $\cdot$ 0027405; 27 $\cdot$ 001; $\cdot$ 007869.
40. 3849 $\cdot$ 82; 27 $\cdot$ 224; 1 $\cdot$ 39594.
41. $\cdot$ 04; $\cdot$ 0009; $\cdot$ 000001.
42. $\cdot$ 000001; $\cdot$ 000000008; 1 $\cdot$ 4641.
43. 271 $\cdot$ 37; 2713 $\cdot$ 7; $\cdot$ 0027137.
44. $\cdot$ 0297; 29700; $\cdot$ 0297. 45. 5 $\cdot$ 31; $\cdot$ 0531; 53100.
46. 2 $\cdot$ 4; 2 $\cdot$ 4; 2400000. 47. 721; 214 $\cdot$ 1; $\cdot$ 00513.
48. 3 $\cdot$ 875; 1 $\cdot$ 18; 5 $\cdot$ 1875. 49. 9; $\cdot$ 017; $\cdot$ 00021,
50. $\cdot$ 000668; 211; 35000.
51. 57362000; $\cdot$ 61; $\cdot$ 000044.
52. 61250; 313 $\cdot$ 6; 120. 53. 375; 1305; 6 $\cdot$ 144.
54. $\cdot$ 48596; 795 $\cdot$ 06; 623000. 55. 2 $\cdot$ 44.
56. 1607 $\cdot$ 29. 57. 1 $\cdot$ 429. 58. 83 $\cdot$ 9016.
59. 2 $\cdot$ 368. 60. 746 $\cdot$ 68770. 61. 44937 $\cdot$ 5 lb.
62. (i) 39370 in.; (ii) 285432 $\cdot$ 5 in. 63. 549 $\cdot$ 9 in.
64. 415 $\cdot$ 152*d*. 65. £26. 11*s*. 6*d*. 66. 1018 Kg.
67. 564 seats. 68. 98 fr. 65 c.
69. 160934 $\cdot$ 4 cm.; 39 $\cdot$ 37 in. 70. $\cdot$ 024 cm.
71. £137265. 8*s*. 9*d*. 72. £22324. 18*s*. 9*d*.

EXAMPLES. XI))

1. 83150 m.; 3 m. 27 cm. 5 mm.; 1936 m. 12 cm.
2. 1 m. 72 cm. 9 mm.; 812 m. 37 cm.; 5000 m.
3. 8000 gm.; 721 $\cdot$ 9 gm.; 8315 $\cdot$ 72 gm.
4. 402 gm.; 102 $\cdot$ 301 gm.
5. 817 fr. 29 $\cdot$ 5 c.; 31 fr. 29 $\cdot$ 5 c.; 7 fr. 8 c.; 7 c.
6. 81 fr. 72 c.; 312 fr. 95 c.; 70 fr. 8 c.

7. 86410 Dl. ; 3075 Dl. ; 7920·56 Dl.
8. 15·716 Km. 9. 13 Hl. 77 l. 10. 117 fr. 18 c.
11. (i) 2352 m. (ii) 7 fr. 50 c. (iii) 98·275 gm. (iv) 90·272 cm.
12. 57 dollars 20 cents ; 274 m. 50 cm.
13. 592 Hl. 85 l. ; 1388 Kg. 310 gm.
14. 244 fr. 35 c. ; 148 dm. 74 mm.
15. 1222 Km. ; 5787 dollars 95 cents.
16. 52 fr. 35 c. ; 128 m.
17. 2 Hl. 21 l. ; 2 dollars 50 cents.
18. 81 m. 5 cm. ; 17 Km. 8 m.
19. 3 fr. 18 c. ; 11 fr. 63 c.
20. 422 m. 5 cm. ; 1 Km. 34 m. 2 cm.
21. 898 fr. 24 c. 22. 28 fr. 50 c. 23. 44 fr. 47 c.
24. 20 fr. 19 c. 25. 4195800 fr. 26. 999000 mm.
27. 914·4 mm. 28. 50 Kg. 803·2 gm.
29. 18930 fr. 30. 539000 l. 31. 1125 fr.
32. 65 c. 33. 215·73 cm. 34. 28 m. 80 cm.
35. 128·65 gm. 36. 16 cu. cm. 37. 8 cm.
38. 05 m. 39. 2·5 cm. 40. 1·6 cm.
41. 2595 tons 84 lb. 42. 22·97... gall.
43. (i) 2s. 1d. ; (ii) 1·66d.
45. 14·090904 Km.

EXAMPLES. XII 12

1. (i) 75 sq. m. ; 650 cm. (ii) 5·03 sq. m. ; 1206 cm.
(iii) 4816 sq. m. ; 122000 cm. (iv) 280035 sq. m. ; 1607200 cm.
(v) 177767 sq. m. ; 614·4 cm.
2. (i) 14·7 sq. cm. (ii) 75·27 sq. mm. (iii) 3575 sq. m.
(iv) 1732 sq. mm. (v) 15·56 sq. cm.
3. (i) 38·62 sq. cm. (ii) 42·42 sq. cm. (iii) 54·04 sq. cm.
(iv) 6·25 sq. cm. (v) 20·68 sq. cm.
4. (i) 28·26 sq. cm. (ii) 63·585 sq. cm. (iii) 00229 sq. m.
(iv) 00785 sq. m. (v) 3615173 sq. mm.
5. 20 cm. 6. 12 cm. 7. 2 m.
8. (i) 30580 sq. m. (ii) 305·8 ares. (iii) 3·058 Ha.
9. 53 m. 10. 25 mm. 11. 230 sq. m.
12. 227·7968 sq. m. 13. 17136. 14. 400.
15. 300 cm. 16. 6 sq. m. 17. 75.
18. 4·63512 sq. m. 19. 6·4516 sq. cm.
20. 1549·9969 sq. in. 21. 174 sq. m.
22. 8100 sq. m. 23. 106·425 sq. m.
24. 7·065 ares. 25. 23770 sq. mm. (approx.)

26. 10·96 sq. cm. 27. 53·4 sq. m.
 28. 340 sq. cm. 29. 87·135 sq. cm.
 30. £6. 14s. 9d.
 31. (i) 30 cu. cm. (ii) 19·125 cu. cm. (iii) 157035 cu. cm.
 (iv) 72000 cu. cm. (v) 17812·5 cu. cm.
 32. (i) 15·7 cu. cm. (ii) 7·065 cu. cm. (iii) ·038151 cu. dm.
 (iv) 3815·1 cu. cm. (v) 7·23456 cu. mm.
 33. (i) 3·23 cu. cm. (ii) 2·355 cu. cm. (iii) ·012717 cu. dm.
 (iv) 1271·7 cu. cm. (v) 2·41152 cu. mm.
 34. (i) 1000 l. (ii) 10 Hl. 35. 5825000 cu. cm.
 36. 510125 cu. cm. 37. 15 blocks.
 38. 50 cm. 39. ·9 sq. mm. 40. 40 cm. 41. 31·4 l.
 42. 49. 43. ·06594 l. 44. 5850 Kg.
 45. 2090 cu. cm. (approx.). 46. 50 cm. 47. ·7968 l.
 48. 19·782 cm.

EXAMPLES. XIII 13

1. 24, 144. 2. 5, 825. 3. 9, 2430. 4. 7, 1155.
 5. 7, $2^3 \times 5 \times 7^2$. 6. 24, $2^4 \times 3^2 \times 5$.
 7. 3, $2 \times 3 \times 5^2 \times 7$. 8. 9, $2^3 \times 3^6 \times 5$.
 9. 11, $2^2 \times 3 \times 5 \times 11^2$. 10. 13, $2^2 \times 3 \times 5 \times 13^2$.
 11. 16, $2^6 \times 5 \times 11$. 12. 17, $2 \times 5^2 \times 11 \times 17^2$.
 13. 23, 12×23^2 . 14. 29, $29 \times 41 \times 101$.
 15. 117, $117 \times 8 \times 825$. 16. 1, 2006×12051 .
 17. 821, $25 \times 43 \times 821$. 18. 37, $37 \times 3011 \times 9900$.
 19. 198, $198 \times 98 \times 165$. 20. 139, $121 \times 139 \times 938$.
 21. 741, $17 \times 57 \times 110 \times 741$. 22. 1, $128 \times 13121 \times 19845$.
 23. 39, $6 \times 39 \times 78 \times 187$.
 24. 811, $1110 \times 121 \times 733 \times 811$.
 25. $2^4 \times 3^2 \times 5 \times 7 \times 11 \times 13 \times 17$.
 26. 102 feet. 27. 10 shillings. 28. 119 lb.
 29. £1. 8s. 30. 11 in. 31. 2.
 32. 6.1 a.m. 33. 23·1 m. 34. 55440.
 35. 27s. 6d. 36. 15 min.

EXAMPLES. XIV 14

1. 4d.; 4s.; £1. 12s.; 9 in.; 6 sec.
 2. 14s.; 45 sec.; 10d.; 2s. 6d.; 4 in.
 3. 5s.; 13s. 4d.; 16s. 8d.; 7s. 6d.; 1s. 8d.; £8. 12s.;
 £7. 2s.; £5. 3s. 9d.
 4. 20 min.; 33 min. 20 sec.; 45 min.; 7 min. 30 sec.;
 11 min. 15 sec.

5. 10 cm., 3 cm., 180 cm., 360 cm.
 6. £3. 7. £4. 8s. 8. 1 lb. 14 oz.
 9. 3 cwt. 84 lb. 10. £11. 0s. 10d. 11. £13. 10s. 8d.
 12. £9687. 10s. 13. 2 m. 3 fur. 2 ch.
 14. 1 Kg. 8 Hg. 2 Dg. 1 gm. 4 dg.
 15. £7. 3s. 9³/₄d. 16. $\frac{1}{80}, \frac{1}{10}, \frac{1}{6}, \frac{1}{8}, \frac{4}{5}, \frac{5}{6}$
 17. $\frac{1}{14}, \frac{1}{8}, \frac{27}{112}, \frac{1}{2}, \frac{3}{1792}$
 18. (i) 2⁶/₁₁. (ii) 4⁸/₈. (iii) 2²⁵/₃₇. (iv) 37³/₅. (v) 12³/₈. (vi) 14⁶/₇₁.
 (vii) 1³⁰⁷⁵/₅₁₃₉. (viii) 19²²¹/₃₆₅. (ix) 2³⁶³⁹/₈₁₉₁. (x) 2770³⁰/₃₆₁.
 19. (i) $\frac{29}{17}$. (ii) $\frac{66}{19}$. (iii) $\frac{295}{21}$. (iv) $\frac{683}{57}$. (v) $\frac{7301}{365}$. (vi) 2909
 1649. (vii) $\frac{6608}{93}$. (viii) $\frac{1559}{52}$. (ix) $\frac{36310}{711}$. (x) $\frac{9364}{57}$.
 22. $\frac{2}{3}, \frac{8}{12}, \frac{16}{24}, \frac{32}{48}$. 24. $\frac{9}{12}, \frac{3}{4}, \frac{54}{72}$.
 25. $\frac{8}{10}, \frac{12}{15}, \frac{20}{25}, \frac{24}{30}$. 26. $\frac{6}{12}, \frac{8}{12}, \frac{9}{12}$.
 27. $\frac{8}{28}, \frac{10}{35}, \frac{14}{49}$. 28. Greatest, $\frac{7}{8}$; least, $\frac{3}{4}$.
 30. $\frac{3}{7}, \frac{2}{5}, \frac{2}{3}, \frac{9}{16}$. 31. $\frac{14}{15}, \frac{15}{16}, \frac{3}{4}, \frac{9}{10}, \frac{11}{13}$.
 32. $1\frac{1}{2}, 2\frac{1}{2}, 2\frac{1}{10}, \frac{6}{11}, 2\frac{3}{4}$. 33. $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{a}$.
 34. $\frac{2}{3}, 1, \frac{a^2}{b}, \frac{3a}{25}$. 35. $2\frac{1}{2}, \frac{13}{40}$.
 36. $\frac{23}{25}, 1\frac{1}{25}$. 37. $\frac{14}{59}, 2$.
 38. $\frac{195}{217}, \frac{71}{264}$. 39. $\frac{1}{468}$.
 40. $\frac{3}{5}$. 41. $\frac{127}{31}$. 42. $1\frac{584}{945}$.

EXAMPLES. XV 15

1. $\frac{7}{8}$. 2. 1 in. 3. $6\frac{3}{10}$ cm. 5. $\frac{11}{24}$.
 6. $2\frac{9}{10}$. 7. $12\frac{10}{27}$. 8. $7\frac{1}{4}$. 9. $2\frac{1}{4}$.
 10. $39\frac{2}{3}$. 11. $\frac{1111}{10000}$. 12. $20\frac{23}{100}$. 13. $\frac{1}{2}$; 1s.
 14. 7 miles. 15. £1200. 16. $64\frac{1}{2}$ yd. 17. £387. 10s.
 17. £12. 3s. $4\frac{1}{2}$ d. 18. 1s. $6\frac{13}{50}$ d. 19. £387. 10s.
 20. £5. 5s. 4d. 21. $3\frac{1}{6}$ oz. 22. $\frac{2}{5}$ gall.
 23. Britannia silver. 24. $\frac{23}{80}, \frac{57}{80}$. 25. $\frac{7}{8}$.
 26. (i) $\frac{5}{16}$. (ii) $\frac{1}{6}$. 27. (i) $\frac{1}{8}$. (ii) $\frac{9}{40}$. (iii) $1\frac{9}{16}$.
 28. $\frac{2}{5}$ cm. 29. $9\frac{1}{4}$. 30. $1\frac{1}{8}$. 31. $\frac{71}{72}$.
 32. $\frac{71}{72}$. 33. $1\frac{15}{16}$. 34. $1\frac{65}{68}$. 35. $\frac{11}{630}$.
 36. $\frac{21}{32}$. 37. $\frac{80}{91}$. 38. $\frac{17}{20}$. 40. $\frac{1}{100}$.
 41. $1\frac{1}{8}$ in. 43. (i) $\frac{1}{18}$ oz.; (ii) $\frac{1}{90}$ oz. 44. 1d.
 45. £2. 12s. 6d. 46. $\frac{59}{375}$ franc. 47. $11\frac{23}{30}$ gm.
 48. Roman carat by $\frac{31}{90000}$ oz. 49. $24\frac{4}{5}$ lb. 50. 111 cm.
 51. $\frac{1}{35}$. 52. $9\frac{1}{24}$. 53. $2\frac{8}{11}$. 54. $17\frac{3}{16}$.
 55. $10\frac{37}{56}$. 56. $2\frac{23}{24}$. 57. $\frac{209}{720}$. 58. $\frac{23}{60}$.
 59. $4\frac{47}{48}$. 60. $5\frac{1508}{2205}$. 61. 1. 62. 480 gall.
 63. 10s. 64. £40. 65. 36. 66. 3 miles.
 67. 25 boys. 68. $\frac{193}{255}$. 69. 264.
 70. £3901. 10s.

EXAMPLES. XVI 16

6. $1\frac{1}{2}$, $1\frac{4}{5}$, 3; $22\frac{11}{12}$, 25, $31\frac{1}{4}$; $4\frac{1}{3}$, $14\frac{5}{6}$, 32.
 7. $7\frac{2}{3}$. 8. $\frac{2\ 0\ 2\ 3}{2\ 0\ 5\ 2}$. 9. $11\frac{5}{8}$. 10. $23\frac{5}{7}$.
 11. 3. 12. 6.
 17. $\frac{1}{3}$, 5, $1\frac{1}{7}$, 1, $\frac{9}{10}$, $\frac{3}{13}$, $2\frac{1}{7}$, $16\frac{3}{8}$, $\frac{2\ 4}{2\ 1\ 7}$, 723.
 18. $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$; $1\frac{1}{18}$, $\frac{3\ 8}{4\ 5}$, $\frac{2}{9}$; $\frac{2}{3}$, $1\frac{1}{3}$, $1\frac{2\ 3}{4\ 2}$.
 19. $\frac{3}{10}$. 20. $12\frac{1}{2}$. 21. $\frac{1\ 0\ 4}{1\ 0\ 5}$. 22. $\frac{1\ 1}{1\ 1\ 0}$.
 23. $6\frac{1\ 8}{1\ 2\ 5}$. 24. 76. 25. $\frac{4}{5}$. 26. $\frac{3}{5}$.
 27. 1. 28. $1\frac{7}{64}$ in. 29. 31995. 31. $\frac{8}{27}$.
 32. $1\frac{2}{3}$. 34. $\frac{1}{4}$ oz. 35. £4. 11s. 6d.
 37. 110. 38. 12. 39. $5\frac{3}{4}$ cwt.; $14\frac{3}{8}$ cwt.; $\frac{2}{5}$.
 40. £11 $\frac{2\ 11}{3\ 1\ 5}$. 41. 10.
 42. $\frac{5}{2\ 8\ 8}$ yd.; $84\frac{1}{36}$ cu. yd. 43. $141\frac{1}{4}$ lb.
 44. $6\frac{4\ 7}{2\ 1\ 0}$ ft. 45. 30.
 46. $\frac{4\ 5}{6\ 4}$ sq. in.; 46080. 47. £1. 18s. 3d.
 48. £9546. 13s. 4d. 49. £205. 10s. 50. 240.
 51. $2\frac{4\ 6\ 3}{8\ 9\ 6}$ tons. 52. 1224; 1428; 2652.
 53. $1\frac{3}{2\ 2}$ lb. 54. $\frac{9}{2\ 5}$. 55. $\frac{2}{5}$.

EXAMPLES. XVII 17

1. $2\frac{7}{9}$; $1\frac{1}{9}$. 2. 3; 3. 3. $\frac{1\ 3}{1\ 6}$; 18. 4. $13\frac{4}{9}$; $3\frac{4}{9}$.
 5. $\frac{1\ 3}{4}$; $1\frac{1}{14}$. 6. $\frac{7}{8}$. 7. 47. 8. $2\frac{4\ 3}{4\ 5}$.
 9. $1\frac{4\ 3}{1\ 4\ 0}$. 10. $18\frac{9}{10}$. 11. $\frac{3}{4}$. 12. 9.
 13. $\frac{1\ 3}{2\ 9}$. 14. $2\frac{2}{5}$. 15. $3\frac{3}{4}$. 16. 1.
 17. $\frac{7}{20}$, $\frac{1}{4}$. 18. $\frac{9}{1\ 3\ 6}$. 19. $\frac{2\ 5}{8\ 4}$. 20. $\frac{4}{1\ 1}$.
 21. $4\frac{9}{10}$. 22. $\frac{1}{80}$. 25. $\frac{10}{8\ 7}$. 27. $\frac{2\ 7}{6\ 8}$.
 28. $1\frac{1}{2}$. 29. $\frac{4\ 2}{4\ 7}$. 30. $\frac{1}{3\ 3\ 1\ 2\ 4}$. 31. $4\frac{1}{2}$.
 33. £2. 13s. 4d.; £5. 16s. 8d.; £17. 16s. 3d.
 34. £2. 5s.; £2. 9s. 4d.; 10s. 35. 1s. 10d.; 9s. 4d.
 36. 5 cwt.; 5 cwt. 3 qr. 11 lb.
 37. 4 tons 11 cwt. 3 qr. 7 lb. 38. £399. 12s.
 39. 2s. 40. £2. 1s. $1\frac{3}{4}$ d.
 41. $\frac{2}{3}$. 42. $\frac{1}{6}$. 43. $1\frac{4\ 7}{6\ 3}$. 44. $56\frac{1\ 2}{2\ 3}$.
 45. $1\frac{4\ 9}{6\ 1}$. 46. $\frac{7}{8}$. 47. $\frac{3}{5}$. 48. $1\frac{5}{9}$.
 49. $\frac{1\ 2\ 4\ 6\ 1}{3\ 8\ 2\ 7\ 8}$. 50. $\frac{8\ 1}{3\ 2\ 0}$. 51. $\frac{1\ 3}{4\ 0\ 0}$. 53. $\frac{1}{5}$.
 54. 5·6875. 55. £17. 10s. 56. 1·034.
 57. 175:144. 58. £200; £150. 59. 2:1.
 60. £1. 1s. $0\frac{1}{2}$ d. 62. 23:22.
 63. 66 marks. 64. 402680 males; 684556 females.
 65. 1262:1313. 66. 1 ft. 5 in.
 67. 112·9975 grains. 68. 311:320.
 69. (i) ·9075. (ii) 1·102... 70. 2:3.

71. 10.38 p.m. 72. $\frac{16}{7565}$.
 74. (i) 2.15. (ii) .15. 75. 4 tons 20 lb.
 78. 63 lb. 79. $\frac{1}{3}$. 80. .03. 81. .091.
 82. 6 gall. 2 pt. 83. £65. 84. 2d.
 85. £51. 8s. 9d.; £34. 5s. 10d.; £25. 14s. 4½d.
 86. 3.2 oz.

EXAMPLES. XVIII 18

1. $\frac{3}{10}, \frac{3}{100}, \frac{3}{1000}, \frac{3}{10000}$. 2. $\frac{3}{20}, \frac{3}{200}, \frac{3}{2000}, \frac{3}{20000}$.
 3. $5\frac{1}{5}, 6\frac{1}{2}, 6\frac{1}{20}, 6\frac{1}{500}$. 4. $3\frac{1}{80}, 7\frac{3}{4}, 82\frac{1}{40}, 82\frac{1}{50}$.
 5. $\frac{1}{625}, \frac{1}{125}, \frac{1}{2500}, \frac{1}{20000}$. 6. $\frac{27}{100}, \frac{11}{40}, 8\frac{1}{16}, 4\frac{1}{32}$.
 7. .5, .25, .2, .125.
 8. 2.3, 2.03, 2.00003, 2.2, 2.6.
 9. 3.35, 4.255, 5.61. 10. 1.125, 2.75, 3.3125.
 11. 5.12, 4.024, 5.0048. 12. 2.00002, 21.296, 8.1004.
 13. .33333..., .11111..., .66667..., .44444....
 14. .28571..., .57143..., .85714..., 1.14286....
 15. .07692..., .23077..., .38462..., .53846....
 16. .16667..., .83333..., .08333..., .58333....
 17. .06383..., .06897..., .76471....
 18. .83797..., .99505..., .39947....
 19. £.75, £.875, £.0125.
 20. £.2125, £.2875, £.35625. 21. £3.2125, £7.91875.
 22. £2.79375, £17.15625. 23. £4.01875, £73.753125.
 24. £3.222, £2.459. 25. £6.740, £17.146.
 26. £1.999, £.230. 27. £.909, £.965.
 28. £20.741, £2.795. 29. 1s. 6d.; 6s. 3d.; 2d.
 30. 14s. 4d.; 18s. 4d.; £1. 31. 3s.; £39. 17s. 8d.
 32. £2. 13s. 9d.; £38. 19s. 9d.
 33. 10s. 2d.; £34. 4s. 8d. 34. £2250. 35. £122. 10s.
 36. £59. 7s. 6d. 37. £50.
 38. £20. 12s. 6d. 39. £65. 12s. 6d.
 40. 12s. 6d. 41. £460. 16s. 8d.
 42. £1911. 2s. 11d. 43. £803. 2s. 6d.
 44. £10191. 11s. 3d. 45. 9.
 46. 4.590. (i) .0459; (ii) 45.9.
 47. £.066796875, £4007. 16s. 3d.
 48. £425. 4s. 2d. 49. 4s. 1d. 50. £27288.
 51. (i) 7s. 9d. (ii) 7s. 8.549d. 52. 9, 11, 7.

EXAMPLES. XIX

1. (i) 91. (ii) 102.25. (iii) .046.
 2. £1. 18s. 3. 12 yr. 1 mo. 4. 5 ft. 10 in.

- | | | |
|---|-----------------------------------|----------------|
| 5. 113. | 6. 2 hr. 2 min. | 7. 324 miles. |
| 8. 30·03. | 9. 64. | 10. 5. |
| 11. 2s. 5 <i>d</i> . | | |
| 12. 18. | 13. $26\frac{23}{160}$ <i>d</i> . | 14. 1 mm. |
| 15. 195046 dollars 71 cents. | 16. 40034043. | 17. 5·5. |
| 18. £2080. | 19. £42. 12s. 3 <i>d</i> . | |
| 20. 16·05 gm. | 21. 28. | 22. £26168216. |
| 23. 2 hr. 36 min. | 24. £11252000, £240000. | |
| 25. 20. | 26. 5 marks (loss). | |
| 27. £19839948. 15s. | 28. 2·414... in. | |
| 29. 576540 acres, 1130668. | 30. 1s. 5 <i>d</i> . | |
| 31. 51·33... miles per hr. ; 145·44... miles. | 32. 58s. 1 <i>d</i> . | |

EXAMPLES. XX 20

- | | | |
|---------------------------------------|--|-----------------------|
| 1. $3\frac{3}{4}$ hr. | 2. 5s. | 3. 165. |
| 4. 495 grains. | 5. £9. 13s. 0½ <i>d</i> . | 6. 187 fr. 50 c. |
| 7. $2\frac{1}{9}$ sec. | 8. 3280 ft. 10 in. | 9. 8 m. 78 cm. |
| 10. 835 sq. m. | 11. 88 ft. per sec. | 12. 8·37 sq. in. |
| 13. £1. 4s. | 14. 32s. 6 <i>d</i> . | 15. £8. 15s. |
| 16. 6 <i>d</i> . | 17. 345 lb. | 18. 699·79 litres. |
| 19. (i) ·017... ft. per hr. | (ii) 149·04... ft. per year. | |
| 21. 23 cwt. | 22. 11·59. | |
| 23. 1018·18... Kg. | 24. 3s. 1½ <i>d</i> . per qt. | |
| 25. £1333. 6s. 8 <i>d</i> . | 26. 4·0375 oz. | |
| 27. £20. 10s. 11 <i>d</i> . | 28. ·0433... | 29. 4 days. |
| 30. 43 min. 12 sec. ; 14 min. 24 sec. | 31. £1. 2s. 6 <i>d</i> . | |
| 32. 85500 gall. | 33. 56·98... miles per hr. | |
| 34. 1s. per cwt. | 35. 10 years 49 weeks. | |
| 36. 5·4 <i>d</i> . | 37. 99. | 38. 6. |
| 39. 6227 gall. | 40. £1. 14s. | 41. 14s. 3 <i>d</i> . |
| 42. 1275·75 gm. | 43. 1284·46 drachma. | |
| 45. 4·70.... | 46. Greater in March. | |
| 47. £3949. 4s. (nearly). | 48. £3196. 0s. 8 <i>d</i> . | |
| 49. 9077·9... tons. | 50. £37. 6s. 3 <i>d</i> . (nearly). | |
| 51. 5s. 9·3 <i>d</i> . | 52. 10·64... knots. | |
| 53. 42·95... miles per hour. | | |
| 54. ·00000104... pint, 43·96... in. | | |
| 55. £2. 11s. 8 <i>d</i> . | 56. £6984. 10s. 6 <i>d</i> . (nearly). | |

EXAMPLES. XXI 2)

- | | |
|---|------------------------------|
| 1. 6480 sq. ft. | 2. 704 sq. ft. |
| 3. (i) $\frac{5}{6}$ sq. ft. ; (ii) 120 sq. in. | 4. 28512. |
| 5. 11s. 8 <i>d</i> . | 6. 294 sq. ft. |
| | 7. $2126\frac{2}{3}$ sq. ft. |

- | | | |
|---------------------------------|--|------------------------------|
| 8. $86\frac{1}{4}$ sq. in. | 9. £6. 14s. 2d. | 10. £3. 4s. 11d. |
| 11. £10; £5. | 12. 10 ft. 3 in. | 13. 288 ft. |
| 14. 909 sq. in. | 15. $2059\frac{1}{38}$ sq. yd. | 16. 12800. |
| 17. £2. 7s. $4\frac{1}{2}$ d. | 18. 10 in. | 19. 289 sq. in. |
| 20. 24 sq. cm. | 21. £16. 19s. 8d. | 22. 1 cm. |
| 23. 813 sq. yd. | 24. 24 sq. in. | 25. $34\cdot9$ sq. yd. |
| 26. $\cdot45\dots$ sq. cm. | 27. $\cdot46$ sq. in. | 28. $3\cdot303\dots$ sq. ft. |
| 29. $59\cdot69\dots$ sq. in. | 30. $\cdot095\dots$ sq. in. | 31. 618. |
| 32. 120 cu. ft. | 33. 7 cu. ft. | 34. 6 in. |
| 35. $\cdot069$ sq. in. | 36. 3 ft. | |
| 37. 35 tons 3 cwt. 14 lb. | | 38. 28125. |
| 39. $46\cdot656$ cu. ft. | 40. 1408 cu. ft. | 41. $1\frac{2}{3}$ sq. ft. |
| 42. £1. 18s. $11\frac{1}{4}$ d. | 43. $62731\cdot54$ gall. | 44. 229. |
| 45. $21\cdot6\dots$ in. | 46. $79\cdot59$ cu. cm. | 47. 8400 cu. ft. |
| 48. 612 cu. in. | 49. $5\cdot85\dots$ cu. cm. | |
| 50. $339\cdot12\dots$ cu. in. | | 51. $\cdot14\dots$ cm. |
| 52. $58\cdot9\dots$ cu. cm. | 53. $1\cdot23$ sq. cm. ; $1\cdot37\dots$ cm. | |
| 54. 2 hr. 33 min. 36 sec. | | 55. 21 ft. |
| 56. 1890 cu. m. | 57. 56 tons 6 cwt. | 58. $33\cdot5\dots$ cu. in. |
| 59. $904\cdot8\dots$ cu. cm. | 60. $476\cdot3\dots$ cu. cm. | |
| 61. $63\cdot6\dots$ sq. ft. | 62. $18\cdot84$ sq. ft. | |

EXAMPLES. XXII 22

- | | |
|--|-----------------------------|
| 1. $2\cdot625$ gm. per cu. cm. | 2. 5 oz. per cu. in. |
| 3. (i) $\cdot81$ gm. per cu. cm. ; (ii) $\cdot81$ Kg. per cu. m. | |
| 4. $27\cdot2$ gm. | 5. $\cdot946$ gm. |
| 7. 1 : 8. | 6. $181\cdot44$ cu. cm. |
| 9. $\cdot9$ gm. per cu. cm. | 8. $1\cdot414$ gm. |
| 11. (i) $1\cdot03$ gm. per cu. cm. ; (ii) $1\cdot03$. | 10. 2 cm. |
| 13. $8\cdot2$. | 12. $40\cdot5$ gm. |
| 16. $616\cdot4\dots$ oz. | 15. $30\cdot2$ cu. cm. |
| 19. $\cdot029\dots$ sq. in. | 17. 15 oz. |
| 22. $60\frac{1}{2}$ lb. | 18. $\cdot87$ cu. ft. |
| 25. $4\cdot4$ cu. cm. | 20. 6 in. |
| 28. $1\cdot015$. | 21. $5\cdot445\dots$ gm. |
| 30. (i) $1\cdot25$; (ii) $1\cdot248$. | 24. 1. |
| 32. $11\cdot33\dots$ gm. per cu. cm. | 27. $1\cdot0147\dots$ |
| 34. $2\cdot5$. | 31. $1\cdot408$. |
| 37. $1\cdot73\dots$ lb. | 33. 8. |
| 39. $4\cdot075\dots$ oz. per cu. in. | 36. 3 cu. cm. |
| 41. $2\cdot71\dots$ gm. | 40. $7\cdot0416$. |
| 43. 2 cu. cm. per gm. | 44. $\cdot112\dots$ cu. cm. |
| 45. 166 lb. per cu. ft. (nearly). | |

46. 9... cu. cm. per gm.

47. 139.3... cu. cm.

48. 4.24... oz.

49. 2.213... lb.

50. 8605.23 cu. in.

EXAMPLES. XXIII 23

1. (i) 2.36 in.; (ii) 3.07 in.
2. (i) 3.63 Kg.; (ii) 16.5 lb.
3. (i) 177 sq. cm.; (ii) 1.6 cm.
4. (i) £1.72; (ii) £.71.
6. (i) 44 marks; (ii) 77 marks.
7. (i) 2.21 lb.; (ii) .6 lb.
8. 6, 48, 90, 135 ft. per sec.
9. (i) 27s. 6d.; (ii) 44s. 6d.
10. (i) .89 cm.; (ii) 1.13 cm.; (iii) 1.32 cm.
11. (i) 15 fr. 75 c.; (ii) 9.8s.
12. (i) 99.4 cm.; (ii) 977.
13. 149°, 6.36.
14. (i) 2 cm.; (ii) 2.47 cm.; (iii) 4.22 cm.
15. 49.
16. 4.36, 6.16, 8.66, 10.39.
17. 1.49 sq. cm.; .42 sq. cm.
18. 6s. 11d.
19. 2.7 cm.
20. -18.7° C., 60.8° F., -40° F., 38.7° C.
21. (i) 2.88 cm.; (ii) 4.53 cm.; (iii) 6.2 cm.
22. £3. 0s. 7½d.; £4. 12s. 6d.
23. 60.37.
24. (i) 12.6 lb.; (ii) 19.7 lb.
25. .165 mm.
26. (i) .26; (ii) 1.95.
27. 1.7 millions; 2.25 millions.
28. 56° C.
29. £1666.
30. 821.5 lb.
31. 21s.; 30s.
32. 29.2 lb. per sec.
33. 11.95 in.
34. 312.5 miles; £29. 14s.; 15.
35. 2.67.
36. 25.
37. 9 ft.; 9 ft.
38. (i) (500 - x) yd.; (ii) (500 - x)x sq. yd.
39. (1000 - 2x) yd.
41. Radius of semicircle, 1.7 ft.
42. £8. 4s. 9d.
43. Side of square, 3 ft.
44. 12.6 knots.
45. 1440 cu. ft.
46. $\frac{4.9}{8.4}$ sq. ft.
47. 63.24 yd.
48. Radius of end, .64 ft.

EXAMPLES. XXIV 24

1. 8, 11, 60, 19.
3. $\frac{2}{3}$, $\frac{3}{8}$, $\frac{7}{11}$, $\frac{5}{8}$.
5. $1\frac{1}{2}$, $1\frac{3}{4}$, $1\frac{5}{7}$, $1\frac{1}{20}$.

2. 13, 17, 100, 1200.
4. .2, .03, 1.7.

6. (a) 21, 25; (b) 24, 150; (c) 121, 350; (d) 26, 66; (e) 315, 3528.
 7. (a) 231, 457; (b) 75·6, 5·03; (c) 6·94, ·0701; (d) 24·5, 71·09.
 8. 3·53, 35·3, ·353, ·00353.
 9. (i) 1·732; (ii) 2·828; (iii) 3·742; (iv) 4·243; (v) 4·472; (vi) 2·646; (vii) 3·317; (viii) 4·899; (ix) 1·414; (x) 4·123.
 11. 126 m. 12. 220 yd. 14. 226·24 m.
 15. 6·36 in. 16. 13 ft. 17. 2·4 ft.
 18. 118·72 miles. 19. 281·8 ft. 20. 31·2 miles.
 21. 1 ft. 10 in. 22. 22·23 m. 23. 1204 miles.
 24. 18·2 m. 25. 74·8 ft. 26. 54 yd.
 27. 2·828 ft. 28. 9·75 ft. 29. 18 in.
 30. 13·6 in. 31. 17·8 cm. 32. 13 in.
 33. 18·76 in. 34. 18 cm., 430 sq. cm.
 35. 31·09 cm. 36. (i) 7·5 in.; (ii) 8·46 in.
 37. 5·74 ft. 38. 8·48 ft. 39. 78·8 miles.
 40. ·15 in. 41. 19·4 in. 42. 9 miles.
 43. 1·225. 44. ·030... 46. (i) 30; (ii) 7.
 48. 7·8 sec. 49. ·00025. 50. ·777.
 51. 26 in.

EXAMPLES. XXV 25

1. 1 hr. 36 min. 2. 962 sq. feet. 3. $7\frac{3}{4}$ sec.
 4. 300·51 cu. cm. 5. 7·14. 6. $54\frac{1}{4}$ mi. per hr.
 7. 188·5 sq. in. 8. 14·1 cu. cm. 9. £60.
 10. £2. 10s. 0d. 11. 30%. 12. ·9933 sec.
 13. 1·785 sq. cm. 14. 22·57 cm. 15. £742. 12s. 7d.
 16. £2. 16s. 3d. 17. 11·1. 18. $4\frac{1}{2}$ yr.
 19. 68·45 sq. in. 20. 5·68 cm. 21. 2289 cu. cm.
 22. 501·335 cu. cm. 23. 15·55°C.; -17·78°C., 0°C.
 24. 140°F., 32°F., 89·6°F. 25. 205 cu. cm.
 26. 19·683 tons. 27. 6720 lb. 28. 19·5 tons.
 29. 13·866 lb. wt. 30. (i) £19. 4s.; (ii) £11. 5s. 7d.

EXAMPLES. XXVI 26

2. 1, 2·8. 4. 25, 20. 5. 12 days.
 6. 3 hr. 20 min. 7. 40 lb.
 8. $25\frac{5}{16}$ miles per hr. 9. 5 less.
 10. 17 hr. 4 min. 11. 40·35 min. 12. £112. 10s.
 13. 43s. 4d. 14. $1\frac{1}{16}$ lb. 15. 45 days.
 16. $2\frac{1}{4}$ lb.; $1\frac{1}{2}$ lb. 17. 1935·36 cu. in. 18. 25.

19. 14s. (nearly). 20. 156·63... Km. per hr.
 21. 20 days. 22. 27 feet. 23. 379·94 sq. cm.
 24. 97·7 gall. 25. 2·5 cm. 26. 381 cu. cm.
 27. $\frac{13}{30}$ amp. 28. 4·19 cu. cm. 29. 192·1875 cm.
 30. 15·9 miles. 31. 1·29 in. 32. £12.
 33. ·0425 lb. 34. $5\frac{5}{9}$ hr. 35. 81 (about).
 36. 54·25 cents. 37. 7·95 cm.
 38. $a = \frac{11}{35}$; $b = \frac{4}{5}$; 48·4. 39. 23·4375 Kg.
 40. 125700 cm. per sec.

EXAMPLES. XXVII 27

1. $\frac{1}{50}$, $\frac{1}{20}$, $\frac{1}{10}$, $\frac{3}{20}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{17}{20}$.
 2. 50%, 33 $\frac{1}{3}$ %, 25%, 20%, 37 $\frac{1}{2}$ %, 10%, 4%, 6 $\frac{1}{4}$ %, 16 $\frac{2}{3}$ %,
 3 $\frac{1}{3}$ %.
 3. (α) £15, £75, £187. 10s., £250; (β) ·3 Km., 1·5 Km.,
 3·75 Km., 5 Km.; (γ) 2*l.*, 10*l.*, 2s. 1*l.*, 2s. 9 $\frac{1}{2}$ *l.*; (δ) 44·8 lb.,
 2 cwt., 5 cwt., 6 cwt. 74 $\frac{2}{3}$ lb.; (ϵ) 5·04*l.*, 2s. 1·2*l.*, 5s. 3*l.*, 7s.
 4. (i) 12 $\frac{1}{2}$ %; (ii) 22 $\frac{2}{9}$ %; (iii) 50%; (iv) $\frac{5}{8}$ %; (v) 20%.
 5. 288. 6. 90%. 7. 160. 8. £280.
 9. 60. 11. £22. 12. 25%. 13. 5·8%.
 14. 75%. 15. £3. 16. 1 $\frac{1}{4}$ %. 17. 82%.
 18. 57·6. 20. 15%.
 21. (i) £35; (ii) 6*l.*; (iii) 588; (iv) 45%.
 22. (i) 74·8 yd.; (ii) 7s. 7·04*l.*; (iii) 33·57%; (iv) 79·31%.
 23. (i) 42·86%; (ii) 33·33%; (iii) 16·67%; (iv) 18·18%;
 (v) 52·94%.
 24. 5%; (i) 2 $\frac{1}{2}$ %; (ii) 12 $\frac{1}{2}$ %; (iii) $\frac{1}{8}$ %; (iv) $\frac{1}{4}$ %;
 (v) 1 $\frac{1}{4}$ %.
 25. £12. 16s. 27. 1 $\frac{1}{9}$ %.
 28. 95% copper, 4% tin, 1% zinc. 29. ·506%.
 30. £9. 15s. 31. 14 $\frac{2}{7}$ %; 2s. 10*l.* 32. 97·5%.
 33. 19 cwt. 90 lb. 34. 718.
 35. (i) 3244372 Roman Catholics; (ii) 70244 others.
 36. ·3571%. 37. 22·39%.
 38. 2 cu. ft. 918 cu. in. 39. 968 tons.
 40. (i) $\frac{100y}{x}$; (ii) $\frac{2y}{x}$. 41. 296·29%.
 42. £928234500; 44·3%. 43. 41·94%.
 44. 24s. 7*d.* 45. 9 $\frac{1}{16}$ %. 46. 12 $\frac{1}{2}$ %.
 47. 44·05%. 48. 30%.
 49. (i) £12. 7s. 6*l.*; (ii) £17. 1s.; (iii) £18. 19s. 6*l.*;
 (iv) £27. 10s.
 50. ·06%. 51. £2920746. 52. 17%.
 53. 25%. 54. 42·15 million pounds.

- | | | |
|------------------------|----------------------------|-------------------|
| 55. 8 gall. | 56. 1200. | 57. £4. 12s. 8d. |
| 58. 2563000. | 59. 2·46 lb. | 60. 50·55 litres. |
| 62. 39·2 %. | 63. 497 (approx.). | |
| 64. 225 gm. of copper. | 65. 21·28 lb. | |
| 66. 5200. | 67. $\frac{ay+ab}{a+a} \%$ | |

EXAMPLES. XXVIII 28

1. (i) Gain 50 %; (ii) gain 30 %; (iii) loss $8\frac{1}{3} \%$; (iv) gain $33\frac{1}{3} \%$; (v) loss $90\frac{10}{11} \%$.
2. (i) Gain 4 %; (ii) loss $11\frac{1}{9} \%$; (iii) gain $33\frac{1}{3} \%$; (iv) loss $33\frac{1}{3} \%$; (v) loss $14\frac{106}{121} \%$.
3. (i) £27; (ii) £2. 10s.; (iii) £15. 1s.; (iv) 2·4d.; (v) 3d.
4. (i) £233. 6s. 8d.; (ii) £1. 12s.; (iii) 13s. 4d.; (iv) 1s. 3d.; (v) $6\frac{3}{4}d.$
5. 50 %.
6. 5s.
7. $33\frac{1}{3} \%$.
8. £35. 18s. 6d.
9. 20 %.
10. 7·97 %.
11. £18. 19s. 6d.
12. 400 %.
13. 25s.
14. 67·55 %.
15. 19·5 %.
16. Gain $3\frac{1}{8} \%$.
17. 25 %.
18. 6s. 6d.
19. Gain $5\frac{1}{8} \%$.
20. 2s. 5d. (approx.).
21. £28.
22. Gain 2 %.
23. 20 %.
24. 2s. 5d. (nearly).
25. 1s.
27. 15·8 %.
28. 200 %.
29. Gain 10 %.
30. 19·23... %.
31. $85\cdot19\ldots \%$.
32. 5 : 4.
33. 42 lb.
34. 168 lb.
35. 20 %.
36. $22\frac{2}{9} \%$.
37. $12\frac{1}{2} \%$.
38. $13\frac{1}{3} \%$.
39. £ $2\frac{4x}{3}$.
40. £ $\frac{100x}{121}$.
41. Loss 34 %.

EXAMPLES. XXIX 29

1. £267. 15s.
2. £18. 3s. 9d.
3. £298. 4s.
4. £111. 1s. 4d.
5. £2550. 15s. 10d.
6. £24. 12s. 9d.
7. £34194. 3s. 6d.
8. £7113. 5s. 7d.
9. £1475. 15s. $7\frac{1}{2}d.$
10. £1244. 6s. 2d.
11. £175. 2s. 4d.
12. £33. 2s. 6d.
13. £504. 9s. $7\frac{1}{2}d.$
14. £482. 8s. 11d.
15. £2279. 3s. 4d.
16. £2205. 15s.
17. 19 tons 12 cwt. 3 qr. 5 lb.
18. £143. 5s.
19. £8. 19s. $4\frac{1}{2}d.$
20. £65. 4s. 9d.
21. £58. 0s. 11d.
22. £33. 7s. 5d.
23. £12. 8s. 4d.
24. £553. 11s. $9\frac{1}{2}d.$
25. £6. 16s. 11d.
26. £26. 0s. 8d.
27. 25 sq. ft. 40 sq. in.
28. 1 sq. ft. 106 sq. in.
29. 152 sq. ft.
30. 446 sq. ft. $49\frac{1}{2} \text{ sq. in.}$
31. £274. 13s. 4d.

- | | |
|--|---------------------|
| 32. £305. 18s. 7½d. | 33. £9180. 9s. 0d. |
| 34. £19203. 5s. 5d. | |
| 35. £50539. 13s. 9d., £2628063. 15s. | |
| 36. £10. 6s. 7d. | 37. £6. 7s. 7d. |
| 38. 9s. 7d. | |
| 39. £187. 5s. 7d. | 40. £25. 11s. 5d. |
| 41. £62. 16s. 4d | |
| 42. £285. 7s. 6d. | 43. £1786. 16s. 2d. |
| 44. £232. 16s. | 45. £16. 10s. 6d. |
| 46. £63564. 2s. 1d. | 47. £26. 3s. 1d. |
| 48. 184 miles. | 49. £9. 0s. 10d. |
| 50. 7 sq. ft. 20 sq. in. | 51. £14. 14s. 3d. |
| 52. £6. 17s. 9d. | |
| 53. 4 tons 1 cwt. 97 lb. (to nearest lb.). | |
| 54. 12° 51' 45"; 560° (approx.). | |
| 55. £3. 7s. 10d. | 56. £35. 4s. 7d. |
| 57. £1543. 7s. 4d. | |

EXAMPLES. XXX 30

- | | | |
|--|-------------------------|-----------------------------|
| 1. 7·67 sq. cm. | 2. 91·4 sq. in. | 3. 9·12 sq. in. |
| 4. 207·1 sq. cm. | 5. 219·78 cu. cm. | 7. 6·88 gm. |
| 6. 6·50 sq. m. | 9. 271·3 sq. yd. | 10. 3·8 sq. in. |
| 11. 5·34 cm. | 12. 8·6 cu. in. | 13. 3. |
| 14. 1·3 gm. | | |
| 15. (i) ·0005168 cm.; (ii) ·05051 yd.; (iii) ·5168 cm.; (iv) 1 Km. | | |
| 16. 3. | 17. 3. | 18. 2. |
| 19. 2. | | |
| 20. 52317 × 10 ⁵ tons | 21. 2. | 22. 2. |
| 24. The first. | 25. ·02 cm., ·005. | 26. ·005. |
| 27. ·004. | 28. ·33 %. | 29. ·6 %. |
| 30. The first. | 31. ·0002. | 41. 50 %. |
| 43. 69·998 mm., ·001 mm. | 44. 25073000 Kg. | |
| 45. £295000. | 46. 5·182 gm.; ·001 mg. | |
| 47. ·5834. | 48. 216000. | 49. 59 × 10 ⁻⁹ . |
| 50. 92 × 10 ⁻⁸ . | 51. 5·7. | 52. ·66. |
| 53. 679000. | 54. 869. | 55. 1100. |
| 56. 5874·54. | 57. ·0260. | 58. 5901 millions. |
| 59. 4038. | 60. ·31051. | 61. 1·020. |
| 62. 17·219. | | |
| 63. 05194. | 64. 83·12. | 65. ·000873. |
| 66. ·06. | | |
| 67. ·6·7246. | 68. ·07088. | 69. 13719. |
| 70. 1089 thousand. | 71. 105·26. | |
| 72. 3592. | 73. 4892. | 74. 48 hundred. |
| 75. 517. | 77. 20510. | 78. 3097. |
| 79. ·333. | | |
| 80. ·411. | 81. 2·718. | 82. 1·21. |
| 83. 94 cents. | | |
| 84. 64 %. | 85. 3·5 %. | 86. £52. |
| 87. 1. | | |
| 88. 18s. 2d. | 90. 25. | 91. 95·8 %. |

92. (i) $\cdot 38\%$; (ii) 34% ; (iii) $1\cdot 1\%$; (iv) 61% .
 93. 2 passengers, 6 seamen. 95. 365 Turks, 311 Greeks.
 96. $\cdot 49$. 98. $25\cdot 73\%$.
 100. $33\cdot 5 \pm \cdot 3$ cu. cm. 101. $\cdot 48$ gm.

EXAMPLES. XXXI 3/

1. $33\cdot 8$ sq. cm. 2. 19 sq. in. 3. 20 cm.
 4. $3\cdot 14$ sq. cm., $1\cdot 73$ sq. in. 5. $25\cdot 9$ cu. ft.
 6. (i) $4\cdot 9$ sq. cm.; (ii) $\cdot 76$ sq. in.
 7. $5\cdot 79$ cu. cm., $49\cdot 77$ gm. 8. 27375 sq. miles.
 9. $\cdot 5$ cu. cm. 10. 56 miles, 56 miles.
 11. 22 cm. 12. $13\cdot 2$ cu. ft. 13. 656 sq. in.
 14. 400 sq. yd. 15. $\cdot 086$ cu. ft. 16. $3\cdot 6$ sq. miles.
 17. (i) $1\cdot 95$ sq. cm.; (ii) $\cdot 3$ sq. in. 18. $\cdot 014$ cu. ft.
 19. 128 triangles, 222 sq. in. 22. 8 miles.

EXAMPLES. XXXII 32

12. $4\cdot 3511$. 13. $\cdot 3511$. 14. $\cdot 7541$. 15. $\bar{1}\cdot 9025$.
 16. $2\cdot 7541$. 17. $2\cdot 2405$. 18. $\bar{1}\cdot 2405$. 19. $\bar{1}\cdot 7861$.
 20. $\bar{3}\cdot 4475$. 21. $\cdot 7704$. 22. $\bar{1}\cdot 6150$.
 23. $\cdot 8451$, $\cdot 2817$, $\cdot 4829$.
 24. (i) 5 ; (ii) $\bar{1}\cdot 8599$; (iii) $\cdot 2788$.
 25. 31 . 26. 23 . 31. $412\cdot 2$. 32. $3\cdot 859$.
 33. $\cdot 7518$. 34. $33\cdot 32$. 35. 5273 . 36. $\cdot 000343$.
 37. $132\cdot 5$. 38. $18\cdot 31$. 39. 55 . 40. $1\cdot 243$.
 41. $\cdot 04454$. 42. 1563×10^{-17} . 43. $41\cdot 54$.
 44. $4\cdot 17$. 45. $\cdot 2913$. 46. $1\cdot 075$. 47. $1\cdot 11$.
 48. $\cdot 02417$. 49. $\cdot 2346$. 50. $2\cdot 043 \times 10^{-33}$.
 51. $\cdot 1625$. 52. $3\cdot 491$. 53. $102\cdot 8$.
 54. $42\cdot 12$. 55. 1113×10^{-7} . 56. $4\cdot 249$.
 57. $70\cdot 39$. 58. $\cdot 002581$. 59. $2\cdot 476$. 60. $4\cdot 482$.
 61. $1\cdot 269$. 62. $574\cdot 1$. 63. 2226000 . 64. 2851 .
 65. $5\cdot 383$. 66. $\cdot 01651$. 67. $3\cdot 659$ sq. cm.
 68. 11490 cu. in. 69. 146600 cu. cm. 70. 1047 sq. in.
 71. $14\cdot 05$ in. 72. 187500 miles per hr.
 73. 1226 cm. 74. $10\cdot 46\%$. 75. 101 .
 76. 5258 . 77. $\cdot 585$. 79. 497900 .
 80. $78\cdot 14$. 81. $1\cdot 334$. 82. $\pounds 875$. $13s. 6d.$
 83. $\cdot 00014$. 84. $2\cdot 092$. 85. $1\cdot 66$.
 86. 3×10^{-6} . 87. $86\cdot 21$ sq. in. 88. 421×10^7 .
 89. $149\cdot 1$ sq. cm. 90. $\cdot 00079$. 91. $50\cdot 9343$.
 92. 1303×10^{-10} . 93. $8\cdot 647$.

EXAMPLES. XXXIII } 3

- | | | | |
|-------------------------|---------------------------|--------------------------|---------|
| 1. £40. | 2. £21. | 3. £36. | 4. £54. |
| 5. £97. 10s. | | 6. 337 dollars 50 cents. | |
| 7. £48. 2s. | 8. £481. 5s. | 9. £86. 0s. 7d. | |
| 10. £7. 6s. | 11. £191. 5s. | 12. £67. 16s. 4d. | |
| 13. £73. 13s. 9d. | 14. £80. 2s. 6d. | 15. £12. 5s. 8d. | |
| 16. £3. 8s. 6d. | 17. £3. 19s. 9d. | 18. £173. 9s. 7d. | |
| 19. £52. 12s. | 20. £470. 18s. 2d. | 21. £10. 9s. 3d. | |
| 22. £605. 14s. 8d. | 23. £30. 4s. | 24. £4. 1s. 4d. | |
| 25. £14. 19s. 11d. | 26. £431. 1s. 7d. | 27. £18. 17s. 8d. | |
| 28. £52. 2s. 9d. | 30. £26. 10s. 11d. | | |
| 31. £19775623. 10s. 7d. | 32. £137789. 12s. 11d. | | |
| 33. £98. 2s. 11d. | 34. £6. 18s. 11d. | | |
| 35. £2518. 16s. 9d. | 36. £550. | | |
| 37. £325. 8s. 4d. | 38. 8 yr. | 39. $3\frac{1}{2}\%$. | |
| 40. $3\frac{1}{2}$ yr. | 41. $6\frac{1}{2}$ yr. | 42. 4% . | |
| 43. 169 days. | 44. £3643. 6s. 8d. | 45. $4\frac{1}{2}\%$. | |
| 46. 40 yr. | 47. $3\frac{1}{4}\%$. | 48. £55. 10s. | |
| 49. £302. 10s. | 50. £284. | 51. 12 %. | |
| 52. £4160. 8s. 5d. | 53. 150 days. | 54. £720. | |
| 55. £7. 14s. 10d. | 56. £1063. 5s. 11d. | | |
| 57. 40 yr. | 58. £189. | 59. £7. 2s. 6d. | |
| 60. 3 %. | 61. 191 dollars 51 cents. | | |
| 62. £22. 10s. | 63. £9. 1s. 2d. | 64. £43. 19s. | |
| 65. £4. 1s. 10d. | 66. £563. 3s. 6d. | | |
| 67. £2494. 4s. 11d. | 68. £1678. 15s. | | |
| 69. £61. 5s. | 70. 4 %. | 71. £311. 7s. 11d. | |
| 72. £345. 8s. 11d. | 73. £22. 15s. 11d. | 74. 5 %. | |
| 75. £877. 13s. 7d. | 76. £15. 19s. 4d. | | |

EXAMPLES. XXXIV } 4

- | | | |
|---------------------|---------------------|--------------------|
| 1. £109. 5s. 5d. | 2. £550. 15s. | 3. £827. 17s. 2d. |
| 4. £299. 4s. 6d. | 5. £837. 9s. 2d. | 6. £6699. 9s. 4d. |
| 7. £113. 7s. 11d. | 8. £1681. 7s. 2d. | |
| 9. £3876. 10s. 7d. | 10. £4023. 2s. 8d. | |
| 11. £4286. 2s. 9d. | 12. £1833. 12s. 3d. | |
| 13. £606. 3s. 3d. | 14. £762. 16s. 5d. | 15. £426. 11s. 9d. |
| 16. £1307. 4s. 4d. | 17. £3153. 0s. 2d. | |
| 18. £819. 18s. 11d. | 19. £930. 6s. | |
| 20. £1209. | 21. £576. 6s. | 22. £12500. |
| 23. £5987. | 24. £6569. | 25. £3328. |
| 26. £750. | 27. £487. 12s. | 28. £537. |

- | | | |
|----------------------------|---------------------------|-----------------------|
| 29. £133. 12s. | 30. £95. 17s. 7d. | 31. £743. |
| 32. £5526. | 33. £53. 17s. 2d. | 34. £3662. |
| 35. 5 %. | 36. $4\frac{1}{2}$ %. | 37. $4\frac{1}{4}$ %. |
| 38. 3 %. | 40. 3 yr. | 41. 16·76 %. |
| 42. £904. 1s. | 43. 2 %. | 44. £200. |
| 45. £10. 11s. | 46. £2. | 47. £365. |
| 48. 18 yr. | 49. $7\frac{1}{4}$ pence. | |
| 50. (i) £1485; (ii) £1500. | | 51. £1. 14s. 1d. |

EXAMPLES. XXXV 3 5

- | | |
|--|---|
| 1. £333. 6s. 8d. stock. | 2. £750 stock. |
| 3. 75 shares. | 4. £1725. |
| 6. £1595. 15s. stock. | 7. £2991. 10s. |
| 8. £4956. | 9. £1305 stock. |
| 11. £6728. 8s. | 10. 19 shares; 5s. |
| 12. £8494. 10s. stock (approx.). | |
| 13. £527. 5s. | 14. £3766. 13s. 4d. stock; £3333. 6s. 8d. |
| 15. £30937. 10s. | 16. £75. |
| 17. £187. 10s. | |
| 18. £4. | 19. £1260. |
| 20. £159. 1s. 9d. | |
| 21. £205. 11s. 10d. | 22. £207. 13s. 4d. |
| 23. £225. 19s. 9d. | 24. £2000 stock, £40. |
| 25. 42 shares, £4. 14s. 6d. | 26. $107\frac{1}{4}$. |
| 27. £234. 7s. 6d. | 28. £38. 8s. |
| 29. 5 % (nearly). | |
| 30. 70. | 31. £1300 stock. |
| 32. 94. | |
| 33. $3\frac{1}{2}$ %. | 34. $6\frac{2}{3}$ %. |
| 35. £252. 16s. 2d. | |
| 36. £4794. 10s. stock (approx.); £119. 17s. 3d. | |
| 37. £1. 1s. | 38. $6\cdot45$ %. |
| 39. $100\frac{1}{2}$; £200. | |
| 40. 425. | 41. £3. 17s. 6d. |
| 42. £1357. 4s.; £1388. 8s. | 43. £400. 16s.; gain £25. 1s. |
| 46. £2. 10s. | 47. Gain £1. 16s. |
| 48. £516. 13s. 4d. | 50. Gain £222. 13s. 6d. |
| 51. £8953. 2s. 6d. | 52. £191. 5s. |
| 53. £2636. 5s. | 54. £70; gain £5. |
| 55. 54. | |
| 56. £1840 in Consols, £1430 in India stock. | |
| 57. Gain 10s. | |
| 58. £1126. 15s. stock (approx.); £28. 3s. 5d. (approx.); | |
| £33. 13s. 1d. | |
| 59. £8085. | 60. £94 $\frac{1}{6}$. |

EXAMPLES. XXXVI 3 6

- | | |
|-----------------------------------|------------|
| 1. 24 min. | 2. 15 sec. |
| 3. $10\frac{10}{11}$ min. past 8. | 4. 3 hr. |

5. 172 yd.
6. $9\frac{3}{4}$ yd.
7. 12 hr.
8. (i) 30 sec.; (ii) 6 sec.
9. $21\frac{9}{11}$ min. past 1, and $54\frac{6}{11}$ min. past 1.
10. 10 yd., 6 yd. (approx.).
11. 10 miles per hr.
12. 210 miles.
13. 6 days.
14. 9 sec. (approx.).
15. 12 midnight a week later.
16. A, $10\frac{4}{5}$ days; B, $13\frac{1}{2}$ days.
17. 5 miles.
18. 20.
19. 4 miles per hr., $2\frac{2}{3}$ miles per hr.
20. 2.21 p.m. (approx.).
21. 11.58 a.m. (approx.).
22. $91\frac{2}{3}$ miles.
23. 5.39 (approx.); 28 miles (nearly).
24. (i) $54\frac{6}{11}$ min. past 4; (ii) $5\frac{5}{11}$ and $38\frac{2}{11}$ min. past 4.
25. 35 men.
26. 12 sec. (approx.).
27. 5 days; 4 days.
28. 3:2:1.
29. 12.1 (nearly).
30. $42\frac{1}{2}$ miles per hr.; $37\frac{1}{2}$ miles per hr.
31. 10 min.; $\frac{1}{2}$ hr.
32. 11 men; £60. 15s. 10d.
33. 64 miles.
34. 18 min. 15 sec.

ANSWERS TO TEST PAPERS

I 1

1. Fifty-nine thousand seven hundred and fifty-eight.
2. £303125.
3. £12 in hand.
4. $3^3 \times 5 \times 11$; $2^4 \times 3^3 \times 5$; 135.
5. £3. 18s. $9\frac{3}{4}d$.
6. £1. 10s. 5d.

II 2

1. L. 553.
2. 15158.
3. £127765. 17s. 4d.
4. 5s. 4d.
5. £8. 6s. 8d.
6. £42. 1s. $0\frac{1}{2}d$.

III 3

1. £37693778, £34483879.
2. 24.
3. £67. 15s. 1d.
4. 14 lb. 13 oz.
5. £171. 1s. $0\frac{1}{2}d$.
6. 1225.

IV 4

1. (a) 58261; (b) 25734931.
2. (i) £5. 8s. 4d.; (ii) £5. 18s. 4d.
3. 22 articles; 8s. 4d.
4. 10.10 a.m.
5. £165. 3s. $1\frac{1}{2}d$.
6. 10.

V 5

1. (a) 21522 tons; (b) £283194.
2. £343940. 8s.
3. 2^3 , 7^3 ; 3^3 , 5^3 ; 2^{12} .
4. 11s. $9\frac{1}{2}d$.
5. £36. 3s. 3d.
6. £1. 14s. 6d.

VI 6

1. (a) £341146; (b) £341146.
2. 113 lb. 1000 gr.
3. 10 weights.
4. 2^6 , 3^2 , 5^2 ; 129600.
5. 9192 gr.
6. 25.

VII 7

1. £5373.
2. 11213, 2043.
3. -12 feet.
4. 63360; £132.
5. £91. 13s. 4d.
6. 1073741824.

VIII 8

1. 33537, 35668, 37316.
2. 191 feet.
3. £645. 9s. $2\frac{1}{4}d$.
4. 8640.
5. 434.
6. 981.

IX 9

1. 324.
2. 3842760.
3. 1584.
4. 4s. 0d.
5. 2^2 , 3^2 , 5^2 , 7^2 ; 2, 3, 5, 7; 210.
6. 2s. 0d.

X 10

1. 6017.
2. 78840000.
3. 3s. $9\frac{1}{2}d$.
4. 39.
5. 2 lb. 8 oz.
6. 23569920 millions of miles.

XI 11

1. 9101120·9 cm., 910·11209 Hm., 3549437·151 in.
2. The first by £16. 5s.
3. 54 sq. cm.
4. $\frac{937}{1008}$.
5. 70·06.
6. 9·11932 miles.

XII 12

1. 1000 times.
2. ·001316, ·003975.
3. 96·75 Km. per hr.
4. 17850.
5. 5; 144500; 10.
6. 1.

XIII 13

1. £3. 19s. 11 $\frac{1}{4}$ d.
2. (i) .175, .2375, .86875, .10625; (ii) .18, .24, .87, .11.
3. 9497.8116 Kg. 4. 41946.
5. (i) 1 $\frac{1}{17}$; (ii) 1 $\frac{53}{816}$. 6. 1.003...

XIV 14

1. £2264666.2625.
2. 20.28 sq. cm.
3. (i) 259.920276; (ii) 7.028.
4. 725.
5. $\frac{2057}{2072}$.
6. £4. 19s. 7d.; £20. 11s. 7d.

XV 15

1. £78. 4s. 7 $\frac{3}{4}$ d.
2. 396.9 Kg.
3. (i) .0048; (ii) .52941; (iii) .01545.
4. £1200. 5. 17.91 sq. cm. 6. $\frac{91}{124}$.

XVI 16

1. .37875.
2. 19875 sq. cm.
3. 25.63888.
4. 23.208... knots; 777.833... feet.
5. $\frac{17}{280}$.
6. £6. 3s.

XVII 17

1. 62.99136 cu. in.
2. 52.9416 cu. cm.
3. (i) .049; (ii) .154; (iii) .019.
4. 8 st. 7 lb.
5. 187475 miles per sec.
6. 1 $\frac{1}{4}$.

XVIII 18

1. $\frac{21541}{21600}$.
2. 50.653 cu. cm.; .11396925 Kg.
3. £.025.
4. (i) $\frac{5}{16}$; (ii) $\frac{72}{125}$; (iii) $\frac{8}{78125}$.
5. (i) £16. 17s. 6d.; (ii) £9. 7s. 6d. 6. 152.

XIX 19

1. 50 yd. 2 ft.
2. 1 cwt. 84 lb. 14 oz.; 42 packages.
3. (i) $\frac{1}{512000}$; (ii) .000001953125.
4. 73542 litres.
5. 348.5... cu. cm.
6. 1 $\frac{1}{2}$.

XX 20

1. 10000; 15000.
2. 30·6514.
3. £1. 3s. 6d.
4. 4195·04 cu. cm.
5. $3^3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 23 \times 37$; 1485.
6. $\frac{7}{8}\tau$.

XXI 21

1. (a) 151811600; (b) 203850500.
2. 804 sq. miles.
3. £1. 1s. 10d.
4. The receipts in 1912 are the greater by £29637. 0s. 7½d.
5. 29·87.
6. 8.

XXII 22

1. 146·5; 172·8.
2. 17½ days.
3. 19·06 gm.
5. 1 hr. 4 min. 32 sec.
6. ·356.

XXIII 23

1. £6. 17s. 9d.
2. £602. 11s. 10½d.; £1150. 8s. 1½d.
3. 3·165 sq. in.
4. 3·46 cm.
5. $\frac{3}{4}d$.
6. 8·2 gm. per cu. cm.

XXIV 24

1. 1·14.
2. 12.
3. 7.
4. ·5 cm.
5. £3000.
6. $3\frac{319}{336}$.

XXV 25

1. 10 yr.
2. (i) $\frac{21}{50}$; (ii) ·09197.
3. £94. 16s. 7d.
4. 1868·3 cu. cm.
5. 773·4 litres.
6. 1s. 4d.

XXVI 26

1. £2. 12s. 6d.
2. Cheaper in France.
3. $\frac{11}{19}$, 2·1, 505.
4. $\frac{1727}{3072}$.
5. 7 in.
6. 10 cm.

XXVII 27

1. $1\frac{230}{407}$.
2. 7; 10; 10; 11.
3. 4.38 p.m.
4. $15\frac{3}{16}$.
6. 6249984.

XXVIII 28

- | | |
|-------------|--|
| 1. 12.9. | 2. (i) 39; (ii) 4679. |
| 3. 8.4 a.m. | 4. .00001 cm. 5. 105 lb. 6. 11.5 ft. |

XXIX 29

- | | | |
|--------------------------------|---------------------------|---------------------------------|
| 1. 3.6 in. | 2. 5.33 p.m. | 3. 6.75 sq. in. |
| 4. $1\frac{4}{8}\frac{3}{0}$. | 5. $20\frac{4}{7}$ miles. | 6. 527; $4\frac{5}{7}$; 2.530. |

XXX 30

- | | |
|-----------------------------|----------------------------------|
| 1. 45. | 2. £46736. 17s. 8d.; 1477841 lb. |
| 3. $125\frac{5}{8}$ cu. in. | 5. (i) .25; (ii) 2.03. |
| 6. $\frac{cxv}{bz}$. | |

XXXI 31

- | | |
|------------------------------|----------------------------------|
| 1. (i) .00099%; (ii) .0266%. | 2. £30. 7s. 9d. |
| 3. 1.20. | 4. £64. 4s. 3d. |
| 5. 100 miles per hr. | 6. $21\frac{9}{11}$ min. past 4. |

XXXII 32

- | | | |
|----------------------|-------------|--------------------|
| 1. 36s. 8d. | 2. 29.002. | 3. £2. 6s. |
| 4. (i) 35; (ii) 100. | 5. .9714... | 6. 3405.72 cu. in. |

XXXIII 33

- | | | |
|-------------------------|--------------|---------------|
| 1. 35%. | 2. .9864 gm. | 3. 16.5; 1.9. |
| 4. £1412; £49. 10s. 9d. | 5. £28 gain. | |
| 6. .0009. | | |

XXXIV 34

- | | | |
|----------------------------|-----------------|------------|
| 1. Gain $4\frac{1}{6}\%$. | 2. 2% (nearly). | 3. 1.21d. |
| 4. 4.326 cm. | 5. £7200. | 6. £20160. |

XXXV 35

- | | | |
|----------------|----------------|-------------|
| 2. 7513 yd. | 3. 9.869... | 4. £4. 15s. |
| 5. 439 cu. in. | 6. 723 cu. in. | |

XXXVI 36

1. 12s. ; 15·4...%
2. 95·6 hr.
3. 25 %.
4. 5 sq. ft. 121 sq. in. ; 10 lb. 10·3 oz.
5. £4. 11s. 6d.
6. 974×10^{-10} .

XXXVII 37

1. $6 \times 10^{-5} \%$; $3 \times 10^{-4} \%$.
2. £453. 10s. $3\frac{3}{4}$ d. ; £522. 13s. 4d.
3. 513 cu. cm.
4. £38. 8s. 1d.
5. -46·1.
6. 1665 cu. in.

XXXVIII 38

1. 28·0 %.
2. (i) £4. 6s. ; (ii) £3.
3. $44\frac{7}{22}$ sec.
4. £43. 15s.
5. Loss £3. 7s. 4d.
6. One.

XXXIX 39

1. 1 : 2.
2. 2·96.
3. 5 min. 45 sec. after 1 o'clock.
4. 4 years.
5. $5\frac{1}{4}$ miles.
6. (i) 13 ; (ii) 1778×10^{-15} .

XL 40

1. 5 days.
2. 30·6 % ; 224734 dollars.
3. 10 miles per hr.
4. £340. 10s.
5. $38\frac{2}{11}$ min. past 7.
6. 8×10^{-8} .

ANSWERS TO MISCELLANEOUS EXAMPLES

1. $\frac{184}{639}$. 2. £14. 11s. 3. 13306. 4. 15·6.
5. Thursday. 6. £10. 18s. 10d. 8. 140 grains.
9. ·884 m., 2·652 m.
10. 33·1 grains; 4·7 grains; 236·2 grains; 425·2 grains.
11. $12\frac{1}{2}\%$. 12. $\frac{5}{21}$. 13. 16·8 Kg.
14. £13. 9s. 15. 48 lb. 16. 40.
17. 11·9%. 18. 49·69; 7·05. 19. 48%.
20. (i) £17. 5s. 2d.; (ii) £17. 12s. 11d.
21. 44·16 sq. in.; ·21 oz. 22. £3031. 5s.
23. (a) 875·616; (b) 2·991. 24. £3. 0s. 10d.
25. £342. 10s.; £657. 10s. 26. 939 times (nearly).
27. 1·1. 28. 3·351; 3·352. 29. 40 ft.; 74 ft.
30. 2·08. 31. 64·9%. 32. £523. 4s.
33. 88 yd. 34. 1·85. 35. 65.
36. 1·011 cu. cm. 37. 23 hr. 16 min.; 5 hr. 35 min.
38. 274625. 39. £2141. 16s. 1d.; £28. 3s. 11d.
40. 13. 41. 36·5%; £3165. 17s. 9d.
42. 19 miles per hr.
43. (i) 32; (ii) 256; (iii) 130; (iv) 68; (v) 40.
44. 213 cwt. 45. 1.16 p.m.
46. 1·0493. 47. The second; 23·2 tons.
48. (i) $\frac{b}{a} \left\{ \frac{100^3}{(100+p)(100+q)} \right\} - 100\%$; (ii) $1:2 - \frac{a}{b} \left\{ \frac{(100+2p)(100+q)}{100^3} \right\}$.
49. 37:155. 50. 27·86. 52. 4050 sq. in.
53. 2805 yd. 54. £5640. 55. 106·25.
56. apy ; bqx . 57. £2. 7s. 10d. 58. £277.
59. ·027.
60. (a) £1339. 5s. 9d.; (b) £1333. 9s. 11d.
61. 3·335... oz. 62. 320. 63. £410.
64. $\frac{c}{2ad-bc}$ shillings; 1s. 9d.
65. Wednesday, 14·5 min. past 1 a.m.
66. 196·8 sq. ft. 67. 29860 tons. 68. 46660 gall.

69. £7590. 70. $1\frac{1}{2}$ miles; 53 min. $5\frac{1}{2}$ sec.
 71. £85085. 72. $\frac{7}{7}\frac{4}{5}$. 73. 53 sec. 74. £80000.
 75. 5s. 11d.; 6s. 3d.; 3s. 8d.; 3s. 11d.; $3\cdot7\%$.
 76. $11\frac{6}{4}$ cu. in. 77. 2 ft. 2 in. 78. 165×10^3 cm.
 79. $67\cdot1\%$. 80. 12 miles per hr.; 15 hr.
 81. 27·39 lb. 82. 2·5.
 83. The pyramid by 54 lb. 84. 701·3 yd.
 85. £1. 11s. 86. $\frac{4}{3}x$.
 87. £700; £1645. 4s.; £1. 4s. 11d.
 88. £122·3; 2581·8 fr.; the French stock.
 89. 2·5 in. 90. 484 sq. cm. (approx.).
 91. $38\frac{2}{11}$ min. past 7; $21\frac{9}{11}$ and $54\frac{6}{11}$ min. past 7.
 92. 4316000; 14·6. 93. 5·165 sq. ft.; 103·3 lb.
 94. 12.15 $\frac{1}{2}$ p.m.; 47, 38, 32; C.
 95. $2\cdot7 \times 10^{10}$. 96. 3·2 ft.; 287. 97. $3\frac{1}{2}$ hr.
 98. 506 in. 99. 30 yr. 100. 4356; $146\frac{2}{3}$ yd.
 101. 03%. 102. 6 yr. 103. 956·8 Kg.
 104. 23·9 cu. in. 105. 7 in. 106. 32 or 33; 31.
 107. £416. 5s. 108. $26\frac{1}{4}$ sec.
 109. 2·116; 1896·2; 896·2. 110. 6·93 in.; 03%.
 111. 1·804 in. 112. 7·5.
 113. £3123. 10s. 7d. in $5\frac{1}{2}\%$; £6179. 9s. 5d. in 3%.
 114. 1 in. 115. (i) 465·35; (ii) 3·29.
 116. 6. 117. 18·3 sq. cm.; $\frac{2xy^3}{x^2+y^3}$.
 118. B:C=5:2; C:A=5:2.
 119. 644·75 cu. in.

Printed by
MORRISON & GIBB LIMITED
Edinburgh

A LIST OF BOOKS

SELECTED FROM

BELL'S EDUCATIONAL CATALOGUE



CONTENTS

	PAGE
LATIN AND GREEK	2
MATHEMATICS	5
ENGLISH	8
MODERN LANGUAGES	12
SCIENCE AND TECHNOLOGY	14
MENTAL AND MORAL SCIENCE	15
HISTORY	15
BOHN'S POPULAR LIBRARY	16

MESSRS. BELL are at all times glad to receive visits from members of the teaching profession, and to avail themselves of the opportunity to discuss matters of mutual interest, to submit their latest publications, and to talk over new methods and ideas.

LONDON: G. BELL AND SONS, LTD.

PORTUGAL STREET, KINGSWAY, W.C.

CAMBRIDGE . . . DEIGHTON, BELL & CO.

NEW YORK . . . THE MACMILLAN COMPANY

BOMBAY . . . A. H. WHEELER & CO.

LATIN AND GREEK

Full Catalogue of Classical Books sent on application

Bell's Illustrated Classics

Edited by E. C. MARCHANT, M.A.

Edited with Introductions, Notes and Vocabularies. With illustrations, maps and plans, 1s. 6d. each; except the GREEK PLAYS, which are 2s. each.

- Cæsar.** Book I. By A. C. LIDDELL, M.A.
 — Book II. By A. C. LIDDELL, M.A.
 — Book III. By F. H. COLSON, M.A., and G. M. GWYTHYR, M.A.
 — Book IV. By Rev. A. W. UPCOTT, D.D.
 — Book V. By A. REYNOLDS, M.A.
 — Books IV. and V., in one volume, 2s. 6d.
 — Book VI. By J. T. PHILLIPSON, M.A.
 — Books V. and VI., in one volume, 2s. 6d.
 — Book VII. By S. E. WINBOLT, M.A.
Cæsar's Invasions of Britain (De Bello Gallico. Lib. IV. XX.—V. XXIII.). By Rev. A. W. UPCOTT, D.D., and A. REYNOLDS, M.A.
Cicero. Speeches against Catiline. I. and II. (1 vol.). By F. HERRING, M.A.
 — Selections. By J. F. CHARLES, B.A.
 — De Amicitia. By H. J. L. J. MASSÉ, M.A.
 — De Senectute. By A. S. WARMAN, B.A.
Cornelius Nepos. Epaminondas, Hannibal, Cato. By H. L. EARL, M.A.
Eutropius. Books I. and II. (1 vol.). By J. G. SPENCER, B.A.
Homer: Iliad. Book I. By L. D. WAINWRIGHT, M.A.
Horace: Odes. Book I. By C. G. BOTTING, B.A.
 — Book II. By C. G. BOTTING, B.A.
 — Book III. By H. LATTER, M.A.
 — Book IV. By H. LATTER, M.A.
Livy. Book IX, cc. i-xix. By W. C. FLAMSTEAD WALTERS, M.A.

- Livy.** Hannibal's First Campaign in Italy. (Selected from Book XXI.) By F. E. A. TRAVES, M.A.
Lucian: Vera Historia. By R. E. YATES, B.A.
Ovid: Metamorphoses. Book I. By G. H. WELLS, M.A.
 — Selection from the Metamorphoses. By J. W. E. PEARCE, M.A.
 — Elegiac Selections. By F. COVERLEY SMITH, B.A.
 — Tristia. Book I. By A. E. ROGERS, M.A.
 — Tristia. Book III. By H. R. WOOLRYCH, M.A.
Phædrus: A Selection. By Rev. R. H. CHAMBERS, M.A.
Stories of Great Men. By Rev. F. CONWAY, M.A.
Virgil: Aeneid. Book I. By Rev. E. H. S. ESCOTT, M.A.
 — Book II. By L. D. WAINWRIGHT, M.A.
 — Book III. By L. D. WAINWRIGHT, M.A.
 — Book IV. By A. S. WARMAN, B.A.
 — Book V. By J. T. PHILLIPSON, M.A.
 — Book VI. By J. T. PHILLIPSON, M.A.
 — Books VII., VIII., IX., X., XI., XII. By L. D. WAINWRIGHT, M.A. 6 vols.
 — Selection from Books VII. to XII. By W. G. COAST, B.A.
 — Georgics. Book IV. By L. D. WAINWRIGHT, M.A.
Xenophon: Anabasis. Books I., II., III. By E. C. MARCHANT, M.A. 3 vols.

GREEK PLAYS (2s. each)

- Aeschylus:** Prometheus Vincitus. By C. E. LAURENCE, M.A.
Euripides: Alcestis. By E. H. BLAKENEY, M.A.
Euripides: Bacchæ. By G. M. GWYTHYR, M.A.
 — Hecuba. By Rev. A. W. UPCOTT, M.A.
 — Medea. By Rev. T. NICKLIN, M.A.

Bell's Illustrated Classics—Intermediate Series

Edited for higher forms, without Vocabularies. With Illustrations and Maps.

- Cæsar:** Seventh Campaign in Gaul, B.C. 52. De Bello Gallico. Lib. VII. By the Rev. W. COOKWORTHY COMPTON, M.A. 2s. 6d. net.
 — De Bello Civili. Book I. By the Rev. W. J. BENSLEY, M.A. 2s. 6d. net.
Livy. Book XXI. Edited by F. E. A. TRAVES, M.A. 2s. 6d. net.
Tacitus: Agricola. By J. W. E. PEARCE, M.A. 2s.
Sophocles: Antigone. By G. H. WELLS, M.A. 2s. 6d. ret.
Homer: Odyssey. Book I. By E. C. MARCHANT, M.A. 2s.
Athenians in Sicily. Being portions of Thucydides, Books VI. and VII. By the Rev. W. COOKWORTHY COMPTON, M.A. 2s. 6d. net.

Bell's Simplified Latin Classics

Edited, with Notes, Exercises, and Vocabulary, by S. E. WINBOLT, M.A., Christ's Hospital, Horsham. Crown 8vo. With numerous Illustrations. 1s. 6d. each.

FIRST LIST OF VOLUMES, NOW READY

Cæsar's Invasions of Britain.
Livy's Kings of Rome.
Vergil's Taking of Troy.
Vergil's Athletic Sports.
Simple Selections from Cicero's Letters.

Cæsar's Fifth Campaign (from *De Bello Gallico*, Book V.).
Tacitus' Agricola.
Sallust's Catiline.

Uniform with 'Bell's Simplified Latin Classics.'

Dialogues of Roman Life. By S. E. WINBOLT, M.A. Fcap. 8vo. Illustrated. With or without Vocabulary. 2s.

Latin and Greek Class Books

Bell's Illustrated Latin Readers.

Edited by E. C. MARCHANT, M.A.

Pott 8vo. With brief Notes, Vocabulary, and numerous Illustrations. 1s. each.

Scalæ Primæ. A Selection of Simple Stories for Translation into English.

Scalæ Mediæ. Short Extracts from Eutropius and Caesar.

Scalæ Tertiæ. Selections in Prose and Verse from Phædrus, Ovid, Nepos and Cicero.

Latin Picture Cards. Edited by Prof. FRANK S. GRANGER, M.A. Sixteen cards printed in colours, with Vocabulary and Exercises. 1s. 3d. net per set.

Bell's Illustrated Latin Course, for the First Year. In three Parts. By E. C. MARCHANT, M.A., and J. G. SPENCER, B.A. With numerous Illustrations. 1s. 6d. each.

Latin Unseens. Selected and arranged by E. C. MARCHANT, M.A. 1s.

Latin Reader (Verse and Prose). By W. KING GILLIES, M.A., and H. J. ANDERSON, M.A. 2s.

Latin of the Empire (Prose and Verses). By W. KING GILLIES, M.A., and A. R. CUMMING, M.A. 4s. 6d.

First Exercises in Latin Prose Composition. By E. A. WELLS, M.A. With Vocabulary. 1s.

Materials for Latin Prose Composition. By the Rev. P. FROST, M.A. 2s. Key, 4s. net.

Passages for Translation into Latin Prose. By Prof. H. NETTLESHIP, M.A. 3s. Key, 4s. 6d. net.

Easy Translations from Nepos, Cæsar, Cicero, Livy, &c., for Retranslation into Latin. By T. COLLINS, M.A. 2s.

Memorabilia Latina. By F. W. LEVANDER, F.R.A.S. 1s.

Test Questions on the Latin Language. By F. W. LEVANDER, F.R.A.S. 1s. 6d.

Latin Syntax Exercises. By L. D. WAINWRIGHT, M.A. Five Parts. 8d. each.

A Latin Verse Book. By the Rev. P. FROST, M.A. 2s. Key, 5s. net.

Latin Elegiac Verse. Easy Exercises in. By the Rev. J. PENROSE. 2s. Key, 3s. 6d. net.

Bell's Concise Latin Course. Part I.

By E. C. MARCHANT, M.A., and J. G. SPENCER, B.A. 2s.

Bell's Concise Latin Course. Part II.

By E. C. MARCHANT, M.A., and S. E. WINBOLT, M.A. 2s. 6d.

Cothurnulus. Three Short Latin Historical Plays. By Prof. E. V. ARNOLD, Litt.D. With or without Vocabulary, 1s. Vocabulary separately, 4d.

Easy Latin Plays. By M. L. NEWMAN. 6d.

Eclogæ Latinæ; or, First Latin Reading Book. With Notes and Vocabulary by the late Rev. P. FROST, M.A. 1s. 6d.

Latin Exercises and Grammar Papers. By T. COLLINS, M.A. 2s. 6d.

Unseen Papers in Latin Prose and Verse. By T. COLLINS, M.A. 2s. 6d.

Foliorum Silvula. Part I. Passages for Translation into Latin Elegiac and Heroic Verse. By H. A. HOLDEN, Litt.D. 7s. 6d.

How to Pronounce Latin. By J. P. POSTGATE, Litt.D. 1s. net

Res Romanæ, being brief Aids to the History, Geography, Literature and Antiquities of Ancient Rome. By E. P. COLERIDGE, M.A. With 3 maps. 2s. 6d.

Climax Prote. A First Greek Reader. With Hints and Vocabulary. By E. C. MARCHANT, M.A. With 30 illustrations. 1s. 6d.

Greek Verbs. By J. S. BAIRD, T.C.D. 2s. 6d.

Analecta Græca Minora. With Notes and Dictionary. By the Rev. P. FROST, M.A. 2s.

Unseen Papers in Greek Prose and Verse. By T. COLLINS, M.A. 3s.

Notes on Greek Accents. By the Rt. Rev. A. BARRY, D.D. 1s.

Res Græcae. Being Aids to the study of the History, Geography, Archæology, and Literature of Ancient Athens. By E. P. COLERIDGE, M.A. With 5 Maps, 7 Plans, and 17 other illustrations. 5s.

Notabilia Quaedam. 1s.

LATIN AND GREEK—continued

Other Editions, Texts, &c.

Anthologia Latina. A Selection of Choice Latin Poetry, with Notes. By Rev. F. ST. JOHN THACKERAY, M.A. 16mo. 4s. 6d.

Anthologia Graeca. A Selection from the Greek Poets. By Rev. F. ST. JOHN THACKERAY, M.A. 16mo. 4s. 6d.

Aristophanis Comœdiae. Edited by H. A. HOLDEN, LL.D. Demy 8vo. 18s.

The Plays separately: *Acharnenses*, 2s.; *Equites*, 1s. 6d.; *Vespæ*, 2s.; *Pax*, 2s.; *Lysistrata*, et *Thesmophoriazusæ*, 4s.; *Aves*, 2s.; *Ranae*, 2s.; *Plutus*, 2s.

Catullus. Edited by J. P. POSTGATE, M.A., LITT.D. Fcap. 8vo. 3s.

Corpus Poetarum Latinorum. Edited by WALKER. 1 thick vol. 8vo. Cloth, 18s.

Mundus Alter et Idem. Edited as a School Reader by H. J. ANDERSON, M.A. 2s.

Horace. The Latin Text, with Conington's Translation on opposite pages. Pocket Edition. 4s. net; or in leather, 5s. net. Also in 2 vols., limp leather. The *Odes*, 2s. net; *Satires and Epistles*, 2s. 6d. net.

Livy. The first five Books. PRENDENVILLE's edition revised by J. H. FREESE, M.A. Books I., II., III., IV., V. 1s. 6d. each.

Lucan. The *Pharsalia*. By C. E. HASKINS, M.A. With an Introduction by W. E. HEITLAND, M.A. Demy 8vo. 14s.

Lucretius. Titi Lucreti Cari de rerum natura libri sex. Edited with Notes, Introduction, and Translation, by the late H. A. J. MUNRO. 3 vols. 8vo. Vols. I. and II. Introduction, Text, and Notes. 18s. Vol. III. Translation, 6s.

Ovid. The Metamorphoses. Book XIII. With Introduction and Notes by Prof. C. H. KEENE, M.A. 2s. 6d.

Ovid. The Metamorphoses. Book XIV. With Introduction and Notes by Prof. C. H. KEENE, M.A. 2s. 6d.

** Books XIII. and XIV. together. 3s. 6d.

Persius. A Persii Flacci Satirarum Liber. Edited, with Introduction and Notes by A. PRETOR, M.A. 3s. 6d. net.

Plato. The Proem to the Republic of Plato. (Book I. and Book II. chaps. 1-10.) Edited, with Introduction, Critical Notes, and Commentary, by Prof. T. G. TUCKER, LITT.D. 6s.

Petronii Cena Trimalchionis. Edited and Translated by W. D. LOWE, M.A. 7s. 6d. net.

Propertius. Sexti Properti Carmina recognovit J. P. POSTGATE, LITT.D. 4to. 3s. net.

Rutilius: Rutilii Claudii Namatiani de Reditu Suo Libri Duo. With Introduction and Notes by Prof. C. H. KEENE, M.A., and English Verse Translation by G. F. SAVAGE ARMSTRONG, M.A. 7s. 6d. net.

Theocritus. Edited with Introduction and Notes, by R. J. CHOLMELEY, M.A. Crown 8vo. 7s. 6d.

Theognis. The Elegies of Theognis, and other Elegies included in the Theognidean Sylloge. With Introduction, Commentary, and Appendices, by J. HUDSON WILLIAMS, M.A. Crown 8vo. 7s. 6d. net.

Thucydides. The History of the Peloponnesian War. With Notes and a Collation of the MSS. By the late R. SHILLETO, M.A. Book I. 8vo. 6s. 6d. Book II. 5s. 6d.

Bell's Classical Translations

Crown 8vo. Paper Covers. 1s. each

Æschylus: Translated by WALTER HEADLAM, LITT.D., and C. E. S. HEADLAM, M.A. Agamemnon—The Suppliants—Choëphoroe—Eumenides—Prometheus Bound—Persians—Seven against Thebes.

Aristophanes: The *Acharnians*. Translated by W. H. COVINGTON, B.A.

—The *Plutus*. Translated by M. T. QUINN, M.A.

Cæsar's Gallic War. Translated by W. A. McDEVITTE, B.A. 2 Vols. (Books I.-IV., and Books V.-VII.).

Cicero: *Friendship and Old Age*. Translated by G. H. WELLS, M.A.

—Orations. Translated by Prof. C. D. YONGE, M.A. 6 vols. *Catiline*, *Murena*, *Sulla* and *Archias* (in one vol.), *Manilian Law*, *Sextius*, *Milo*.

Demosthenes on the Crown. Translated by C. RANN KENNEDY.

Euripides. Translated by E. P. COLERIDGE, M.A. 14 vols. *Medea*—*Alcestis*—*Heracleidæ*—*Hippolytus*—*Suppliants*—*Troades*—*Ion*—*Andromache*—*Bacchæ*—*Hecuba*—*Hercules Furens*—*Orestes*—*Iphigenia in Tauris*.

Homer's Iliad. Books I. and II., Books III.-IV., Books V.-VI., Books VII.-VIII., Books IX.-X., Books XI.-XII., Books XIII.-XIV., Books XV. and XVI., Books XVII. and XVIII., Books XIX. and XX. Translated by E. H. BLAKENEY, M.A. 10 vols. —Book XXIV. Translated by E. H. BLAKENEY, M.A.

Horace. Translated by A. HAMILTON BRYCE, LL.D. 4 vols. *Odes*, Books I. and II.—*Odes*, Books III. and IV., *Carmen Seculare* and *Epodes*—*Satires*—*Epistles* and *Ars Poetica*.

Bell's Classical Translations—continued

Livy. Books I., II., III., IV. Translated by J. H. FREESE, M.A. With Maps. 4 vols.
 — Books V. and VI. Translated by E. S. WEYMOUTH, M.A. Lond. With Maps. 2 vols.
 — Book IX. Translated by FRANCIS STORR, M.A. With Map.
 — Books XXI., XXII., XXIII. Translated by J. BERNARD BAKER, M.A. 3 vols.
Lucan: The Pharsalia. Book I. Translated by FREDERICK CONWAY, M.A.
Ovid's Fasti. Translated by HENRY T. RILEY, M.A. 3 vols. Books I. and II.—Books III. and IV.—Books V. and VI.
Ovid's Tristia. Translated by HENRY T. RILEY, M.A.
Plato: Apology of Socrates and Crito (1 vol.), Phædo, and Protagoras. Translated by H. CARY, M.A. 3 vols.

Plautus: Trinummus. Aulularia, Menæchmi, Rudens, and Captivi. Translated by HENRY T. RILEY, M.A. 4 vols.
Sophocles. Translated by E. P. COLERIDGE, M.A. 7 vols. Antigone—Philoctetes—Edipus Rex—Edipus Coloneus—Electra—Trachiniae—Ajax.
Thucydides. Book VI. Translated by E. C. MARCHANT, M.A.
 — Book VII. Translated by E. C. MARCHANT, M.A.
Virgil, Translated by A. HAMILTON BRUCE, LL.D. 6 vols. Bucolics—Georgics—Æneid, 1-3—Æneid, 4-6—Æneid, 7-9—Æneid, 10-12.
Xenophon's Anabasis. Translated by the Rev. J. S. WATSON, M.A. With Map. 3 vols. Books I. and II.—Books III., IV., and V.—Books VI. and VII.
 — Hellenics. Books I. and II. Translated by the Rev. H. DALE, M.A.

* * *For other Translations from the Classics, see the Catalogue of Bohn's Libraries, which will be forwarded on application*

MATHEMATICS

Full Catalogue of Mathematical Books post free on application

Cambridge Mathematical Series

Public School Arithmetic. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 3s. 6d. Or with Answers, 4s. 6d.
The Student's Arithmetic. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. With or without Answers. 2s. 6d.
New School Arithmetic. By C. PENDLEBURY, M.A., and F. E. ROBINSON, M.A. With or without Answers. 4s. 6d. In Two Parts. 2s. 6d. each.
 Key to Part II., 8s. 6d. net.
New School Examples in a separate volume, 3s. Or in Two Parts, 1s. 6d. and 2s.
Arithmetic, with 8000 Examples. By C. PENDLEBURY, M.A. 4s. 6d. In Two Parts. 2s. 6d. each. Key to Part II., 7s. 6d. net.
Examples in Arithmetic. Extracted from the above. 3s. Or in Two Parts 1s. 6d. and 2s.
Commercial Arithmetic. By C. PENDLEBURY, M.A., and W. S. BEARD, F.R.G.S. 2s. 6d. Part I. separately, 1s. Part II., 1s. 6d.
Arithmetic for Indian Schools. By C. PENDLEBURY, M.A., and T. S. TAIT. 3s.
Examples in Arithmetic By C. O. TUCKEY M.A. With or without Answers. 3s.
Junior Practical Mathematics. By W. J. STAINER, B.A. 2s., with Answers, 2s. 6d. Part I., 1s. 4d., with Answers, 1s. 6d. Part II., 1s. 4d.

Elementary Algebra. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. New and Revised Edition. 4s. 6d. Also Part I., 2s. 6d., or with Answers, 3s. Part II., with or without Answers, 2s. 6d. Key 10s. net; or in 2 Parts, 5s. net each.
A Shorter Algebra. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 2s. 6d.
Examples in Algebra. Extracted from above. With or without Answers, 3s. Or in Two Parts. Part I., 1s. 6d., or with Answers, 2s. Part II., with or without Answers, 2s.
Examples in Algebra. By C. O. TUCKEY, M.A. With or without Answers. 3s.
 — **Supplementary Examples.** 6d. net.
Elementary Algebra for use in Indian Schools. By J. T. HATHORNTHWAITHE, M.A. 2s.
Choice and Chance. By W. A. WHITWORTH, M.A. 7s. 6d.
 — **DCC Exercises,** including Hints for the Solution of all the Questions in "Choice and Chance." 6s.
Euclid. Books I.—VI., and part of Book XI. By HORACE DEIGHTON, M.A. 4s. 6d., or in separate books.
Introduction to Euclid. By HORACE DEIGHTON, M.A., and O. EMTAGE, B.A. 1s. 6d.

Cambridge Mathematical Series - *continued*

- Euclid.** Exercises on Euclid and in Modern Geometry. By J. McDOWELL, M.A. 6s.
- Elementary Graphs.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 6d. net.
- A New Geometry.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. Crown 8vo. 2s. 6d. Also Books I.-III. separately, 1s. 6d.
- Elementary Geometry.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 4s. 6d. Or in Parts. Answers, 6d. net. Key, 6s. net.
- Examples in Practical Geometry and Mensuration.** By J. W. MARSHALL, M.A., and C. O. TUCKEY, M.A. 1s. 6d.
- Geometry for Schools.** By W. G. BORCHARDT, M.A., and the Rev. A. D. PERROTT, M.A. Complete, 4s. 6d.; also Vol. I., 1s.; Vol. II., 1s. 6d.; Vol. III., 1s.; Vols. I.-III., 2s. 6d.; Vol. IV., 1s.; Vols. I.-IV., 3s.; Vol. V., 1s.; Vol. VI., 1s. 6d.; Vols. I.-V., 3s. 6d.; Vols. IV.-V., 2s.
- A New Trigonometry for Schools.** By W. G. BORCHARDT, M.A., and Rev. A. D. PERROTT, M.A. 4s. 6d. Or Two Parts, 2s. 6d. each. Key, 10s. net; or 2 Parts, 5s. net each.
- Numerical Trigonometry.** By W. G. BORCHARDT, M.A., and the Rev. A. D. PERROTT, M.A. 2s. 6d.
- Junior Trigonometry.** By W. G. BORCHARDT, M.A., and the Rev. A. D. PERROTT, M.A. 3s. 6d.
- Elementary Trigonometry.** By CHARLES PENDLEBURY, M.A., F.R.A.S. 4s. 6d.
- Short Course of Elementary Plane Trigonometry.** By CHARLES PENDLEBURY. 2s. 6d.
- Elementary Trigonometry.** By J. M. DYER, M.A., and the Rev. R. H. WHITCOMBE, M.A. 4s. 6d.
- Algebraic Geometry.** By W. M. BAKER, M.A. 6s. Part I. (The Straight Line and Circle), 2s. 6d. Key, 7s. 6d. net.
- Practical Solid Geometry.** By the Rev. PERCY UNWIN, M.A. 4s. 6d.
- Analytical Geometry for Beginners.** By Rev. T. G. VVYVAN, M.A. Part I. The Straight Line and Circle. 2s. 6d.
- Conic Sections, treated Geometrically.** By W. H. BESANT, Sc.D., F.R.S. 4s. 6d. Key, 5s. net.
- Elementary Conics, being the first 8 chapters of the above.** 2s. 6d.
- Conics, the Elementary Geometry of.** By Rev. C. TAYLOR, D.D. 5s.
- Calculus for Beginners.** By W. M. BAKER, M.A. 3s.
- Differential Calculus for Beginners.** By A. LODGE, M.A. With Introduction by Sir OLIVER LODGE. 4s. 6d.
- Integral Calculus for Beginners.** By A. LODGE, M.A. 4s. 6d.
- Roulettes and Glisettes.** By W. H. BESANT, Sc.D., F.R.S. 5s.
- Geometrical Optics.** An Elementary Treatise by W. S. ALDIS, M.A. 4s.
- Practical Mathematics.** By H. A. STERN, M.A., and W. H. TOPHAM, 6s.; or Part I., 2s. 6d.; Part II., 3s. 6d.
- Elementary Hydrostatics.** By W. H. BESANT, Sc.D. 4s. 6d. Solutions, 5s. net.
- Elements of Hydrostatics.** By C. M. JESSOP, M.A., and G. W. CAUNT, M.A. 2s. 6d.
- Elementary Mechanics.** By C. M. JESSOP, M.A., and J. H. HAVELOCK, M.A., D.Sc. 4s. 6d.
- Experimental Mechanics for Schools.** By FRED CHARLES, M.A., and W. H. HEWITT, B.A., B.Sc. 3s. 6d.
- The Student's Dynamics.** Comprising Statics and Kinetics. By G. M. MINCHIN, M.A., F.R.S. 3s. 6d.
- Elementary Dynamics.** By W. M. BAKER, M.A. New Revised Edition, 4s. 6d. Key, 10s. 6d. net.
- Elementary Dynamics.** By W. GARNETT, M.A., D.C.L. 6s.
- Dynamics, A Treatise on.** By W. H. BESANT, Sc.D., F.R.S. 10s. 6d.
- Heat, An Elementary Treatise on.** By W. GARNETT, M.A., D.C.L. 4s. 6d.
- Elementary Physics, Examples and Examination Papers in.** By W. GALLATLY, M.A. 4s.
- Mechanics, A Collection of Problems in Elementary.** By W. WALTON, M.A. 6s.

Uniform Volume

Geometrical Drawing. For Army and other Examinations. By R. HARRIS. 3s. 6d.

The Junior Cambridge Mathematical Series.

- A Junior Arithmetic.** By C. PENDLEBURY, M.A., and F. E. ROBINSON, M.A. 1s. 6d. With Answers, 2s.
- Examples from a Junior Arithmetic.** Extracted from the above. 1s. With Answers, 1s. 6d.
- A First Algebra.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 1s. 6d.; or with Answers, 2s.
- A First Geometry.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. With or without Answers. 1s. 6d.
- Elementary Mensuration.** By W. M. BAKER, M.A., and A. A. BOURNE, M.A. 1s. 6d.

Other Mathematical Works

The Mathematical Gazette. Edited by W. J. GREENSTREET, M.A. (Jan., March, May, July, Oct. and Dec.) 1s. 6d. net.

The Teaching of Elementary Mathematics, being the Reports of the Committee of the Mathematical Association. 6d. net.

The Teaching of Elementary Algebra and Numerical Trigonometry. Being the Report of the Mathl. Assoc. Committee, 1911. 3d. net.

A New Shilling Arithmetic. By C. PENDLEBURY, M.A., and F. E. ROBINSON, M.A. 1s.; or with Answers, 1s. 4d.

A Shilling Arithmetic. By CHARLES PENDLEBURY, M.A., and W. S. BEARD, F.R.G.S. 1s. With Answers, 1s. 4d.

Elementary Arithmetic. By CHARLES PENDLEBURY, M.A. With or without Answers. 1s. 6d.

A Preparatory Arithmetic. By CHARLES PENDLEBURY, M.A. With or without Answers. 1s. 6d.

Problem Papers for Preparatory Schools (Arithmetic). By T. COOPER SMITH, B.A. 1s. 6d.

Bell's Indoor and Outdoor Experimental Arithmetic. By H. H. GOODACRE, F.R.G.S. Parts I.-III., paper, 3d. each, cloth, 4d. each; Parts IV. and V., paper, 4d. each, cloth, 6d. each. Teachers' Book, 3s. 6d. net.

Graduated Arithmetic, for Junior and Private Schools. By the same Authors. Parts I., II., and III., 3d. each; Parts IV., V., and VI., 4d. each; Part VII., 6d. Answers to Parts I. and II., 4d. net; Parts III.-VII., 4d. net each.

Arithmetic for the Standards (Scheme B). Standard I., sewed, 2d., cloth, 3d.; II., III., IV., and V., sewed, 3d. each, cloth, 4d. each; VI. and VII., sewed, 4d. each, cloth, 6d. each. Answers to each Standard, 4d. net each.

Exercises and Examination Papers in Arithmetic, Logarithms and Mensuration. By C. PENDLEBURY, M.A. 2s. 6d. New Edition.

Test Cards in Arithmetic (Scheme B) By C. PENDLEBURY, M.A. For Standards II, III., IV., V., VI. and VII. 1s. net each.

Public School Examination Papers in Mathematics. Compiled by P. A. OPENSHAW, B.A. 1s. 6d.

Bell's New Practical Arithmetic. By W. J. STAINER, M.A. 1st, 2nd, 3rd, 4th, 5th and 6th Years, paper, 3d. each, cloth, 4d. each; 7th Year, paper, 4d., cloth, 6d. Teachers' Books, 8d. net each Year.

Bell's New Practical Arithmetic Test Cards, for the 2nd, 3rd, 4th, 5th, 6th, and 7th years. 1s. 3d. net each.

Graduated Exercises in Addition (Simple and Compound). By W. S. BEARD. 1s.

Algebra for Elementary Schools. By W. M. BAKER, M.A., and A. A. BOURNE, M.A. Three stages, 6d. each. Cloth, 8d. each. Answers, 4s. net each.

A First Year's Course in Geometry and Physics. By ERNEST YOUNG, M.A., B.Sc. 2s. 6d. Parts I. and II. 1s. 6d.; or Part III. 1s.

Trigonometry, Examination Papers in. By G. H. WARD, M.A. 2s. 6d. Key, 5s. net.

Euclid, The Elements of. The Enunciations and Figures. By the late J. BRASSE, D.D. 1s. Without the Figures, 6d.

Hydromechanics. By W. H. BESANT, Sc.D., and A. S. RAMSEY, M.A. Part I., Hydrostatics. 7s. 6d. net. Part II., Hydrodynamics. By A. S. RAMSEY, M.A. 10s. 6d. net.

Hydrodynamics and Sound, An Elementary Treatise on. By A. B. BASSET, M.A., F.R.S. 8s.

The Geometry of Surfaces. By A. B. BASSET, M.A., F.R.S. 10s. 6d.

Elementary Treatise on Cubic and Quartic Curves. By A. B. BASSET, M.A., F.R.S. 10s. 6d.

Analytical Geometry. By Rev. T. G. VVYAN, M.A. 4s. 6d.

Book-keeping

Book-keeping by Double Entry, Theoretical, Practical, and for Examination Purposes. By J. T. MEDHURST, A.K.C., F.S.S. 1s. 6d.

Book-keeping, Examination Papers in. Compiled by JOHN T. MEDHURST, A.K.C., F.S.S. 3s. Key, 2s. 6d. net.

Book-keeping, Graduated Exercises and Examination Papers in. Compiled by P. MURRAY, F.S.S.S., F.Sc.S. (Lond.). 2s. 6d.

Text-Book of the Principles and Practice of Book-keeping and Estate-Office Work. By Prof. A. W. THOMSON, B.Sc. 5s.

ENGLISH

Full Catalogue of English Books post free on application.

- Mason's New English Grammars.** Revised by A. J. ASHTON, M.A.
 A Junior English Grammar. 1s.
 Intermediate English Grammar. 2s.
 Senior English Grammar. 3s. 6d.
- Works by C. P. MASON, B.A., F.C.P.
First Notions of Grammar for Young Learners. 1s.
First Steps in English Grammar, for Junior Classes. 1s.
Outlines of English Grammar, for the Use of Junior Classes. 2s.
English Grammar; including the principles of Grammatical Analysis. 3s. 6d.
A Shorter English Grammar. 3s. 6d.
Practice and Help in the Analysis of Sentences. 2s.
English Grammar Practice. 1s.
- Elementary English Grammar through Composition.** By J. D. ROSE, M.A. 1s.
Advanced English Grammar through Composition. By JOHN D. ROSE, M.A. 2s. 6d.
Aids to the Writing of English Composition. By F. W. BOWSER, B.A. 1s. net.
Preparatory English Grammar. By W. BENSON, M.A. New Edition. 1s. net.
Rudiments of English Grammar and Analysis. By ERNEST ADAMS, Ph.D. 1s.
Examples for Analysis in Verse and Prose. Selected by F. EDWARDS. 1s.
The Paraphrase of Poetry. By EDMUND CANDLER. 1s.
Essays and Essay-Writing, for Public Examinations. By A. W. READY, B.A. 3s. 6d.
Précis and Précis-Writing. By A. W. READY, B.A. 3s. 6d. Or without Key, 2s. 6d.
Matriculation Précis. By S. E. WINBOLT, M.A. 1s. net. Key, 6d. net.
Elements of the English Language. By ERNEST ADAMS, Ph.D. Revised by J. F. DAVIS, M.A., D.Lit. 4s. 6d.
History of the English Language. By Prof. T. R. LOUNSBURY. 5s. net.
The Teaching of English Literature in the Secondary School. By R. S. BATE, M.A. 2s. 6d. net.
An Outline History of English Literature. By W. H. HUDSON. 2s. 6d. net.
Representative Extracts from English Literature. By W. H. HUDSON. 2s. 6d. net.
- Ten Brink's Early English Literature**
 3 vols. 3s. 6d. each.
- Introduction to English Literature.**
 By HENRY S. PANCOAST. 5s. net.
- A First View of English Literature.** By HENRY S. PANCOAST and PERCY VAN DYKE SHELLY. Crown 8vo. 5s. net.
- Introduction to American Literature.**
 By H. S. PANCOAST. 4s. 6d. net.
- The Foreign Debt of English Literature.**
 By T. G. TUCKER, Litt.D. Post 8vo. 6s. net.
- Handbooks of English Literature.**
 Edited by Prof. HALES. 3s. 6d. net each.
- The Age of Alfred.** (660-1154). By F. J. SNELL, M.A.
The Age of Chaucer. (1346-1400.) By F. J. SNELL, M.A.
The Age of Transition. (1400-1580.) By F. J. SNELL, M.A. 2 vols.
The Age of Shakespeare. (1579-1631.) By THOMAS SECCOMBE and J. W. ALLEN. 2 vols. Vol. I. Poetry and Prose. Vol. II. Drama.
The Age of Milton. (1632-1660.) By the Rev. J. H. B. MASTERMAN, M.A., with Introduction, etc., by J. BASS MULLINGER, M.A.
The Age of Dryden. (1660-1700.) By R. GARNETT, LL.D., C.B.
The Age of Pope. (1700-1744.) By JOHN DENNIS.
The Age of Johnson. (1744-1798.) By THOMAS SECCOMBE.
The Age of Wordsworth. (1793-1832.) By Prof. C. H. HERFORD, Litt.D.
The Age of Tennyson. (1830-1870.) By Prof. HUGH WALKER.
- Notes on Shakespeare's Plays.** By T. DUFF BARNETT, B.A. 1s. each.
 Midsummer Night's Dream.—Julius Cæsar.—The Tempest.—Macbeth.—Henry V.—Hamlet.—Merchant of Venice.—King Richard II.—King John.—King Richard III.—King Lear.—Coriolanus. Twelfth Night.—As You Like It.—Much Ado About Nothing.
- Principles of English Verse.** By C. M. LEWIS. 5s. net.
- Introduction to Poetry.** By RAYMOND M. ALDEN. 5s.
- General Intelligence Papers.** With Exercises in English Composition. By G. BLUNT. 2s. 6d.

Bell's English Texts for Secondary Schools

Edited by A. GUTHKELCH, M.A.

Browning's The Pied Piper, and other Poems. Edited by A. GUTHKELCH. 8d.

Fairy Poetry. Selected and edited by R. S. BATE, M.A. 1s.

Hawthorne's Wonder Book and Tanglewood Tales. Selected and Edited by H. HAMPSHIRE, M.A. 1s.

Kingsley's Heroes. Edited by L. H. POND, B.A. With 2 maps. 1s.

Lamb's Tales from Shakespeare. Selected and edited by R. S. BATE, M.A. 10d.

Lamb's Adventures of Ulysses. Selections. Edited by A. C. DUNSTAN, PH.D. 8d.

Stories of King Arthur, from Malory and Tennyson. Edited by R. S. BATE, M.A. 1s.

The Story of Enid, from Tennyson and The Mabinogion. By H. A. TREBLE, M.A. 10d.

Scott's A Legend of Montrose. Abridged and edited by F. C. LUCKHURST. 1s.

Charles Reade's The Cloister and the Hearth. Abridged and edited by the Rev. A. E. HALL, B.A. 1s.

Coleridge's The Ancient Mariner; and Selected Old English Ballads. Edited by A. GUTHKELCH, M.A. 1s.

Hakluyt's Voyages. A Selection edited by the Rev. A. E. HALL, B.A. 1s.

Selections from Boswell's Life of Johnson. Edited by E. A. J. MARSH. 1s.

Selections from Ruskin. Edited by H. HAMPSHIRE, M.A. 1s.

Lockhart's Life of Scott. Selections edited by A. BARTER, LL.A. 1s.

Charles Lamb's Selected Essays and Letters. Edited by A. GUTHKELCH, M.A. With Map of London. 1s. 4d.

Selections from Carlyle. Edited by ELIZABETH LEE. 1s.

English Odes. Edited by E. A. J. MARSH, M.A. 1s.

Bell's English Classics

Bacon's Essays. (Selected.) Edited by A. E. ROBERTS, M.A. 1s.

Browning Selections from. Edited by F. RYLAND, M.A. 1s. 6d.

— **Strafford.** Edited by E. H. HICKEY. 1s. 6d.

Burke's Conciliation with America. By Prof. J. MORRISON. 1s. 6d.

Burke's Letters on a Regicide Peace. I. and II. Edited by H. G. KEENE, M.A., C.I.E. 1s. 6d.

Byron's Siege of Corinth. Edited by P. HORDERN. 1s.

Byron's Childe Harold. Edited by H. G. KEENE, M.A., C.I.E. 2s. Also Cantos I. and II., sewed, 1s. Cantos III. and IV., sewed, 1s.

Carlyle's Hero as Man of Letters. Edited by MARK HUNTER, M.A. 1s. 6d.

— **Hero as Divinity.** By MARK HUNTER, M.A. 1s. 6d.

Chaucer's Minor Poems, Selections from. Edited by J. B. BILDERBECK, M.A. 1s. 6d.

De Quincey's Revolt of the Tartars and the English Mail-Coach. Edited by CECIL M. BARROW, M.A., and MARK HUNTER, M.A. 2s.

* * Revolt of the Tartars, separately. 1s.

— **Opium Eater.** Edited by MARK HUNTER, M.A. 2s. 6d.

Goldsmith's Good-Natured Man and She Stoops to Conquer. Edited by K. DEIGHTON. Each 1s.

* * The two plays together, 1s. 6d.

— **Traveller and Deserted Village.** Edited by the Rev. A. E. WOODWARD, M.A. Cloth, 1s. 6d., or separately, sewed, 10d. each.

Irving's Sketch Book. Edited by R. G. OXENHAN, M.A. Sewed, 1s. 6d.

Johnson's Life of Addison. Edited by F. RYLAND, M.A. 1s.

— **Life of Pope.** Edited by F. RYLAND, M.A. 2s.

* * The Lives of Swift and Pope, together, sewed, 2s. 6d.

Johnson's Life of Milton. Edited by F. RYLAND, M.A. 1s. 6d.

— **Life of Dryden.** Edited by F. RYLAND, M.A. 1s. 6d.

* * The Lives of Milton and Dryden, together, sewed, 2s. 6d.

— **Life of Swift.** Edited by F. RYLAND, M.A. 1s.

— **Lives of Prior and Congreve.** Edited by F. RYLAND, M.A. 1s.

Kingsley's Heroes. Edited by A. E. ROBERTS, M.A. Illus. 1s. 6d. Sewed, 1s.

Lamb's Essays. Selected and Edited by K. DEIGHTON. 1s. 6d.

Longfellow, Selections from, including Evangeline. Edited by M. T. QUINN, M.A. 1s. 6d.

* * Evangeline, separately, sewed, 10d.

Bell's English Classics —continued

Macaulay's Lays of Ancient Rome. Edited by P. HORDERN. 1s. 6d.

— **Essay on Clive.** Edited by CECIL BARROW. 1s. 6d.

— **War of the Spanish Succession.** Edited by A. W. READY. 1s. 6d.

Massinger's A New Way to Pay Old Debts. Edited by K. DEIGHTON. 1s. 6d.

Milton's Paradise Lost. Books III. and IV. Edited by R. G. OXENHAM, M.A. 1s. ; or separately, sewed, 10d. each.

Milton's Paradise Regained Edited by K. DEIGHTON. 1s

Pope's Essay on Man. Edited by F. RYLAND, M.A. 1s.

Pope, Selections from. Edited by K. DEIGHTON. 1s. 6d.

Scott's Lady of the Lake. Edited by the Rev. A. E. WOODWARD, M.A. 2s. 6d. The Six Cantos separately, sewed, 6d. each.

Shakespeare's Julius Cæsar. Edited by T. DUFF BARNETT, B.A. (Lond.). 1s. 6d.

— **Merchant of Venice.** Edited by T. DUFF BARNETT, B.A. (Lond.). 1s. 6d.

— **Tempest.** Edited by T. DUFF BARNETT, B.A. (Lond.). 1s. 6d.

Wordsworth's Excursion. Book I. Edited by M. T. QUINN, M.A. Sewed, 1s.

Bell's Sixpenny English Texts

Bound in limp cloth, 6d. each.

***Poems by John Milton.**

***Spenser's 'Faerie Queene.'** Book I.

***Poems by Tennyson.**
Selections from Byron.

***Macaulay's 'History of England.'**
Chapter III.

Gibbon's 'Decline and Fall.' Chapters I. to III.

Selections from Pope.

Poems by Gray and Cowper.

Plutarch's Lives of Cæsar and Cicero.

***English Elegiacs.**

***Selections from Chaucer.**

Kingsley's Heroes.

The volumes marked with an asterisk are supplied interleaved and bound in cloth boards, 1s.

English Readings. 16mo.

Burke: Selections. Edited by BLISS PERRY. 2s. 6d.

Byron: Selections. Edited by F. I. CARPENTER. 2s. 6d.

Coleridge: Prose Selections. Edited by HENRY A. BEERS. 2s.

Dryden: Essays on the Drama, Edited by WILLIAM STRUNK. 2s.

Johnson: Prose Selections. Edited by C. G. OSGOOD. 3s.

Milton: Minor English Poems. Edited by MARTIN W. SAMPSON. 2s. 6d.

Swift: Prose Selections. Edited by FREDERICK C. PRESCOTT. 2s. 6d.

Tennyson: The Princess. Edited by L. A. SHERMAN. 2s.

Thackeray: English Humourists. Edited by WILLIAM LYON PHELPS. 2s. 6d.

Readers

The Story of Peter Pan (as told in "The Peter Pan Picture Book.") With 16 Illustrations and Songs from the Play in Tonic Solfa and Old Notation. 9d.

Alice in Wonderland. By LEWIS CARROLL. Illustrated by ALICE B. WOODWARD. 9d.

Thrift. A Common Sense Book for Girls. By F. FOOT. 8d. net.

York Readers. A new series of Literary Readers, with Coloured and other Illustrations.

Primer I. 3d. Primer II. 4d.

York Readers—continued.

Infant Reader. 6d.

Introductory Reader. 8d.

Reader, Book I., 9d. Book II., 10d. Book III., 1s. Book IV., 1s. 3d. Book V., 1s. 6d.

York Poetry Books. 3 Books. Paper covers, 6d. each; cloth, 8d. each.

Bell's Poetry Books. In Seven Parts. Price 3d. each Part, paper covers; or 4d. cloth covers.

Poetry for Upper Classes. Selected by E. A. HELPS. 1s. 6d.

Readers—continued

Books for Young Readers. Illustrated. 6d. each.

Æsop's Fables. | Tot and the Cat, etc.
The Old Boat-House, etc.
The Cat and the Hen, etc.
The Two Parrots. | The Lost Pigs.
The Story of Three Monkeys.
The Story of a Cat.
Queen Bee and Busy Bee. | Gull's Crag.

Bell's Continuous Readers. Bound in Cloth. 9d. each.

Suitable for Standard III.

The Story of Peter Pan.
The Adventures of a Donkey
The Life of Columbus.
The Three Midshipmen.

Suitable for Standard IV.

Alice in Wonderland.
The Water Babies.
The Parables from Nature.
Uncle Tom's Cabin.
Robinson Crusoe.

Suitable for Standard V.

Tom Brown's Schooldays.
The Last of the Mohicans.
Feats on the Fiord.
The Little Duke.
Hereward the Wake.

Suitable for Standards VI. and VII.

The Last Days of Pompeii.
Oliver Twist.
The Tale of Two Cities.
Ivanhoe.
Lamb's Tales from Shakespeare.

Bell's Reading Books and Literature Readers. Strongly bound in Cloth. Illustrated. 1s. each.

Suitable for Standard III.

Adventures of a Donkey.
Great Deeds in English History.
Grimm's German Tales.
Andersen's Danish Tales.
Great Englishmen.
Great Irishmen.
Life of Columbus.
The Three Midshipmen.

Suitable for Standard IV.

Great Scotsmen.
Uncle Tom's Cabin.
Swiss Family Robinson.
Great Englishwomen.
Children of the New Forest.
Settlers in Canada.
Edgeworth's Tales.
The Water Babies.
Parables from Nature.

Suitable for Standard V.

Lyrical Poetry.
The Story of Little Nell.
Masterman Ready.
Gulliver's Travels.
Robinson Crusoe.
Poor Jack.
Arabian Nights.

Bell's Reading Books, &c.—continued.

The Last of the Mohicans.
Feats on the Fiord.
The Little Duke.

Suitable for Standards VI. and VII.

The Talisman. | Ivanhoe.
Woodstock. | Oliver Twist.
The Vicar of Wakefield.
Lamb's Tales from Shakespeare.
Southey's Life of Nelson.
Sir Roger de Coverley.
Deeds that Won the Empire.
Six to Sixteen. | Fights for the Flag.

Bell's Supplementary Readers. Crown 8vo. Illustrated. Limp Cloth. 6d. net each.

Suitable for Standards III. and IV.

Anderson's Danish Tales.
Great Deeds in English History
Grimm's Tales.
Adventures of a Donkey.
Great Englishmen.
Life of Columbus.

Suitable for Standards IV. and V.

Parables from Nature.
Uncle Tom's Cabin.
Swiss Family Robinson.
Great Englishwoman.
Settlers in Canada.

Suitable for Standards V. and VI

Masterman Ready.
Robinson Crusoe.
Children of the New Forest.

Suitable for Standards VI. and VII.

The Talisman. | Ivanhoe.
Oliver Twist. | Woodstock.

Bell's Geographical Readers. By M. J. BARRINGTON-WARD, M.A.

The Child's Geography. Illustrated. 6d.
The Round World. (Standard II.) 1s.
About England. (Stand. III.) Illus. 1s. 4d.

The Care of Babies. A Reading Book for Girls' Schools. Illustrated. Cloth, 1s.

Bell's History Readers on the Concentric Method. Fully Illustrated.

First Lessons in English History. 10d.
A Junior History of England. 1s. 6d.
A Senior History of England. 2s.

Abbey History Readers. Revised by the

Rt. Rev. F. A. GASQUET, D.D. Illustrated.
Early English History (to 1066). 1s.
Stories from English History (1066-1485). 1s. 3d.
The Tudor Period (1485-1603). 1s. 3d.
The Stuart Period (1603-1714). 1s. 6d.
The Hanoverian Period (1714-1837). 1s. 6d.

Bell's History Readers. Illustrated.

Early English History (to 1066). 1s.
Stories from English History (1066-1485). 1s. 3d.
The Tudor Period (1485-1603). 1s. 3d.
The Stuart Period (1603-1714). 1s. 6d.
The Hanoverian Period (1714-1837). 1s. 6d.

MODERN LANGUAGES

French and German Class Books

Bell's French Course. By R. P. ATHERTON, M.A. Illustrated. 2 Parts. 1s. 6d. each. Key to the Exercises, Part I., 6d. net; Part II., 1s. net.

Bell's First French Reader. By R. P. ATHERTON, M.A. Illustrated. 1s.

The Direct Method of Teaching French. By D. MACKAY, M.A., and F. J. CURTIS, Ph.D.

First French Book. 1s. net.

Second French Book. 1s. 6d. net.

Teacher's Handbook. 1s. net.

Subject Wall Picture (Coloured). 7s. 6d. net.

Bell's French Picture-Cards. Edited by H. N. ADAIR, M.A. Two Sets of Sixteen Cards. Printed in Colours, with questionnaire on the back of each. 1s. 3d. net each.

Bell's Illustrated French Readers. Pott 8vo. Fully Illustrated.

** Full List on application.

French Historical Reader. By H. N. ADAIR, M.A. New Composition Supplement, 2s.; or without Supplement, 1s. 6d. Supplement separately, 6d. net.

Simple French Stories. By MARC CEPPI. Fcap. 8vo. With or without Vocabulary and Notes. 1s.

Contes Français. Edited, with Introduction and Notes, by MARC CEPPI. With or without Vocabulary, 1s. 6d. Handbook of Exercises and Questionnaires, 6d.

Tales from Molière. By MARC CEPPI. Fcap. 8vo. With Vocabulary and Notes, 2s. Text only, 1s. 6d.

A French Dramatic Reader. By MARK CEPPI. With Notes. Fcap. 8vo. 2s.

Contes d'Hier et d'Aujourd'hui. First Series. By J. S. NORMAN, M.A., and CHARLES ROBERT-DUMAS. Illustrated. 1s. 6d. Second Series. 2s.

Le Français de France. By Madame VALETTE VERNET. With Illustrations. 2s.

Grammaire Pratique. Pour "Le Français de France." By Madame VALETTE VERNET. 10d.

Stories and Anecdotes for Translation into French. By CARL HEATH. 1s.

French Composition. By M. KENNEDY, M.A. Cloth, 8d.

Vocabulaire Français. French Vocabularies for Repetition. By J. P. R. MARI-CHAL. 1s. 6d.

Gasc's French Course

First French Book. 1s.

Second French Book. 1s. 6d.

Key to First and Second French Books. 1s. 6d. net.

French Fables for Beginners. 1s.

Histoires Amusantes et Instructives. 1s.

Practical Guide to Modern French Conversation. 1s.

French Poetry for the Young. With Notes. 1s.

Materials for French Prose Composition. 3s. Key, 2s. net.

Prosateurs Contemporains. 2s.

Le Petit Compagnon; a French Talk-Book for Little Children. 1s.

By the Rev. A. C. Clapin

French Grammar for Public Schools. 2s. 6d. Key, 3s. 6d. net.

A French Primer. 1s.

Primer of French Philology. 1s.

English Passages for Translation into French. 2s. 6d. Key, 4s. net.

A German Grammar for Public Schools. 2s. 6d.

A Spanish Primer. 1s.

Bell's First German Course. By L. B. T. CHAFFEY, M.A. 2s.

Bell's First German Reader. By L. B. T. CHAFFEY, M.A. Illustrated. 2s.

German Historical Reader. By J. E. MALLIN, M.A. 2s.

Buddenbrook: Ein Schultag eines Realuntersekundaners. Edited by J. E. MALLIN, M.A. Illustrated. 2s. 6d.

Materials for German Prose Composition. By Dr. C. A. BUCHHEIM. 4s. 6d. A Key to Parts I. and II., 3s. net. Parts III. and IV., 4s. net.

First Book of German Prose. Being Parts I. and II. of the above, with Vocabulary. 1s. 6d.

Kurzer Leitfaden der Deutschen Dichtung. By A. E. COP. 2s. 6d.

Gasc's French Dictionaries

FRENCH ENGLISH AND ENGLISH-FRENCH DICTIONARY. New Edition with Supplement of New Words. Large 8vo 12s. 6d.

CONCISE FRENCH DICTIONARY. Medium 16mo. 3s. 6d. Or in Two Parts 2s. each.

POCKET DICTIONARY OF THE FRENCH AND ENGLISH LANGUAGES. 16mo. 2s. 6d.

LITTLE GEM FRENCH DICTIONARY. Narrow 8vo. 1s. net. Limp Leather, 2s. net.

French and German Annotated Editions

Bell's French Plays. (Based on Gombert's French Drama.) Edited by MARC CEPPI. Paper, 6d.; cloth, 8d.

First Volumes:

Molière. Le Tartuffe. — L'Avarc. — Le Misanthrope.

Racine. Les Plaideurs.

Voltaire. Zaïre.

Corneille. Le Cid.

Gombert's French Drama. Re-edited, with Notes, by F. E. A. GASC. Sewed, 6d. each.

Molière. Le Misanthrope. — L'Avarc. — Le Bourgeois Gentilhomme. — Le Tartuffe. — Le Malade Imaginaire. — Les Femmes Savantes. — Les Fourberies de Scapin. — Les Précieuses Ridicules. — L'Ecole des Femmes. — L'Ecole des Maris. — Le Médecin Malgré Lui.

Racine. La Thébàïde. — Les Plaideurs. — Iphigénie. — Britannicus. — Phèdre. — Esther. — Athalie.

Corneille. Le Cid. — Horace. — Cinna. — Polyeucte.

Voltaire. Zaïre.

Fénelon. Aventures de Télémaque. By C. J. DELILLE. 2s. 6d.

La Fontaine. Select Fables. By F. E. A. GASC. 1s. 6d.

Lamartine. Le Tailleur de Pierres de Saint-Point. By J. BOÏELLE, R.-ès-L. 1s. 6d.

Saintine. Picciola. By Dr. DUBUC. 1s. 6d.

Voltaire. Charles XII. By L. DIREY. 1s. 6d.

German Ballads from Uhland, Goethe, and Schiller. By C. L. BIELEFELD. 1s. 6d.

Goethe. Hermann und Dorothea. By E. BELL, M.A., and E. WÜLFEL. 1s. 6d.

Lessing. Minna von Barnhelm. By Prof. A. B. NICHOLS. 2s. 6d.

Schiller. Wallenstein. By Dr. BUCHHEIM. 5s. Or the Lager and Piccolomini, 2s. 6d. Wallenstein's Tod, 2s. 6d.

— Maid of Orleans. By Dr. W. WAGNER. 1s. 6d.

— Maria Stuart. By V. KASTNER. 1s. 6d.

Bell's Modern Translations

A Series of Translations from Modern Languages, with Memoirs, Introductions, etc. Crown 8vo. 1s. each.

Dante. Inferno. Translated by the Rev. H. F. CARY, M.A.

— Purgatorio. Translated by the Rev. H. F. CARY, M.A.

— Paradiso. Translated by the Rev. H. F. CARY, M.A.

Goethe. Egmont. Translated by ANNA SWANWICK.

— Iphigenia in Tauris. Translated by ANNA SWANWICK.

— Goetz von Berlichingen. Translated by Sir WALTER SCOTT.

— Hermann and Dorothea. Translated by E. A. BOWRING, C.B.

Hauff. The Caravan. Translated by S. MENDEL.

— The Inn in the Spessart. Translated by S. MENDEL.

Lessing. Laokoon. Translated by E. C. BEASLEY.

— Minna von Barnhelm. Translated by ERNEST BELL, M.A.

Lessing. Nathan the Wise. Translated by R. DILLON BOYLAN.

Molière. Translated by C. HERON WALL. 8 vols. The Misanthrope. — The Doctor in Spite of Himself. — Tartuffe. — The Miser. — The Shopkeeper turned Gentleman. — The Affected Ladies. — The Learned Women. — The Impostures of Scapin.

Racine. Translated by R. BRUCE BOSWELL, M.A. 5 vols. Athalie. — Esther. — Iphigenia. — Andromache. — Britannicus.

Schiller. William Tell. Translated by Sir THEODORE MARTIN, K.C.B., LL.D. *New Edition, entirely revised.*

— The Maid of Orleans. Translated by ANNA SWANWICK.

— Mary Stuart. Translated by J. MELLISH.

— Wallenstein's Camp and the Piccolomini. Translated by J. CHURCHILL and S. T. COLERIDGE.

— The Death of Wallenstein. Translated by S. T. COLERIDGE.

* * For other Translations from Modern Languages, see the Catalogue of Bohn's Libraries, which will be forwarded on application.

SCIENCE AND TECHNOLOGY

Detailed Catalogue sent on application

- Elementary Botany.** By PERCY GROOM, M.A., D.Sc., F.L.S. With 275 Illustrations. 3s. 6d.
- Elementary Botany.** By G. F. ATKINSON, Ph.B. 6s.
- Botany for Schools and Colleges.** By G. F. ATKINSON. Illustrated. 4s. 6d. net.
- Practical Plant Physiology.** By FREDERICK KEEBLE, M.A. Crown 8vo. 3s. 6d.
- A Laboratory Course in Plant Physiology.** By W. F. GANONG, Ph.D. 7s. 6d. net.
- The Botanist's Pocket-Book.** By W. R. HAYWARD. Revised by G. C. DRUCE. 4s. 6d.
- An Introduction to the Study of the Comparative Anatomy of Animals.** By G. C. BOURNE, M.A., D.Sc. With numerous Illustrations. 2 Vols.
Vol. I. Animal Organization. The Protozoa and Coelenterata. Revised Edition. 6s.
Vol. II. The Coelomata. 6s.
- A Manual of Zoology.** By RICHARD HERTWIG. Translated by Prof. J. S. KINGSLEY. Illustrated. 12s. 6d. net.
- Injurious and Useful Insects.** An Introduction to the Study of Economic Entomology. By Prof. L. C. MIALI, F.R.S. With 100 Illustrations. 3s. 6d.
- Civil Service Examination Papers: Chemistry Papers, Theoretical and Practical.** By A. P. NEWTON. 1s.
- A First Year's Course of Chemistry.** By JAMES SINCLAIR. 1s. 6d.
- An Introduction to Chemistry.** By D. S. MACNAIR, Ph.D., B.Sc. 2s.
- Elementary Inorganic Chemistry.** By Prof. JAMES WALKER, D.Sc. 3s. 6d.
- Introduction to Inorganic Chemistry.** By Dr. ALEXANDER SMITH. 7s. 6d. net.
- Laboratory Outline of General Chemistry.** By Dr. ALEXANDER SMITH. 2s. 6d.
- General Chemistry for Colleges.** By Dr. ALEXANDER SMITH. 6s. 6d. net.
- An Experimental Course in Physical Chemistry.** By J. F. SPENCER, D.Sc., Ph.D. Crown 8vo. 2 vols. 3s. 6d. each.
- A Text-book of Organic Chemistry.** By WM. A. NOYES. 6s. net.
- A Three Years' Course in Practical Physics.** By JAMES SINCLAIR. 3 vols. 1s. 6d. each.
- A College Text-Book of Physics.** By A. L. KIMBALL, Ph.D. Illustrated. 10s. 6d. net.
- The Principles of Physics.** By W. F. MAGIE. Illustrated. 7s. 6d. net.
- Practical Electricity and Magnetism.** First Year's Course. By R. E. STEEL. 2s.
- A Text-Book of Gas Manufacture for Students.** By JOHN HORNBY. Revised and Enlarged. 7s. 6d. net.
- Turbines.** By W. H. STUART GARNETT. 8vo. 5s. net.
- Electrons.** By Sir OLIVER LODGE. 6s. net.
- Engines and Boilers.** By W. MCQUADE. Crown 8vo. Numerous Illus. 3s. 6d. net.
- Exercises in Metal Work.** By A. T. J. KERSEY, A.R.C.Sc. Crown 8vo. 1s. 6d. net.
- Practical Wood Carving for Technical Classes.** By F. P. DRURY. 2s. 6d.

Technological Handbooks

Edited by Sir H. TRUEMAN WOOD

Specially adapted for candidates in the examinations of the City and Guilds Institute. Illustrated

- Woollen and Worsted Cloth Manufacture.** By Prof. ROBERTS BEAUMONT. *[New Edition in preparation.]*
- Soap Manufacture.** By W. LAWRENCE GADD, F.I.C., F.C.S. 5s.
- Plumbing: Its Principles and Practice.** By S. STEVENS HELLYER. 5s.
- Silk-Dyeing and Finishing.** By G. H. HURST, F.C.S. 7s. 6d.
- Printing.** A Practical Treatise. By C. T. JACOBI. 7s. 6d.
- Cotton Spinning: Its Development, Principles, and Practice.** By R. MARS-DEN. 6s. 6d.
- Cotton Weaving: Its Development, Principles, and Practice.** By R. MARS-DEN. 10s. 6d.
- Coach Building.** By JOHN PHILIPSON, M.Inst.M.E. 6s.
- Bookbinding.** By J. W. ZAEHNSDORF. 5s.
- The Principles of Wool Combing.** By HOWARD PRIESTMAN. 6s.

Music

- Music, A Complete Text-Book of.** By Prof. H. C. BANISTER. New and Cheaper Edition. 3s. 6d.
- Music, A Concise History of.** By Rev. H. G. DONAVIA HUNT, Mus. Doc. New and Cheaper Edition. 2s. net.

HISTORY

Catalogue of Historical Books sent post free on application

Lingard's History of England. Abridged and Continued by DOM H. N. BIRT. With a Preface by ABBOT GASQUET, D.D. New Edition. With Maps. 3s. 6d.; or in 2 vols. Vol. I. (to 1485), 2s. Vol. II. (1485-1912), 2s.

An Introduction to English Industrial History. By HENRY ALLSOPP, B.A. 2s.

English History Source Books. Edited by S. E. WINBOLT, M.A., and KENNETH BELL, M.A. 1s. net each.

449-1066. The Welding of the Race. Edited by Rev. JOHN WALLIS, M.A.

1066-1154. The Normans in England. Edited by A. E. BLAND, M.A.

1154-1216. The Angevins and the Charter. Edited by S. M. TOYNE, M.A.

1216-1307. The Growth of Parliament. Edited by W. D. ROBISON.

1307-1400. War and Misrule. Edited by A. A. LOCKE.

1399-1485. The Last of Feudalism. Edited by W. GARMON JONES, M.A.

1485-1547. The Reformation and the Renaissance. Edited by F. W. BEWSHER

1547-1603. The Age of Elizabeth. Edited by ARUNDELL ESDAILE, M.A.

1603-1660. Puritanism and Liberty. Edited by KENNETH BELL, M.A.

1660-1714. A Constitution in Making. Edited by G. B. PERRETT, M.A.

1714-1760. Walpole and Chatham. Edited by K. A. ESDAILE.

1760-1801. American Independence and the French Revolution. Edited by S. E. WINBOLT, M.A.

1801-1815. England and Napoleon. Edited by S. E. WINBOLT, M.A.

1815-1837. Peace and Reform. Edited by A. C. W. EDWARDS.

1856-1876. From Palmerston to Disraeli. Edited by EWING H. ARDING, B.A.

1876-1887. Imperialism and Mr. Gladstone. By R. H. GRETTON, M.A.

1535-1913. Canada. By JAMES MUNRO.

Mediæval England: 1066-1485. A Framework of History. By S. M. TOYNE, M.A. Crown 8vo. 1s. net.

First Lessons in English History. Illustrated. 1s.

A Junior History of England. By E. NIXON. Illustrated. 1s. 6d.

A Senior History of England. By A. MCKILLIAM, M.A. Crown 8vo. Illus. 2s.

Highways of the World. By A. E. MCKILLIAM, M.A. Second Edition, Revised. Crown 8vo. With Maps and Illustrations. 1s. 6d.

A Social History of England. By GEORGE GUEST. Crown 8vo. With many Illustrations. 1s. 6d.

English Church History to A.D. 1000. By W. H. FLECKER, M.A., D.C.L. 1s. 6d.

Civil Service Examination Papers: History Questions. By A. PERCIVAL NEWTON, M.A. 1s.

Ancient History for Schools. By E. NIXON and H. R. STEEL. 2s.

Strickland's Lives of the Queens of England. 6 vols. 5s. each.

* * * Abridged edition for Schools, 6s. 6d.

Landmarks in the History of Europe. By E. M. RICHARDSON, B.A. Crown 8vo. 2s.

Building of the British Empire. By E. M. RICHARDSON, B.A. 1s. 6d.

An Atlas of European History. By EARLE W. DOW. 6s. net.

The Foundations of Modern Europe. By Dr. EMIL REICH. 5s. net.

Dyer's History of Modern Europe. Revised throughout by ARTHUR HASSALL, M.A. 6 vols. With Maps. 3s. 6d. each.

Life of Napoleon I. By JOHN HOLLAND ROSE, Litt.D. 2 vols. 10s. net.

Carlyle's French Revolution. Edited by J. HOLLAND ROSE, Litt.D. 3 vols. With numerous illustrations. 1s. net each.

Mignet's History of the French Revolution, from 1789 to 1814. 1s. net.

Select Historical Documents of the Middle Ages. Translated and edited by ERNEST F. HENDERSON, Ph.D. 5s.

Menzel's History of Germany. 3 vols. 3s. 6d. each.

Ranke's History of the Popes. Translated by E. FOSTER. New Edition. Revised. 3 vols. 1s. net each.

Ranke's History of the Latin and Teutonic Nations. Revised Translation by G. R. DENNIS, B.A. With an Introduction by EDWARD ARMSTRONG, M.A. 6s. net.

Bohn's Popular Library

THE PIONEER SERIES OF CHEAP REPRINTS IN NEW AND
DISTINCTIVE FORMAT.

First List of 40 Volumes.

Strongly bound in Cloth.

One Shilling Net.

1. SWIFT (J.) GULLIVER'S TRAVELS.
- 2-4. MOTLEY (J. L.) RISE OF THE DUTCH REPUBLIC. 3 vols.
- 5-6. EMERSON (R. W.) WORKS: Vol. I.—Essays and Representative Men. Vol. II.—English Traits, Nature, and Conduct of Life.
- 7-8. BURTON (Sir R.) PILGRIMAGE TO AL-MADINAH AND MECCA. 2 vols.
9. LAMB (C.) ESSAYS OF ELIA and LAST ESSAYS OF ELIA.
10. HOOPER (G.) WATERLOO: The Downfall of the First Napoleon.
11. FIELDING (H.) JOSEPH ANDREWS.
- 12-13. CERVANTES. DON QUIXOTE. 2 vols.
14. CALVERLEY (C. S.) THE IDYLLS OF THEOCRITUS with THE ECLOGUES OF VIRGIL.
15. BURNEY (F.) EVELINA.
16. COLERIDGE (S. T.) AIDS TO REFLECTION.
- 17-18. GOETHE. POETRY AND TRUTH FROM MY OWN LIFE. 2 vols.
19. EBERS (Georg). AN EGYPTIAN PRINCESS.
20. YOUNG (Arthur). TRAVELS IN FRANCE.
- 21-22. BURNEY (F.) THE EARLY DIARY OF FRANCES BURNEY (Madame D'Arblay), 1768-1778. 2 vols.
- 23-25. CARLYLE'S HISTORY OF THE FRENCH REVOLUTION. Introduction and Notes by J. Holland Rose, Litt.D. 3 vols.
- 26-27. EMERSON (R. W.) WORKS. Vol. III.—Society and Solitude; Letters and Social Aims; Addresses. Vol. IV.—Miscellaneous Pieces.
- 28-29. FIELDING (H.) TOM JONES. 2 vols.
30. JAMESON (Mrs.) SHAKESPEARE'S HEROINES.
31. MARCUS AURELIUS ANTONINUS, THE THOUGHTS OF. Translated by George Long, M.A.
32. MIGNET'S HISTORY OF THE FRENCH REVOLUTION, from 1789 to 1814.
- 33-35. MONTAIGNE. ESSAYS. Cotton's Translation. 3 vols.
- 36-38. RANKE. HISTORY OF THE POPES. Mrs. Foster's Translation. 3 vols.
39. TROLLOPE (Anthony). THE WARDEN. Introduction by Frederic Harrison.
40. TROLLOPE (Anthony). BARCHESTER TOWERS.

Others in active preparation.

WRITE TO-DAY for a copy of the prospectus containing a history of the famous Bohn's Libraries from their inauguration to the present day.

G. BELL AND SONS, LTD., PORTUGAL STREET, LONDON, W.C.

7.2	7.2	2592
7.2	7.2	
4	144	1296
28	140	324
1.9	2990	162
5	044	

7.2343	3.00	1.732	5184
7.2343	1		
1029	200		3456
4	182		
	1106		27
	1022		

3462 28

139

6924

2100

6224

176

142	51.84	7.2
28 ²	4.9	
144	284	

$W = 2 \times 2 \times 2 \times 3 \times 3$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$2^2 \times 2^2 \times 2^2 \times 3^3$

148	61,62,25	785	148
28	12 62		148
1565	0078 25	150	148

2225	45	625	25	13.31
7825		225		
		225		
		8		

~~1000~~

121
121
1210

168
347

13314	88
1344	
1344	
8	

~~125~~

65

3375	35
0075	

7225	85
------	----

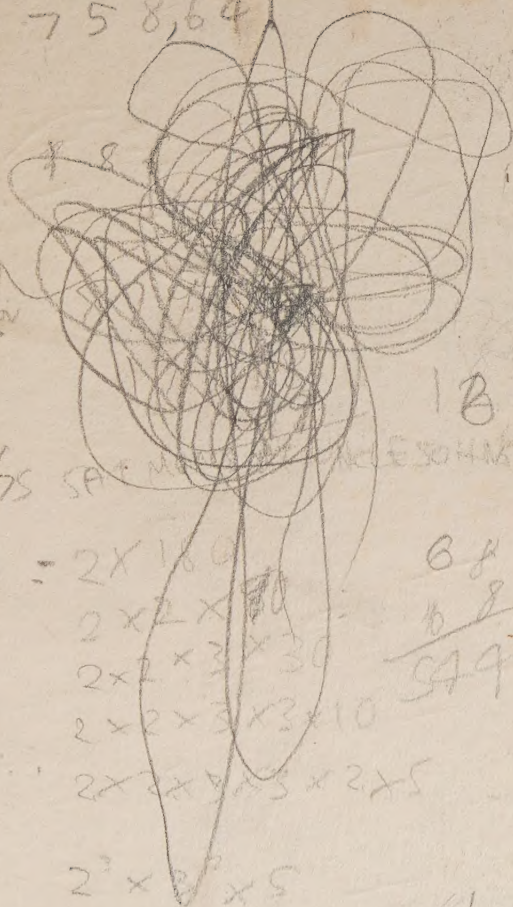
Ex. 5 Numb

on 21/6/2

7/6/75 P. 96

758,64

4.1



1175
147

871

119

120
218

P. 16 360 = 2 x 180

2 x 2 x 90

2 x 2 x 3 x 30

2 x 2 x 3 x 3 x 10

2 x 2 x 3 x 3 x 2 x 5

2³ x 3³ x 5

68

68

549

767

447

5269

11
47
119

5000

5368

231

